On the formation of Pareto-improving trading club without income transfer

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Abstract

Constructing a multi-country general equilibrium model, we show that a Pareto-improving coordinated tariff reforms by a subset of countries (a trading club) is possible without intra-club income transfer, if for each good traded between club member countries there are two groups of members such that one group adjusts a tariff/subsidy on its net import while the other adjusts it on its net export.

1 Introduction

Consider a trading world that consists of an arbitrary number of countries, say $n+1$, such that there is a tariff-ridden world equilibrium for given tariff vectors imposed by those countries. Suppose that a part of the countries, say n countries, form a trading club by adjusting their tariff rates. This paper studies under what conditions the formation of the trading club be Pareto-improving in the sense that, as a result of the adjustments of their tariffs, (1) at least one member country is better-off and (2) no country, whether it is a member or non-member, is worse-off. We show that it is possible to form a Pareto-improving trading club without any international income transfer if for each good traded between club member countries there are two groups of members such that one group adjusts a tariff/subsidy on its net import while the other adjusts it on its net export.

[We need to review the literature. One recent paper which is closely relatyed to this paper would be

- "Non-preferential trading clubs" by Raimond Moller and Alan Woodland, CEPR Discussion paper No. 3772. (www.cepr.org/pubs/dps/DP3572.asp) The main difference between them and this paper is that they assume intra-club income transfer.)]

Section 2 sets up the model. Section 3 shows the main theorem in a general setting. Section 4 applies the theorem to a case such that the Armington Assumption holds in a three-good and three-country framework. Section 5 provides concluding remarks.

2 The Model

We consider a multi-country tariff-ridden general equilibrium model which consists of $n+1$ countries and $m+1$ tradable goods. The countries and goods are indexed as Country 0, Country $1, ...,$ Country n, and Good 0,Good $1, ...,$Good m, respectively. Country $1, ...,$ and Country n form a trading club. Good 0 is the numeraire and Country 0 represents "the rest of the world". Using revenue and expenditure functions, we can describe the multi-country model as follows.

$$E^0(P, u^0) = F^0(P) \tag{1}$$

$$E^i(P+\Lambda^i, u^i) = F^i(P+\Lambda^i)+\Lambda^i[E^i_P(P+\Lambda^i, u^i)-F^i_P(P+\Lambda^i)], \quad i = 1, ..., n \tag{2}$$

$$-[E^0_P(P, u^0) - F^0_p(P)] = \sum_{i=1}^{n}[E^i_P(P + \Lambda^s, u^s) - F^i_P(P + \Lambda^s)], \tag{3}$$

where $P \equiv (p_1, ..., p_m)^T$ and $\Lambda^i \equiv (\tau^i_1, ..., \tau^i_n)^T$ are the international price vector and the import tariff/export tax vector imposed by country i, respectively.[1] $P^i = P+\Lambda^i$, where $P^i \equiv \left(p^i_1, ..., p^i_m\right)^T$ is the domestic price vector in country i. u^i, $i = 0, 1, ..., n$, is the community utility level of Country i. The above system determines the international price of each good and the community utilty level for given tariff rates, τ^i_j, $i = 1, ..., n$, $j = 1, ..., m$, are given. $E^i_P(P + \Lambda^i, u^i) \equiv (E^i_{p_1}, ..., E^i_{p_m})^T$ and $F^i_P(P + \Lambda^i) \equiv (F^i_{p_1}, ..., F^i_{p_m})^T$, where $E^i_{p_j} \equiv \frac{\partial}{\partial p^i_j} E^i$ and $F^i_{p_j} \equiv \frac{\partial}{\partial p^i_j} F^i$, $j = 1, ..., m$.

[1] The superscript T attached to vectors denotes the transpose of them. We assume that vectors without the super script are column vectors.

3 The Main Theorem

First, let us list the main assumptions.

