# The Costs of Fiscal Inflexibility

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Abstract: Extending Gali and Monacelli (*2004*), we build an N-country open economy model, where each economy is subject to sticky wages and prices and, potentially, has access to sales and income taxes as well as government spending as fiscal instruments. We examine an economy either as a small open economy operating under flexible exchange rates or as a member of a monetary union. In a small open economy when all three fiscal instruments are freely available, we show analytically that the welfare impact of technology and mark-up shocks can be completely eliminated (in the sense that policy can replicate the efficient flex price equilibrium), whether policy acts with discretion or commitment. However, once any one of these fiscal instruments is excluded as a stabilisation tool, costs can emerge. Using simulations, we find that the useful fiscal instrument in this case (in the sense of reducing the welfare costs of the shock) is either income taxes or sales taxes. In constrast, having government spending as an instrument contributes very little. In the case of mark-up shocks tax instruments which can offset the impact of the shock directly are highly effective, while other fiscal instruments are less useful.

The results for an individual member of a monetary union facing an idiosyncratic technology shock (where monetary policy in the union does not respond) are very different. First, even with all fiscal instruments freely available, the technology shock will incur welfare costs. Government spending is potentially useful as a stabilisation device, because it can act as a partial substitute for monetary policy. Finally, sales taxes are more effective than income taxes at reducing the costs of a technology shock under monetary union. If all three taxes are available, they can reduce the impact of the technology shock on the union member by around a half, compared to the case where fiscal policy is not used.

Finally we consider the robustness of these results to two extensions. Firstly, introducing government debt, such that policy makers take account of the debt consequences of using fiscal instruments as stabilisation devices, and, secondly, introducing implementation lags in the use of fiscal instruments. We find that the need for debt sustainability has a very limited impact on the use of fiscal instruments for stabilisation purposes, while implementation lags can reduce, but not eliminate, the gains from fiscal stabilisation.

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# 1 Overview

There has been a wealth of recent work deriving optimal monetary policy for both closed and open economies utilising New Classical Keynesian Synthesis models where the structural model and the description of policy makers' objectives are consistently microfounded. (See for example, Woodford (2003) for a comprehensive treatment of the closed economy case, and Clarida *et al* (2001) for its extension to the open economy case.) More recently, some papers have extended this analysis to include various forms of active fiscal policy, although only a few in the context of open economies or a monetary union.[1] Even when fiscal policy has been analysed, however, the number of active fiscal instruments considered has tended to be small (generally one, occasionally two), and these instruments are assumed to be as flexible as interest rates.

The focus on monetary policy rather than a combination of monetary and fiscal policy probably reflects three factors. The first is that, when the only nominal inertia in the economy involves price setting, optimal monetary policy can completely offset the impact of technology or preference shocks (by reproducing the flex price equilibrium) if exchange rates are flexible. However, this is no longer the case if there is also inertia in nominal wage setting, and we allow for both forms of nominal inertia in this paper. As we shall show, this introduces an important potential role for using tax as a stabilisation instrument. Second, there is much less flexibility in moving fiscal policy instruments, although this inflexibility varies between countries (and instruments), and may not be immutable. In this paper we explicitly examine the costs of this inflexibility, either by introducing implementation lags, or by ruling out the use of particular instruments completely. A third concern may be that using fiscal instruments for stabilisation may compromise the control of public sector debt. Although this may involve political economy concerns which are outside the scope of this paper, we do generalise our model to include public sector debt.

We consider open economies in which there are three potential fiscal instruments alongside monetary policy: government spending, income taxes and sales taxes. As well as the small open economy case, we also consider the case of

[1] For example, Sutherland (2004) and Beetsma and Jensen (2004).

an individual member of a monetary union, using a framework set out in Gali and Monacelli (2004) (henceforth GM). We examine optimal policies when all fiscal instruments are available and fully flexible (under commitment or discretion), and then look at the impact on welfare if there are lags in using these instruments, or if only a subset of instruments are available for short term stabilisation.

Our benchmark regime is for a small open economy, when all three fiscal instruments are freely available. Here we can show analytically that the first-best solution can be achieved. However, once any one of these fiscal instruments is excluded as a stabilisation tool, significant costs emerge. Using simulations, we find that the useful fiscal instrument in this case (in the sense of reducing the welfare costs of a technology shock) is either income taxes or sales taxes. In constrast, having government spending as an instrument contributes very little. This is also true of mark-up shocks, where only a tax instrument which can directly offset the inflationary pressures created by the shock is effective in dealing with the shock.

The results for an individual member of a monetary union facing an idiosyncratic technology shock (where monetary policy in the union has no reason to respond) are very different. First, even with all fiscal instruments freely available, the technology shock will imply that variables deviate from their efficient levels, implying welfare costs. Government spending is potentially useful as a stabilisation device, because it can act as a partial substitute for monetary policy. Finally, sales taxes are more effective than income taxes at reducing the costs of a technology shock under monetary union. For both a small open economy and a monetary union member, we find that implementation/reaction lags significantly reduce, but do not eliminate, the welfare benefits of fiscal stabilisation.