Assumption 1: All revenue and expenditure functions satisfy the standard textbook properties. Income effects are always normal in the sense that

$$E_{uP}^{i} \equiv (E_{up_0}^{i}, E_{up_1}^{i}, ..., E_{up_m}^{i})^T > 0_{m+1},$$

where $E_{up_j}^{i} \equiv \frac{\partial^2}{\partial p_j^i \partial u^i} E^i$. Moreover, for any $i = 1, ..., n$, the second derivatives $E_{PP}^{i}(P + \Lambda^i, u^i) - F_{PP}^{i}(P + \Lambda^i)$ are non-singular, where

$$E_{PP}^{i}(P + \Lambda^i, u^i) \equiv \begin{bmatrix} E_{p_1 p_1}^{i} & \cdots & E_{p_1 p_m}^{i} \\ \vdots & \ddots & \vdots \\ E_{p_m p_1}^{i} & \cdots & E_{p_m p_m}^{i} \end{bmatrix}, \ E_{p_j p_h}^{i} \equiv \frac{\partial^2 E^i}{\partial p_j \partial p_h}$$

and

$$F_{PP}^{i}(P + \Lambda^i, u^i) \equiv \begin{bmatrix} F_{p_1 p_1}^{i} & \cdots & F_{p_1 p_m}^{i} \\ \vdots & \ddots & \vdots \\ F_{p_m p_1}^{i} & \cdots & F_{p_m p_m}^{i} \end{bmatrix}, \ F_{p_j p_h}^{i} \equiv \frac{\partial^2 F^i}{\partial p_j \partial p_h}$$

Assumption 2: There exists a unique pre-club equilibrium, ($\bar{P}$, $\bar{u}^j$, $j = 0, 1, ..., n$) for given tariff rates, $i = 1, ..., n$, where $\bar{P} \equiv (\bar{p}_1, ..., \bar{p}_m)^T > -\Lambda^i$, $i = 1, ..., n$. Moreover, a pre-club equilibrium uniquely exists for any tariff rates in a neighborhood of the given tariff rates.

Assumption 3: In the pre-club equilibrium, for any good j, $j = 1, ..., m$, there are two types of club countries such that the first type, say Country $i(j)$, is to impose a positive tariff $\tau_j^{i(j)} > 0$ on the net import of Good j and the second type, say Country $i^*(j)$, is to impose a non-negative tariff, $\tau_j^{i^*(j)} \leq 0$ on the net export of it[2].

In what follows, we denote the sets of the first type countries and the second type countries by Δ and Δ^*, respectively.

Let us state the main theorem.

Theorem 1 *Under Assumptions 1-3, if the negative tariff rates in the pre-club equilibrium are not very large in their absolute values,* $\left|\tau_j^{i^*(j)}\right|$, $j = 1, ..., m$, *then it is possible for n countries to form a trading club that undertakes a differential and non-discriminatory reform of tariffs insuch a way that at least some club countries are better off without hurting all other club countries and the rest of the world.*

[2] Thus, if the net import is negative, a positive (resp. negative), $\tau_j^{i(j)} > 0$ (resp. $\tau_j^{i^*(j)} < 0$) means export (import) subsidy.

Proof. Let us consider the following tariff policies.

$$\begin{aligned}\Lambda^i(\varepsilon^i) \equiv\ & \Lambda^i - \{E^i_{PP}(\bar{P}+\Lambda^i,\bar{u}^i) - F^i_{PP}(\bar{P}+\Lambda^i)\}^{-1} \\ & \times\{E^i_u(\bar{P}+\Lambda^i,\bar{u}^i) - (\Lambda^i)^T E^i_{uP}(\bar{P}+\Lambda^i,\bar{u}^i)\}\varepsilon^i, \end{aligned} \tag{4}$$

where $\varepsilon^i \equiv (\varepsilon^i_1, ..., \varepsilon^i_m)^T$. Totally differentiating (2) and (3) with respect to Λ^i and u^i, $i = 1, ..., n$, around the pre-club equilibrium in such a way that both P and u^0 are left unchanged, and considering (4), we have.

$$du^i = \frac{(\Lambda^i)^T[E^i_{PP}(\bar{P}+\Lambda^i,\bar{u}^i) - F^i_{PP}(\bar{P}+\Lambda^i)]d\Lambda^i}{E^i_u(\bar{P}+\Lambda^i,\bar{u}^i) - (\Lambda^i)^T E^i_{pu}(\bar{P}+\Lambda^i,\bar{u}^i)}, \quad i = 1, ..., n, \tag{5a}$$

$$\begin{aligned}0_m &= \sum_{s=1}^{n}[\{E^s_{PP}(\bar{P}+\Lambda^s,\bar{u}^s) - F^s_{PP}(\bar{P}+\Lambda^s)\}d\Lambda^s + E^s_{uP}(\bar{P}+\Lambda^s,\bar{u}^s)du^s] \\ &= -\sum_{s=1}^{n}[\{E^s_{PP}(\bar{P}+\Lambda^s,\bar{u}^s) - F^s_{PP}(\bar{P}+\Lambda^s)\} \\ &\quad + \frac{E^s_{uP}(\bar{P}+\Lambda^s,\bar{u}^s)(\Lambda^i)^T\{E^i_{PP}(\bar{P}+\Lambda^i,\bar{u}^i) - F^i_{PP}(\bar{P}+\Lambda^i)\}}{E^i_u(\bar{P}+\Lambda^i,\bar{u}^i) - (\Lambda^i)^T E^i_{pu}(\bar{P}+\Lambda^i,\bar{u}^i)}]d\Lambda^s(\varepsilon^s) \end{aligned} \tag{6}$$

where $0_m \equiv (0, ..., 0)^T$, an m-dimensional zero vector, and, from (4),

$$\begin{aligned}d\Lambda^i(\varepsilon^i) =\ & -[E^i_{PP}(\bar{P}+\Lambda^i,\bar{u}^i) - F^i_{PP}(\bar{P}+\Lambda^i)]^{-1} \\ & \times\{E^i_u(\bar{P}+\Lambda^i,\bar{u}^i) - (\Lambda^i)^T E^i_{uP}(\bar{P}+\Lambda^i,\bar{u}^i)\}d\varepsilon^i \end{aligned} \tag{7}$$

The substitution of (7) into (5a) and (6) yields, respectively,

$$du^i = -(\Lambda^i)^T d\varepsilon^i \tag{8}$$

and

$$
\begin{aligned}
0 &= -\sum_{s=1}^{n}[\{E_u^s(\bar{P}+\Lambda^s,\bar{u}^s)-(\Lambda^s)^T E_{uP}^s(\bar{P}+\Lambda^s,\bar{u}^s)\}I_{m,m} \\
&\quad +E_{uP}^s(\bar{P}+\Lambda^s,\bar{u}^s)(\Lambda^s)^T]d\varepsilon^s \\
&= -\sum_{s=1}^{n}\begin{bmatrix} E_u^s-\Sigma_{j\neq 1,j=2}^{m}\tau_j^s E_{up_j}^s & \tau_2^s E_{up_1}^s & \cdots & \tau_m^s E_{up_1}^s \\ \tau_1^s E_{up_2}^s & \ddots & & \tau_m^s E_{up_2}^s \\ \vdots & & \ddots & \vdots \\ \tau_1^s E_{up_m}^s & \tau_2^s E_{up_m}^s & \cdots & E_u^s-\Sigma_{j=1,j\neq m}^{m}\tau_j^s E_{up_j}^s \end{bmatrix} d\varepsilon^s \\
&= -\sum_{s=1}^{n}\sum_{h=1}^{m}\begin{pmatrix} \tau_h^s E_{up_1}^s \\ \vdots \\ \tau_h^s E_{up_{h-1}}^s \\ E_u^s-\Sigma_{j=1,j\neq h}^{m}\tau_j^s E_{up_j}^s \\ \tau_h^s E_{up_{h+1}}^s \\ \vdots \\ \tau_h^s E_{up_m}^s \end{pmatrix} d\varepsilon_h^s \qquad (9)
\end{aligned}
$$

where $I_{m,m}$ is the m-dimensional identity matrix. ■

Let us assume that for Good j, $j=1,...,m$, a club country $i(j)$ in Δ that imports Good j reduces $\varepsilon_j^{i(j)}$ (i.e., $d\varepsilon_j^{i(j)}<0$) and a club country $i^*(j)$ in Δ^* that exports Good j raises $\varepsilon_j^{i^*(j)}$ (i.e., $d\varepsilon_j^{i^*(j)}>0$), while all other ε_j^i's are kept to be zero. It follows from (8) and Assumption 3 that $du^i>0$ for any $i\in\Delta\cup\Delta^*$, while $du^i=0$ for any $i\in\{0,1,...,n\}-\Delta\cup\Delta^*$.