Initially, our analysis assumes the existence of a lump sum tax whose sole purpose is to balance the budget each period. As Ricardian Equivalence holds, changes in this tax have no impact on the economy, but allow us to ignore the government's budget constraint in our analysis. In an extention to our model, we consider the case where lump-sum taxes are not available to offset the consequences for the government's budget constraint of using fiscal instruments as stabilisation devices. Allowing for the impact of changes in policy on debt has only a small impact on our results. This is because it is optimal either to accomodate the impact of fiscal shocks on debt (i.e. debt has a random walk character, as in Benigno and Woodford (2005)), or that the optimal speed for correcting debt disequilibrium is slow.

Our next section derives the model. Section 3 outlines the social planner's problem such that we can write our model in 'gap' form. This representation of the model can also be used to derive a quadratic approximation to welfare. In section 4 we derive the optimal pre-commitment policies for the open economy and for a continuum of economies participating in monetary union. Section 5 simulates such economies to quantify the relative contribution of alternative fiscal instruments to macroeconomic stability. In this section we also consider the importance of implementation lags in relation to fiscal variables. Section

6 adds government debt to the model and assesses the importance of the constraints imposed by the need for fiscal solvency. A conclusion summarises the main results.

# 2 The Model

This section outlines our model. As noted above this is similar in structure to GM, but we allow for the existence of sticky wages as well as prices and introduce distortionary sales and income taxes. The model is further extended by introducing government debt in section 6.

## 2.1 Households

There are a continuum of households of size one, who differ in that they provide differentiated labour services to firms in their economy. However, we shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint and make the same consumption plans even if they face different wage rates due to stickiness in wage-setting. As a result the typical household will seek to maximise the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N(k)_t, G_t) \tag{1}$$

where C,G and N are a consumption aggregate, a public goods aggregate, and labour supply respectively. Here the only notation referring to the specific household, $k$, indexes the labour input, as full financial markets will imply that all other variables are constant across households.

The consumption aggregate is defined as

$$C = \frac{C_H^{1-\alpha} C_F^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} \tag{2}$$

where, if we drop the time subscript, all variables are commensurate. $C_H$ is a composite of domestically produced goods given by

$$C_H = (\int_0^1 C_H(j)^{\frac{\varepsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where j denotes the good's type or variety. The aggregate $C_F$ is an aggregate across countries $i$

$$C_F = (\int_0^1 C_i^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}} \tag{4}$$

where $C_i$ is an aggregate similiar to (3). Finally the public goods aggregate is given by

$$G = (\int_0^1 G(j)^{\frac{\varepsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

which implies that public goods are all domestically produced. The elasticity of substitution between varieties $\epsilon > 1$ is common across countries. The parameter $\alpha$ is (inversely) related to the degree of home bias in preferences, and is a natural measure of openness.

The budget constraint at time $t$ is given by

$$\begin{aligned} & \int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t\{Q_{t,t+1}D_{t+1}\} \\ = \quad & \Pi_t + D_t + W_t N(k)_t(1-\tau_t) - T_t \end{aligned} \tag{6}$$

where $P_{i,t}(j)$ is the price of variety $j$ imported from country $i$ expressed in home currency, $D_{t+1}$ is the nominal payoff of the portfolio held at the end of period $t$, $\Pi$ is the representative household's share of profits in the imperfectly competitive firms, $W$ are wages, $\tau$ is an wage income tax rate, and $T$ are lump sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. Optimisation of expenditure for any individual good implies the demand functions given below,

$$C_H(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\epsilon} C_H \tag{7}$$

$$C_i(j) = \left(\frac{P_i(j)}{P_i}\right)^{-\epsilon} C_i \tag{8}$$

where we have price indices given by

$$P_H = \left(\int_0^1 P_H(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{9}$$

$$P_i = \left(\int_0^1 P_i(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{10}$$

It follows that

$$\int_0^1 P_H(j)C_H(j)dj = P_H C_H \tag{11}$$

$$\int_0^1 P_i(j)C_i(j)dj = P_i C_i \tag{12}$$

Optimisation across imported goods by country implies

$$C_i = \left(\frac{P_i}{P_F}\right)^{-\eta} C_F \tag{13}$$

where

$$P_F = \left(\int_0^1 P_i^{1-\eta} di\right)^{\frac{1}{1-\eta}} \tag{14}$$

This implies

$$\int_0^1 P_i C_i di = P_F C_F \tag{15}$$

Optimisation between imported and domestically produced goods implies

$$P_H C_H = (1-\alpha)PC \tag{16}$$

$$P_F C_F = \alpha PC \tag{17}$$

where

$$P = P_H^{1-\alpha} P_F^{\alpha} \tag{18}$$

is the consumer price index (CPI). The budget constraint can therefore be rewritten as

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1}\} = \Pi_t + D_t + W_t N(k)_t (1-\tau_t) - T_t \tag{19}$$

### 2.1.1 Households' Intertemporal Consumption Problem

The first of the household's intertemporal problems involves allocating consumption expenditure across time. For tractability assume (following GM) that (1) takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t + \chi \ln G_t - \frac{(N(k)_t)^{1+\varphi}}{1+\varphi}) \tag{20}$$

In addition, assume that the elasticity of substitution between the baskets of foreign goods produced in different countries is $\eta = 1$ (this is equivalent to adopting logarithmic utility in the aggregation of such baskets).