Thus, what remains is to show that there exists two vectors,

$d\Xi^T\equiv(d\varepsilon_1^{i(1)},d\varepsilon_2^{i(2)}...,d\varepsilon_m^{i(m)})^T<0_m$ and $(d\Xi^*)^T\equiv(d\varepsilon_1^{i^*(1)},d\varepsilon_2^{i^*(2)}...,d\varepsilon_m^{i^*(m)})^T>0_m$,

that satisfy (9), i.e.,

$$\Gamma d\Xi+\Gamma^* d\Xi^*=0, \qquad (10)$$

where

$$
\begin{aligned}
\Gamma &\equiv \begin{bmatrix} E_u^{i(1)}-\Sigma_{j\neq 1,j=2}^{m}\tau_j^{i(1)}E_{up_j}^{i(1)} & \tau_2^{i(2)}E_{up_1}^{i(2)} & \cdots & \tau_m^{i(m)}E_{up_1}^{i(m)} \\ \tau_1^{i(1)}E_{up_2}^{i(1)} & \ddots & & \tau_m^{i(m)}E_{up_2}^{i(m)} \\ \vdots & & \ddots & \vdots \\ \tau_1^{i(1)}E_{up_m}^{i(1)} & \tau_2^{i(2)}E_{up_m}^{i(2)} & \cdots & E_u^{i(m)}-\Sigma_{j=1,j\neq m}^{m}\tau_j^{i(m)}E_{up_j}^{i(m)} \end{bmatrix} \\
\Gamma^* &\equiv \begin{bmatrix} E_u^{i^*(1)}-\Sigma_{j\neq 1,j=2}^{m}\tau_j^{i^*(1)}E_{up_j}^{i^*(1)} & \tau_2^{i^*(2)}E_{up_1}^{i^*(2)} & \cdots & \tau_m^{i^*(m)}E_{up_1}^{i^*(m)} \\ \tau_1^{i^*(1)}E_{up_2}^{i^*(1)} & \ddots & & \tau_m^{i^*(m)}E_{up_2}^{i^*(m)} \\ \vdots & & \ddots & \vdots \\ \tau_1^{i^*(1)}E_{up_m}^{i^*(1)} & \tau_2^{i^*(2)}E_{up_m}^{i^*(2)} & \cdots & E_u^{i^*(m)}-\Sigma_{j=1,j\neq m}^{m}\tau_j^{i^*(m)}E_{up_j}^{i^*(m)} \end{bmatrix}
\end{aligned}
$$

Since we assume away inferior goods, it is clear from $\tau_j^{i(j)} > 0$ that $\tau_h^{i(j)} E_{up_h}^{i(j)} > 0$ for any $j, h = 1, ..., m$. Moreover, we see from the linear homogeneity of $E_u^i(p_0, p_1 + \tau_1^i, ..., p_m + \tau_m^i, u_i)$ with respect to $(p_0, p_1, ..., p_m)$ that

$$\begin{aligned} & E_u^{i(h)} - \Sigma_{j \neq h, j=1}^m \tau_j^{i(h)} E_{up_j}^{i(h)} \\ = & [p_0 E_{up_0}^{i(h)} + \Sigma_{j=1}^m (\bar{p}_j + \tau_j^{i(h)}) E_{up_j}^{i(h)}] - -\Sigma_{j \neq h, j=1}^m \tau_j^{i(h)} E_{up_j}^{i(h)} \\ = & 1 \times E_{up_0}^{i(h)} + (\bar{p}_h + \tau_h^{i(h)}) E_{up_h}^{i(h)} + \Sigma_{j \neq h, j=1}^m \bar{p}_j E_{up_j}^{i(h)} > 0 \end{aligned}$$

Note that $p_0 = 1$, since p_0 is the price of the numeraire good. Therefore, Γ is a strictly positive matrix.

Next, let us consider the matrix Γ^*. It is clear that all the diagonal elements are positive while all off-diagonal elements are negative. Now, take the hth column of the matrix and sum all its elemets.

$$\begin{aligned} & [E_u^{i^*(h)} - \Sigma_{j \neq h, j=1}^m \tau_j^{i^*(h)} E_{up_j}^{i^*(h)}] + \tau_h^{i^*(h)} \Sigma_{j \neq h, j=1}^m E_{up_j}^{i^*(h)} \\ = & [1 \times E_{up_0}^{i^*(h)} + \Sigma_{j=1}^m (\bar{p}_j + \tau_j^{i^*(h)}) E_{up_j}^{i^*(h)}] - -\Sigma_{j \neq h, j=1}^m \tau_j^{i^*(h)} E_{up_j}^{i^*(h)} \\ & + \tau_h^{i^*(h)} \Sigma_{j \neq h, j=1}^m E_{up_j}^{i^*(h)} \\ = & 1 \times E_{up_0}^{i^*(h)} + \Sigma_{j=1}^m (\bar{p}_j + \tau_h^{i^*(h)}) E_{up_j}^{i^*(h)}, \end{aligned}$$

which is positive if $\left| \tau_h^{i^*(h)} \right|$ is smaller than $\bar{p}_j$ for any $j, h = 1, ...m$. It follows from the Frobenius Theorem (e.g., Takayama (1984), Theorem 4.C.9 on page 387) that Γ^* is non-singular with the positive inverse matrix $(\Gamma^*)^{-1} > 0_{m,m}$. Therefore, for any negative vector $d\Xi$,

$$d\Xi^* = -\bar{\Gamma}^{-1} \Gamma d\Xi > 0_m$$

That is, there exists a pair $(-d\Xi, d\Xi^*) > (0_m, 0_m)$ that satisfies (10), as was to be proved. (QED)

4 An Example: 3 X 3 Model

4.1 The Assumptions

Let me construct a 3 by 3 model satisfying the following assumptions.

Assumption 4: Country A and Country B are going to form a trading club and Country C is the rest of the world.

Assumption 5: There are three goods, a, b, c, and good c serves as the numeraire good. Country A exports good a and imports good b and good c. Country B exports good b and imports good a and good c. Country C exports good c and imports good a and good b.

Assumption 6: Initially, Country A imposes tariff on imports of good b and Country B imposes tariff on imports of good a. More specifically, we assume that at the pre-club equilibrium

- Country A imposes positive import tariffs on good b. Let us denote the tariff rate by t_b^A. Country B imposes positive import tariffs on good a. Let us denote the tariff rate by t_a^B.

- Country A and Country B impose zero tax/subsidy on their exports, i.e., $t_a^A = t_b^B = 0$.

- Country C is assumed to be a free-trade country.

Assumption 7: The income effect of each good is positive.

4.2 The Model

Let us describe the model as.

$$E^A(p_a+t_a^A, p_b+t_b^A, 1, u^A) - F^A(p_a+t_a^A, p_b+t_b^A, 1) = t_a^A[E_a^A - F_a^A] + t_b^A[E_b^A - F_b^A] \tag{11}$$

$$E^B(p_a+t_a^B, p_b+t_b^B, 1, u^B) - F^B(p_a+t_a^B, p_b+t_b^B, 1) = t_a^B[E_a^B - F_a^B] + t_b^B[E_b^B - F_b^B] \tag{12}$$

$$E^C(p_a, p_b, 1, u^C) = F^C(p_a, p_b, 1) \tag{13}$$

$$E_a^A - F_a^A + E_a^B - F_a^B + E_a^C - F_a^C = 0 \tag{14}$$

$$E_b^A - F_b^A + E_b^B - F_b^B + E_b^C - F_b^C = 0 \tag{15}$$

where $E_j^i = \frac{\partial E^i}{\partial p_j^i}$, $F_j^i = \frac{\partial F^i}{\partial p_j^i}$. The five equations (11)-(15) determine the five unknowns, $u^i, i = A, B, C$, and $p_j, j = a, b$, for given initial tariff rates, t_a^A, t_b^A, t_a^B,and t_b^B.

Starting from a given set of tariffs$\{t_a^A, t_b^A, t_a^B, t_b^B\}$, where $t_a^A = t_b^B = 0$ initially (See Assumption 6), we can derive the above system. The above system is the starting point of our tariff reform analysis.

In order to avoid a possible confusion, we shall denote the initial levels of tariffs and equilibrium prices before forming a trading club by

$$t_a^{Ae}, t_b^{Ae}, t_a^{Be}, t_b^{Be}, p_a^e, p_b^e$$

4.3 A Pareto-Improving Trading Club

Since Assumption 6 means that

$$t_a^{Ae} = 0, \quad t_b^{Ae} > 0, \;\; t_a^{Be} > 0, \quad t_b^{Be} = 0,$$

Given the pre-club equilibrium, Country A and Country B form a club and adjust their import and export tariffs. The tariff adjustment scheme is as follows