We can then maximise utility subject to the budget constraint (19) to obtain the optimal allocation of consumption across time,

$$\beta(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}}) = Q_{t,t+1} \tag{21}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\{(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}})\} = 1 \tag{22}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one period bond paying off a unit of domestic currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

A log-linearised version of (22) can be written as

$$c_t = E_t\{c_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) \tag{23}$$

where lowercase denotes logs (with an important exception for $g$ noted below), $\rho = \frac{1}{\beta} - 1$, and $\pi_t = p_t - p_{t-1}$ is consumer price inflation.

### 2.1.2 Households' Wage-Setting Behaviour

We now need to consider the wage-setting behaviour of households. We assume that firms need to employ a CES aggregate of the labour of all households in the domestic production of consumer goods. This is provided by an 'aggregator' that aggregates the labour services of all households in the economy as,

$$N = \left[\int_0^1 N(k)^{\frac{\epsilon-1}{\epsilon_w}} dk\right]^{\frac{\epsilon_w}{\epsilon_w-1}} \tag{24}$$

where $N(k)$ is the labour provided by household $k$ to the aggregator. We allow the degree of labour differentiation to vary in response to iid shocks which introduce the possibility of wage mark-up shocks. Accordingly the demand curve facing each household is given by,

$$N(k) = \left(\frac{W(k)}{W}\right)^{-\epsilon_w} N \tag{25}$$

where $N$ is the CES aggregate of labour services in the economy which also equals the total labour services employed by firms,

$$N = \int_0^1 N(j)dj \tag{26}$$

where $N(j)$ is the labour employed by firm $j$. The price of this labour is given by the wage index,

$$W = \left[\int_0^1 W(k)^{1-\epsilon_w} dk\right]^{1-\epsilon_w} \tag{27}$$

The household's objective function for the setting of its nominal wage is given by,

$$E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[\Lambda_{t+s}\frac{W(k)_t}{P_{t+s}}(1-\tau_{t+s})N(k)_{t+s} - \frac{(N(k)_{t+s})^{1+\varphi}}{1+\varphi}\right]\right) \tag{28}$$

where $\Lambda_{t+s} = C_{t+s}^{-1}$ is the marginal utility of real post-tax income and $N(k) = \left(\frac{W(k)}{W}\right)^{-\epsilon_w} N$ is the demand curve for the household's labour. The first order condition from this problem can be combined with the aggregate wage index (see Leith and Wren-Lewis (2005b)), to give a log-linearised expression for wage-inflation dynamics,

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{\lambda_w}{(1+\varphi\bar{\epsilon}_w)}(\varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t) + \ln(\mu_t^w)) \tag{29}$$

where $\lambda_w = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w}$ and $\mu_t^w$ is the wage-markup in the absence of wage stickiness[2]. Note that the forcing variable in this New Keynesian Phillips curve (NKPC) is a log-linearsed measure of the extent to which wages are not at the level implied by the labour supply decision that would hold under flexible wages.

[2] A time subscript has been added to what would otherwise be the steady-state wage mark-up to reflect that fact that we shall subject this variable to iid mark-up shocks below.

## 2.2 Price and Exchange Rate Identities

The bilateral terms of trade are the price of country i's goods relative to home goods prices,

$$S_i = \frac{P_i}{P_H} \tag{30}$$

The effective terms of trade are given by

$$\begin{aligned} S &= \frac{P_F}{P_H} && (31) \\ &= \exp \int_0^1 (p_i - p_H) di && (32) \end{aligned}$$

Recall the definition of consumer prices,

$$P = P_H^{1-\alpha} P_F^{\alpha} \tag{33}$$

Using the definition of the effective terms of trade this can be rewritten as,

$$P = P_H S^{\alpha} \tag{34}$$

or in logs as

$$p = p_H + \alpha s \tag{35}$$

where $s = p_F - p_H$ is the logged terms of trade. By taking first-differences it follows that,

$$\pi_t = \pi_{H,t} + \alpha(s_t - s_{t-1}) \tag{36}$$

There is assumed to be free-trade in goods, such that the law of one price holds for individual goods at all times. This implies,

$$P_i(j) = \varepsilon_i P_i^i(j) \tag{37}$$

where $\varepsilon_i$ is the bilateral nominal exchange rate and $P_i^i(j)$ is the price of county i's good j expressed in terms of country i's currency. Aggregating across goods this implies,

$$P_i = \varepsilon_i P_i^i \tag{38}$$

where $P_i^i = \left(\int_0^1 P_i^i(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

From the definition of $P_F$ we have,

$$\begin{aligned} P_F &= \left(\int_0^1 P_i^{1-\eta} di\right)^{\frac{1}{1-\eta}} && (39) \\ &= \left(\int_0^1 \left(\varepsilon_i P_i^i\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}} && (40) \end{aligned}$$

In log-linearised form,

$$\begin{aligned} p_F &= \int_0^1 (e_i + p_i^i) di && (41) \\ &= e + p^* && (42) \end{aligned}$$

where $e = \int_0^1 e_i di$ is the log of the nominal effective exchange rate, $p_i^i$ is the logged domestic price index for country i, and $p^* = \int_0^1 p_i^i di$ is the log of the world price index. For the world as a whole there is no distinction between consumer prices and the domestic (world) price level.

Combining the definition of the terms of trade and the result just obtained gives

$$\begin{aligned} s &= p_F - p_H && (43) \\ &= e + p^* - p_H && (44) \end{aligned}$$

Now consider the link between the terms of trade and the real exchange rate. (Note that although we have free trade and the law of one price holds for individual goods, our economies do not exhibit PPP since there is a home bias in the consumption of home and foreign goods. PPP only holds if we eliminate this home bias and assume $\alpha = 1$ since this implies that the share of home goods in consumption is the same as any other country's i.e. infinitesimally small.) The bilateral real exchange rate is defined as,

$$Q_i = \frac{\varepsilon_i P_i}{P} \tag{45}$$

where $P_i$ and $P$ are the two countries respective CPI price levels. In logged form we can define the real effective exchange rate as,

$$\begin{aligned} q_t &= \int_0^1 (e_i + p^i - p) di && (46) \\ &= e + p^* - p && (47) \\ &= s + p_H - p && (48) \\ &= (1 - \alpha) s && (49) \end{aligned}$$

## 2.3 International Risk Sharing

Assuming symmetric initial conditions (e.g. zero net foreign assets, structural similar economies etc) and equating the first order conditions (focs) for consumption between two economies yields,

$$\mathcal{Q}_{i,t+1} \left( \frac{C_{t+1}^i}{C_{t+1}} \right) = \mathcal{Q}_{i,t} \left( \frac{C_t^i}{C_t} \right) \tag{50}$$

where the real exchange rate between home and country i is, $\mathcal{Q}_{i,t} = \frac{\varepsilon_{it} P_t^*}{P_t}$, implying

$$C_t = z^i C_t^i \mathcal{Q}_{i,t} \tag{51}$$

where $z^i$ is a constant which depends upon initial conditions. Loglinearising and integrating over all countries yields,

$$c = c^* + q \tag{52}$$

where $c^* = \int_0^1 c^i di$,or using the relationship between the terms of trade and the real exchange rate,

$$c = c^* + (1-\alpha)s \tag{53}$$

## 2.4 Allocation of Government Spending

The allocation of government spending across goods is determined by minimising total costs, $\int_0^1 P_H(j)G(j)dj$. Given the form of the basket of public goods this implies,

$$G(j) = (\frac{P_H(j)}{P_H})^{-\epsilon} G \tag{54}$$

## 2.5 Firms

The production function is linear, so for firm $j$

$$Y(j) = AN(j) \tag{55}$$

where $a = \ln(A)$ is time varying and stochastic. The demand curve they face is given by,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon}[(1-\alpha)(\frac{PC}{P_H}) + \alpha \int_0^1 (\frac{\varepsilon_i P^i C^i}{P_H})di + G] \tag{56}$$

which we rewrite as,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon} Y \tag{57}$$

where $Y = \left[\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ . The objective function of the firm is given by,

$$\sum_{s=0}^{\infty} (\theta)^s Q_{t,t+s} \left[ (1-\tau^s_{t+s}) \frac{P_H(j)_t}{P_{t+s}} Y(j)_{t+s} - \frac{W_{t+s}}{P_{t+s}} \frac{Y(j)_{t+s}(1-\varkappa)}{A} \right] \tag{58}$$

where $\varkappa$ is an employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition and distortionary sales and income taxes (assuming there is a lump-sum tax available to finance such a subsidy) and $\tau^s$ is a sales tax. $1-\theta$ is the probability of price change in a given period. Leith and Wren-Lewis (2005b) detail the derivation of the NKPC based on this optimisation, which is given by,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda(mc_t + \ln(\mu_t)) \tag{59}$$

where$\lambda = \frac{(1-\theta\beta)(1-\theta)}{\theta}$ and $mc = -a + w - p_H - \ln(1-\tau^s) - v$ are the real log-linearised marginal costs of production, and $v = -\ln(1-\varkappa)$. In the absence of sticky prices profit maximising behaviour implies, $mc = -\ln(\mu)$ where $\mu$ is the price mark-up, which will be subject to iid shocks below.

## 2.6 Equilibrium

Goods market clearing requires, for each good $j$,

$$Y(j) = C_H(j) + \int_0^1 C_H^i(j)di + G(j) \tag{60}$$

Symmetrical preferences imply,

$$C_H^i(j) = \alpha(\frac{P_H(j)}{P_H})^{-\epsilon}(\frac{P_H}{\varepsilon_i P^i})^{-1}C^i \tag{61}$$

which allows us to write,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon}[(1-\alpha)(\frac{PC}{P_H}) + \alpha\int_0^1 (\frac{\varepsilon_i P^i C^i}{P_H})di + G] \tag{62}$$