$$\begin{aligned} \begin{pmatrix} t_a^i(\varepsilon_a^i, \varepsilon_b^i) \\ t_b^i(\varepsilon_a^i, \varepsilon_b^i) \end{pmatrix} &\equiv \begin{pmatrix} t_a^{ie} \\ t_b^{ie} \end{pmatrix} \\ & -(E_u^i - t_a^{ie} E_{ua}^i - t_b^{ie} E_{ub}^i) \begin{pmatrix} E_{aa}^i - F_{aa}^i & E_{ab}^i - F_{ab}^i \\ E_{ba}^i - F_{ba}^i & E_{bb}^i - F_{bb}^i \end{pmatrix}^{-1} \begin{pmatrix} \varepsilon_a^i \\ \varepsilon_b^i \end{pmatrix} \\ i &= A, B, \end{aligned} \tag{16}$$

Totally differentiating $t_a^i(\varepsilon_a^i, \varepsilon_b^i)$ and $t_b^i(\varepsilon_a^i, \varepsilon_b^i)$ with respect to ε_a^i and ε_b^i at $(\varepsilon_a^i, \varepsilon_b^i) = (0, 0)$, we derive

$$\begin{aligned} \begin{pmatrix} dt_a^i \\ dt_b^i \end{pmatrix} &= -(E_u^i - t_a^{ie} E_{ua}^i - t_b^{ie} E_{ub}^i) \begin{pmatrix} E_{aa}^i - F_{aa}^i & E_{ab}^i - F_{ab}^i \\ E_{ba}^i - F_{ba}^i & E_{bb}^i - F_{bb}^i \end{pmatrix}^{-1} \begin{pmatrix} d\varepsilon_a^i \\ d\varepsilon_b^i \end{pmatrix} \\ i &= A, B, \end{aligned} \tag{17}$$

Remark 1: Note that both the inverse matrix

$$\begin{pmatrix} E_{aa}^i - F_{aa}^i & E_{ab}^i - F_{ab}^i \\ E_{ba}^i - F_{ba}^i & E_{bb}^i - F_{bb}^i \end{pmatrix}^{-1} \tag{18}$$

and the term $(E_u^i - t_a^{ie} E_{ua}^i - t_b^{ie} E_{ub}^i)$ are evaluated at the pre-club equilibrium domestic prices and utilites. Therfore, those terms do not depend on ε_a^i and ε_b^i, which means that the tariff adjustment mechanism of Country X, (16), is a linear function of ε_a^i and ε_b^i.

Now, making a parallel argument to the calculations for the n by m case, we obtain

$$du^A = -\begin{pmatrix} t_a^{Ae} & t_b^{Ae} \end{pmatrix} \begin{pmatrix} d\varepsilon_a^A \\ d\varepsilon_b^A \end{pmatrix} \tag{19}$$

$$du^B = -\begin{pmatrix} t_a^{Be} & t_b^{Be} \end{pmatrix} \begin{pmatrix} d\varepsilon_a^B \\ d\varepsilon_b^B \end{pmatrix} \tag{20}$$

$$
\begin{aligned}
&\begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
= &\begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A & e_b^{Ae} E_{ua}^A \\ t_a^{Ae} E_{ub}^A & E_u^A - t_a^{Ae} E_{ua}^A \end{pmatrix} \begin{pmatrix} d\varepsilon_a^A \\ d\varepsilon_b^A \end{pmatrix} \\
&+ \begin{pmatrix} E_u^B - t_b^{Be} E_{ub}^B & t_b^{Be} E_{ua}^B \\ t_a^{Be} E_{ub}^B & E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix} \begin{pmatrix} d\varepsilon_a^B \\ d\varepsilon_b^B \end{pmatrix} \\
= &\begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A \\ t_a^{Ae} E_{ub}^A \end{pmatrix} d\varepsilon_a^A + \begin{pmatrix} t_b^{Ae} E_{ua}^A \\ E_u^A - t_a^{Ae} E_{ua}^A \end{pmatrix} d\varepsilon_b^A \\
&+ \begin{pmatrix} E_u^B - t_b^{Be} E_{ub}^B \\ t_a^{Be} E_{ub}^B \end{pmatrix} d\varepsilon_a^B + \begin{pmatrix} t_b^{Be} E_{ua}^B \\ E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix} d\varepsilon_b^B
\end{aligned} \tag{21}
$$

Note that the adjustments $d\varepsilon_j^i$, $i = A, B, j = a, b$, has to satisfy (21) in order that the adjustments keep the trade volumes of three goods with Country C unchanged, in which case the international prices are also unchanged and so is the welfare level of Country C.