Defining aggregate output as

$$Y = [\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}}dj]^{\frac{\epsilon}{\epsilon-1}} \tag{63}$$

allows us to write

$$\begin{aligned} Y &= (1-\alpha)\frac{PC}{P_H} + \alpha\int_0^1 (\frac{\varepsilon_i P^i C^i}{P_H})di + G & (64)\\ &= S^\alpha[(1-\alpha)C + \alpha\int_0^1 \mathcal{Q}_i C_i di] + G & (65)\\ &= CS^\alpha + G & (66) \end{aligned}$$

Taking logs implies

$$\begin{aligned} \ln(Y-G) &= c + \alpha s & (67)\\ &= y + \ln(1 - \frac{G}{Y}) & (68)\\ &= y - g & (69) \end{aligned}$$

where we define $g = -\ln(1-\frac{G}{Y})$. As this condition holds for all countries, we can write world (log) output as

$$y^* = \int_0^1 (c^i + g^i + \alpha s^i)di \tag{70}$$

However $\int_0^1 s^i di = 0$, so we have

$$y^* = \int_0^1 (c^i + g^i)di = c^* + g^* \tag{71}$$

We can use these relationships to rewrite (23) as

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} - \alpha E_t\{s_{t+1} - s_t\} \\ &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} \end{aligned} \tag{72}$$

While wage inflation dynamics are determined by,

$$\pi^w_{H,t} = \beta E_t \pi^w_{H,t+1} + \frac{\lambda_w}{(1+\varphi\epsilon_w)}(\varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t) + \ln(\mu^w_t)) \tag{73}$$

Here the forcing variable captures the extent to which the consumer's labour supply decision is not the same as it would be under flexible wages. Define this variable as $mc^w = \varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t)$. This can be manipulated as follows,

$$\begin{aligned} mc^w &= \varphi n - w + p_H + c + p - p_H - \ln(1-\tau) && (74) \\ &= \varphi n - w + p_H + c + \alpha s - \ln(1-\tau) && (75) \\ &= \varphi y - (w - p_H) + c^* + s - \ln(1-\tau) - \varphi a && (76) \end{aligned}$$

From above we had

$$y = c^* + g + s \tag{77}$$

so we can also write marginal costs appropriate to wage inflation as

$$mc^w = (1+\varphi)y - (w - p_H) - \ln(1-\tau) - g - \varphi a \tag{78}$$

## 2.7 Summary of Model

We are now in a position to summarise our model. On the demand side we have an Euler equation for consumption,

$$y_t = E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} \tag{79}$$

On the supply side there are equations for price inflation,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(mc_t + \ln(\mu)) \tag{80}$$

where $\lambda = [(1-\beta\theta)(1-\theta)]/\theta$ and $mc = -a + w - p_H - \ln(1-\tau^s) - v$. There is a similar expression for wage inflation,

$$\pi^w_{H,t} = \beta E_t \pi^w_{H,t+1} + \frac{\lambda_w}{(1+\varphi\epsilon_w)}((1+\varphi)y_t - (w_t - p_{H,t}) - \ln(1-\tau_t) - g_t - \varphi a_t + \ln(\mu^w)) \tag{81}$$

which together determine the evolution of real wages,

$$w_t - p_{H,t} = \pi^w_{H,t} - \pi_{H,t} + w_{t-1} - p_{H,t-1} \tag{82}$$

The model is then closed by the policy maker specifying the appropriate values of the fiscal and monetary policy variables. However, although this represents a fully specified model it is often recast in the form of 'gap' variables which are more consistent with utility-based measures of welfare.

## 2.8 Gap variables

Define the natural level of (log) output $y^n$ as the level that would occur in the absence of nominal inertia and conditional on the optimal choice of government spending, steady-state tax rates and the actual level of world output. Define the output gap as

$$y^g = y - y^n \tag{83}$$

With flexible prices and wages we have $mc^n = -\ln(\mu)$ and $mc^{w,n} = -\ln(\mu^w)$ which can be solved (see Leith and Wren-Lewis (2005b)) for the natural level of output,

$$y^n = a + g^n/(1+\varphi) + (v + \ln(1-\overline{\tau}) - \ln(\mu) - \ln(\mu^w))/(1+\varphi) \tag{84}$$

where $\overline{\tau}$ is the steady-state income tax rate. We can then write the forcing variable for wage inflation in 'gap' form as,

$$\begin{aligned} mc^{w,g} &= mc^w + \ln(\mu_t^w) && (85)\\ &= (1+\varphi)y - (w - p_H) - \ln(1-\tau) - g - \varphi a + \ln(\mu_t^w) && (86)\\ &= (1+\varphi)y^g - g^g - (w^g - p_H^g) - \ln(1-\tau)^g && (87) \end{aligned}$$

where $\ln(1-\tau)^g = \ln(1-\tau) - \ln(1-\overline{\tau})$. Substituting this into the Phillips curve for wage inflation gives,

$$\pi_{H,t}^w = \beta E_t \pi_{H,t+1}^w + \frac{\lambda_w}{(1+\varphi\epsilon_w)}((1+\varphi)y^g - g^g - (w^g - p_H^g) - \ln(1-\tau)^g) \tag{88}$$

A similar expression for price inflation is given by,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda[(w_t^g - p_{H,t}^g) - \ln(1-\tau_t^s)^g] \tag{89}$$

where the 'gapped' real wage evolves according to,

$$w_t^g - p_{H,t}^g = \pi_{H,t}^w - \pi_{H,t} + w_{t-1}^g - p_{H,t-1}^g - \Delta a_t \tag{90}$$

We can also write (72) for natural variables as

$$y_t^n = E_t\{y_{t+1}^n\} - (r_t^n - \rho) - E_t\{g_{t+1}^n - g_t^n\} \tag{91}$$

so

$$r_t^n = \rho + E_t\{y_{t+1}^n - y_t^n\} - E_t\{g_{t+1}^n - g_t^n\} \tag{92}$$

This allows us to write (72) for gap variables as

$$y_t^g = y_t - y_t^n = E_t\{y_{t+1}^g\} - (r_t - E_t\{\pi_{H,t+1}\} - r_t^n) - E_t\{g_{t+1}^g - g_t^g\} \tag{93}$$

Note that, given (84), the real natural rate of interest depends - like natural output - only on the productivity shock, the steady-state levels of distortionary taxation and the optimal level of government spending.

# 3 Optimal policy

## 3.1 The Social Planner's Problem in a Small Open Economy.

The social planner simply decides how to allocate consumption and production of goods within the economy, subject to the various constraints implied by operating as part of a larger group of economies e.g. IRS. Since they are concerned with real allocations, the social planner ignores market prices and, therefore, nominal inertia in describing optimal policy. GM demonstrate that the solution to this problem is given by,

$$N = (1-\alpha+\chi)^{\frac{1}{1+\varphi}} \tag{94}$$
$$G = \frac{Y\chi}{1-\alpha+\chi} \tag{95}$$

which implies the optimal value for $g$,

$$g = \ln(1+\frac{\chi}{1-\alpha}) \tag{96}$$

## 3.2 Flexible Price Equilibrium

Profit-maximising behaviour implies that firms will operate at the point at which marginal costs equal marginal revenues,

$$\left(1-\frac{1}{\epsilon}\right)\left(1-\frac{1}{\epsilon_w}\right) = \frac{(1-\varkappa)}{(1-\tau^s)(1-\tau)}(N^n)^{(1+\varphi)}(1-\frac{G^n}{Y^n})$$

Now if $G^n$ is given by the optimal rule (96), then

$$1-\frac{G^n}{Y^n} = \frac{1-\alpha}{1-\alpha+\chi} \tag{97}$$

and if the subsidy $\varkappa$ is given by

$$(1-\varkappa) = (1-\frac{1}{\epsilon})(1-\frac{1}{\epsilon_w})(1-\tau^s)(1-\tau)/(1-\alpha) \tag{98}$$

then

$$N^n = (1-\alpha+\chi)^{\frac{1}{1+\varphi}} \tag{99}$$

is identical to the optimal level of employment above. Here the subsidy has to overcome the distortions due to monopoly pricing in the goods and labour markets, as well as the distortionary income and sales taxes.

### 3.3 The Social Planner's Problem in a Monetary Union

Here the social planner maximises utility across all countries subject to

$$\begin{aligned} Y^i &= A^i N^i && (100) \\ Y^i &= C_i^i + \int_0^1 C_i^j dj + G^i && (101) \end{aligned}$$

Recall that utility for country i at time t is

$$\ln C_t^i + \chi \ln G_t^i - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \tag{102}$$

and

$$C^i = (Y^i - G^i)^{1-\alpha}[\int_0^1 C_i^j dj]^\alpha \tag{103}$$

Again GM demonstrate that optimisation implies

$$\begin{aligned} N^i &= (1+\chi)^{\frac{1}{1+\varphi}} && (104) \\ C^i &= (\frac{1-\alpha}{1+\chi})Y^i && (105) \\ C_i^j &= (\frac{\alpha}{1+\chi})Y^i \qquad j \neq i && (106) \\ G^i &= \frac{\chi}{1+\chi}Y^i = \frac{\chi A^i}{(1+\chi)^{\frac{\varphi}{1+\varphi}}} && (107) \end{aligned}$$

The latter implies $g^i = \ln(1+\chi)$ which is a different fiscal rule than in the case of the small open economy. Why? In the small open economy case governments have an incentive to increase government spending (which is devoted solely to domestically produced goods) to induce an appreciation in the terms of trade (see the discussion in GM). In aggregate this cannot happen, but it leaves government spending inefficiently high. The government spending rule under monetary union eliminates this externality. This also has implications for the derivation of union and national welfare which are discussed below.

### 3.4 Social Welfare

Leith and Wren-Lewis (2005b) derive the quadratic approximation to utility across member states to obtain a union-wide objective function.

$$\begin{aligned} \Gamma = & -\frac{(1+\chi)}{2}\sum_{t=0}^{\infty}\beta^t \int_0^1 [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2]di \\ & +tip + o\left(\|a\|^3\right) \end{aligned} \tag{108}$$

where $\widetilde{\lambda}_w = \frac{\lambda_w}{1+\varphi\epsilon_w}$. It contains quadratic terms in price and wage inflation reflecting the costs of price and wage dispersion induced by price and wage inflation in the presence of nominal inertia, as well as terms in the output gap and government spending gap. The weights attached to each element are a function of model parameters. The key to obtaining this quadratic specification is the employment subsidy which eliminates the distortions caused by imperfect competition in labour and product markets as well as the impact of distortionary sales and income taxes. It is also important to note that it is assumed that national fiscal authorities have internalised the externality caused by their desire to appreciate the terms of trade through excessive government expenditure.