Lemma 1: If

$$
p_z^e + t_z^{ie} > 0 \text{ and } p_j^e + t_k^{ie} > 0, \quad i = A, B, \ j, k = a, b, \ j \neq k, \tag{22}
$$

then each diagonal element and column sums of the two matrices in (21) are positive,

$$
E_u^i - t_j^{ie} E_{uj}^i > 0, \quad i = A, B, \ j, k = a, b, \ j \neq k
$$

Proof: Since E_u^i is linearly homogeneous in three prices, we have

$$
E_u^i = (p_j^e + t_j^{ie}) E_{ju}^i + (p_k^e + t_k^{ie}) E_{ku}^i + 1 \cdot E_{cu}^i
$$

Therefore,

$$
\begin{aligned}
E_u^i - t_j^{ie} E_{uj}^i &= (p_j^e + t_j^{ie}) E_{ju}^i + (p_k^e + t_k^{ie}) E_{ku}^i + 1 \cdot E_{cu}^i - t_j^{ie} E_{uj}^i \\
&= p_j^e E_{ju}^i + (p_k^e + t_k^{ie}) E_{ku}^i + 1 \cdot E_{cu}^i,
\end{aligned}
$$

which is positive as long as positive income effects prevail and under (22). (QED)

Now, we know that in the present case

$$
t_a^{Be} > 0, \ t_b^{Ae} > 0, \tag{23}
$$

and

$$
t_a^{Ae} = 0, \ t_b^{Be} = 0 \tag{24}
$$

Having these sign patterns in mind, let me rearrange (21) in the following way,

$$
\begin{aligned}
(21) \quad = \quad & \begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A \\ 0 \end{pmatrix} d\varepsilon_a^A + \begin{pmatrix} t_b^{Ae} E_{ua}^A \\ E_u^A \end{pmatrix} d\varepsilon_b^A \\
& + \begin{pmatrix} E_u^B \\ t_a^{Be} E_{ub}^B \end{pmatrix} d\varepsilon_a^B + \begin{pmatrix} 0 \\ E_u^B -_a^{Be} E_{ua}^B \end{pmatrix} d\varepsilon_b^B \\
= \quad & \left[\begin{pmatrix} E_u^B \\ t_a^{Be} E_{ub}^B \end{pmatrix} d\varepsilon_a^B + \begin{pmatrix} t_b^{Ae} E_{ua}^A \\ E_u^A \end{pmatrix} d\varepsilon_b^A \right] \\
& + \left[\begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A \\ 0 \end{pmatrix} d\varepsilon_a^A + \begin{pmatrix} 0 \\ E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix} d\varepsilon_b^B \right] \\
= \quad & \begin{pmatrix} E_u^B & t_b^{Ae} E_{ua}^A \\ t_a^{Be} E_{ub}^B & E_u^A \end{pmatrix} \begin{pmatrix} d\varepsilon_a^B \\ d\varepsilon_b^A \end{pmatrix} \\
& + \begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A & 0 \\ 0 & E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix} \begin{pmatrix} d\varepsilon_a^A \\ d\varepsilon_b^B \end{pmatrix} \qquad (25)
\end{aligned}
$$

That is, we derive

$$
\begin{aligned}
& \begin{pmatrix} E_u^B - t_b^{Be} E_{ub}^B & t_b^{Ae} E_{ua}^A \\ t_a^{Be} E_{ub}^B & E_u^A - e t_a^{Ae} E_{ua}^A \end{pmatrix} \begin{pmatrix} d\varepsilon_a^B \\ d\varepsilon_b^A \end{pmatrix} \\
= \quad & - \begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A & t_b^{Be} E_{ua}^B \\ t_a^{Ae} E_{ub}^A & E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix} \begin{pmatrix} d\varepsilon_a^A \\ d\varepsilon_b^B \end{pmatrix}, \qquad (26)
\end{aligned}
$$

which corresponds to (10). It follows Lemma 1 that all elements of the matrix at the LHS of (26) are positive, and all elements of the inverse matrix

$$
\begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A & 0 \\ 0 & E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix}^{-1}
$$