In deriving national welfare for an economy outside of monetary union this externality is not corrected. It can be shown that the objective function becomes,

$$\begin{aligned}\Psi^i &= -\frac{(1-\alpha+\chi)}{2}\sum_{t=0}^{\infty}\beta^t[\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2] \\ &\quad +tip + o\left(\|a\|^3\right) \qquad (109)\end{aligned}$$

which is in the same form as the union-wide welfare function. However it differs in the first term multiplying the objective function and in the definiton of the efficient steady-state around which the 'gapped' variables are defined, which reflects the externality which is accepted as a fact of life outside of EMU, but which we assume is eliminated within EMU.

# 4 Precommitment Policy

In this section we shall consider precommitment policies for the various variants of our model.

## 4.1 Precommitment in the Small Open Economy

We shall initially consider policy in an economy not participating in monetary union. Aside from a direct interest in assessing the potential role for stabilising fiscal policy within a small open economy under flexible exchange rates, this is also informative as union-wide monetary policy will be of the same form as national monetary policy in the open economy. In the small open economy case the lagrangian associated with the policy problem is given by,

$$\begin{aligned}L_t &= \sum_{t=0}^{\infty}\beta^t[\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\ &\quad +\lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (rw_t^{i,g}) - \ln(1-\tau_t^i)^g + u_t^{i,w}) \\ &\quad +\lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw^{i,g} - \ln(1-\tau_t^{i,s})^g + u_t^{i,p}])\end{aligned}$$

$$+\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t^i - r_t^{i,n}))$$
$$+\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t^i)]$$

where the langrange multipliers $\lambda_t^{\pi^w,i}$, $\lambda_t^{\pi,i}$, $\lambda_t^{y,i}$ and $\lambda_t^{rw,i}$ are associated with the constraints given by the NKPC for wage inflation, the NKPC for price inflation, the euler equation for consumption and the evolution of real wages, respectively. The first-order condition for the interest rate is

$$\lambda_t^{y,i} = 0 \tag{110}$$

When there is a national monetary policy it is as if the monetary authorities have control over consumption such that the consumption Euler equation ceases to be a constraint. The foc for the sales tax gap, $\ln(1-\tau^{i,s})^g$, is

$$\lambda\lambda_t^{\pi,i} = 0 \tag{111}$$

i.e. the price Phillips curve ceases to be a constraint on maximising welfare - sales tax changes can offset the impact on any other variables driving price inflation. Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w\lambda_t^{\pi^w,i} = 0 \tag{112}$$

The remaining focs are for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t\lambda_{t+1}^{rw,i} = 0 \tag{113}$$

inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{114}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{115}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{116}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{117}$$

Combinations of these first order conditions define the target criteria for a variety of cases, such that alternative fiscal regimes are modelled by retaining or dropping the focs associated with a specific fiscal instrument. In deriving precommiment policy we consider the general solution to the system of focs after the initial time period, which gives us a set of target criteria which policy must achieve. In the initial period we have two ways of solving the system of focs. We can derive a set of initial values for lagrange multipliers dated at time t=-1, such that the target criteria are also followed in the initial period - this constitutes what is known as the policy from a 'timeless perspective' (see Woodford 2003). Alternatively we can allow policy makers to exploit the fact that expectations are fixed in the initial period and utilise the discretionary solution for the initial period only. This amounts to setting the time t=-1 dated lagrange multipliers to zero (see Currie and Levine (1993)). Although we adopt the latter approach in simulations, we do not report the focs associated with the initial period since these do not provide any additional economic intuition.

#### 4.1.1 Small Open Economy - All Fiscal Instruments

Let us consider the case where the fiscal authorities have access to government spending and both tax instruments in order to stabilise their economy, when operating alongside the national monetary authorities. Leith and Wren-Lewis (2005b) detail the derivation of target criteria in this case which are, for government spending,

$$g_t^{i,g} = 0 \tag{118}$$

the output gap,

$$y_t^{i,g} = 0 \tag{119}$$

price inflation,

$$\pi_{i,t} = 0 \tag{120}$$

and wage inflation,

$$\pi_{i,t}^w = 0 \tag{121}$$

In other words the effects of shocks on these gap variables are completely offset and do not have any welfare implications. Since these target criteria are all static, it will also be the case that the optimal discretionary policy will be the same as this precommitment policy. In terms of policy assignments, monetary policy ensures the output gap is zero. Wage inflation is eliminated by the following rule for income taxes,

$$\ln(1-\tau_t^i)^g = -rw_t^{i,g} + u_t^{i,w} \tag{122}$$

while a similar form of rule (but of the opposite sign) for sales taxes eliminates price inflation,

$$\ln(1-\tau_t^{i,s})^g = rw_t^{i,g} + u_t^{i,p} \tag{123}$$

This shows that with appropriate fiscal instruments available for stabilisation purposes cost push-shocks become trivial to deal with, in contrast to the standard case where they are the shocks that imply the monetary authorities face a trade-off in stabilising output and inflation (see, Clarida et al (1999) for example).