are also non-negative. Since

$$
\begin{pmatrix} d\varepsilon_a^A \\ d\varepsilon_b^B \end{pmatrix} = - \begin{pmatrix} E_u^A - t_b^{Ae} E_{ub}^A & 0 \\ 0 & E_u^B - t_a^{Be} E_{ua}^B \end{pmatrix}^{-1} \begin{pmatrix} E_u^B & t_b^{Ae} E_{ua}^A \\ t_a^{Be} E_{ub}^B & E_u^A \end{pmatrix} \begin{pmatrix} d\varepsilon_a^B \\ d\varepsilon_b^A \end{pmatrix}, \qquad (27)
$$

it follows that if $d\varepsilon_a^A$ and $d\varepsilon_b^B$ are chosen so that (27) is satisfied for any $d\varepsilon_a^B < 0$ and $d\varepsilon_b^A < 0$, then $d\varepsilon_a^A > 0$ and $d\varepsilon_b^B > 0$ and the tariff adjustments leave the club's trade volumes with Country C unchanged and the international prices do not change, which means that Country C's welfare is not affected by the tariff adjustments. Moreover, combining

$$
d\varepsilon_a^B < 0, \quad d\varepsilon_b^A < 0, \quad d\varepsilon_a^A > 0, \quad d\varepsilon_b^B > 0
$$

with

$$
t_a^{Be} > 0, \quad t_b^{Ae} > 0, \quad t_a^{Ae} = 0, \quad t_b^{Be} = 0,
$$

we see that

$$\begin{aligned} du^A &= -[t_a^A d\varepsilon_a^A + t_b^A d\varepsilon_b^A] \\ &= -[(0)(+) + (+)(-)] \\ &> 0 \end{aligned}$$

$$\begin{aligned} du^B &= -[t_a^{Be} d\varepsilon_a^B + t_b^{Be} d\varepsilon_b^B] \\ &= -[(+)(-) + (0)(+)] \\ &> 0 \end{aligned}$$

Hence both du^A and du^B are positive.

Proposition: If the initial tariff-ridden equilibrium satisfies (??) and (??), then the implementation of the tariff adjustment scheme (??) (or one could say (17)) makes Country A and Country B better off without hurting Country C.

Remark 2: Since

$$\begin{pmatrix} t_a^i(0,0) \\ t_b^i(0,0) \end{pmatrix} \equiv \begin{pmatrix} t_a^{ie} \\ t_b^{ie} \end{pmatrix},$$

the tariff adjustments are expressed by a small change in tariffs $(d\varepsilon_a^i, d\varepsilon_b^i)$ from their pre-club levels.

Remark 3 Consider the direction of tariff adjustment, determined by

$$\begin{aligned} \begin{pmatrix} dt_a^i \\ dt_b^i \end{pmatrix} &\equiv \begin{pmatrix} dt_a^i(\varepsilon_a^i, \varepsilon_b^i) \\ dt_b^i(\varepsilon_a^i, \varepsilon_b^i) \end{pmatrix} \\ &= -(E_u^i - t_a^i E_{ua}^i - t_b^i E_{ub}^i) \begin{pmatrix} E_{aa}^i - F_{aa}^i & E_{ab}^i - F_{ab}^i \\ E_{ba}^i - F_{ba}^i & E_{bb}^i - F_{bb}^i \end{pmatrix}^{-1} \begin{pmatrix} d\varepsilon_a^i \\ d\varepsilon_b^i \end{pmatrix} \end{aligned}$$

If

$$\begin{pmatrix} E_{aa}^i - F_{aa}^i & E_{ab}^i - F_{ab}^i \\ E_{ba}^i - F_{ba}^i & E_{bb}^i - F_{bb}^i \end{pmatrix}^{-1} = \begin{pmatrix} (-) & (+) \\ (+) & (-) \end{pmatrix}$$

then we find $dt_a^A > 0$, $dt_b^A < 0$, $dt_a^B < 0$, $dt_b^B > 0$, that is, tariffs are adjusted in the direction to level them (recall that $t_a^A = 0$, $t_b^A > 0$, $t_a^B > 0$, $t_b^B = 0$).

However, in general, the signs of elements in this inverse matrix are ambiguous, and so the signs of $d\varepsilon_j^i$ $(i = A, B; j = a, b)$.