Appendix I details the target criteria for other sub-sets of fiscal instruments[3]. A key result to note is that as we remove fiscal instruments the need to add

[3] For details of the derivation of these criteria see Leith and Wren-Lewis (2005b).

inertia into target criteria grows. If all fiscal instruments are removed, we are left with the mixture of forward and backward-looking target criteria found in Woodford (2003, Chapter 7) in the case where both prices and wages are sticky.

## 4.2 Optimal Precommitment Under EMU:

The Lagrangian associated with the EMU case is given by,

$$
\begin{aligned}
L_t \quad = \quad & \int_0^1 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\
& +\lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (rw_t^{i,g}) - \ln(1-\tau_t^i)^g + u_t^{i,w}) \\
& +\lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g] + u_t^{i,p}) \\
& +\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t - r_t^{i,n})) \\
& +\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t)]di
\end{aligned}
$$

The key difference between this and the previous problem is that we now have a union-wide interest rate and welfare is integrated across all member states. As a result, we no longer have a foc for the national interest rate, but the foc for the union-wide interest rate is given by,

$$\int_0^1 \lambda_t^{y,i} di = 0 \tag{124}$$

However, since all economies in our model are symmetrical in structure, we can aggregate focs across our economies which delivers, in terms of union-wide aggregates, an identical set of focs as we find in the small open economy case above. Therefore, the target criterion for the ECB will take the same form as that attributed to the national monetary authority, but re-specified in terms of union-wide aggregates.

In terms of national focs, these are identical to conditions (111)-(117) above.

### 4.2.1 EMU Case - All Fiscal Instruments

With all fiscal instruments, but with the loss of the monetary policy instrument, we can no-longer eliminate the welfare effects of idiosyncratic shocks. Therefore our policy configuration is no longer trivial. Solving focs (111)-(117) yields the following target criteria. Firstly there is a government spending rule,

$$(1+\varphi)y_t^{i,g} + \frac{1}{\chi}g_t^{i,g} = 0 \tag{125}$$

which ensures the optimal composition of output. There is an income tax rule,

$$(1+\varphi)y^{i,g} - g^{i,g} - rw^{i,g} - \ln(1-\tau_i^i)^g + u_t^{i,w} = 0 \tag{126}$$

which replicates the labour supply decision that would emerge under flexible wages and thereby eliminates wage inflation, and a sales tax rule,

$$(1+\varphi)y_t^{i,g} + \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g} + u_t^{i,p}) = 0 \tag{127}$$

which achieves the appropriate balance between output and inflation while recognising that competitiveness will need to be restored once any shock has passed. Again mark-up shocks are trivially dealt with by the appropriate tax instrument.

With these fiscal rules in place in each member state, the ECB will act to ensure the average output gap within the union is zero,

$$\int_0^1 y_t^{i,g} di = y_t^g = 0 \tag{128}$$

which will imply that the average government spending gap and rates of price and wage inflation will all be zero in the union.

Leith and Wren-Lewis (2005b) detail the target criteria that emerge using sub-sets of fiscal instruments and these results are summarised in Appendix II. As before, as we eliminate fiscal instruments the target criteria to be achieved by the remaining policy instruments become more dynamic reflecting the need to adopt an inertial policy under commitment in order to anchor expectations in a welfare improving way.

# 5 Optimal Policy Simulations

In this section we examine the optimal policy response to a technology shock both within and outside monetary union. We consider discretionary and commitment policies and compute the welfare benefits of employing our various fiscal instruments as stabilisation devices. In this section we outline the response of the model to a series of shocks. Following GM we adopt the following parameter set, $\varphi = 1$, $\mu = 1.2$, $\epsilon = 6$, $\theta = 0.75$, $\beta = 0.99$, $\alpha = 0.4$, and $\gamma = 0.25$. The ratio of government spending to gdp of 0.25 implies that $\chi = \frac{\gamma}{1-\gamma} = 1/3$ in the EMU case[4]. Additionally, since we have sticky wages we need to adopt a measure of the steady-state mark-up in the labour market. Following evidence in Leith and Malley (2005), we choose $\mu^w = 1.2$ (which implies $\epsilon_w = 6$), and a degree of wage stickiness given by $\theta_w = 0.75$, which means that wage contracts last for, on average, one year. The productivity shock follows the following pattern,

$$a_t = \rho_a a_{t-1} + \xi_t \tag{129}$$

where we adopt a degree of persistence in the productivity shock of $\rho_a = 0.6$, although we consider the implications of greater persistence below.

## 5.1 Small Open Economy Simulations

We begin by considering the response of a small open economy to a 1% technology shock with the degree of persistence described above, when no use is

[4] In the small open economy case, $\gamma = \frac{\chi}{1-\alpha+\chi}$ such that fixing the share of government spending requires a rescaling of $\chi$ to take account of the incentive to excessive government spending which is assumed to be eliminated within the union. In the simulations, to facilitate comparisons, we fix $\chi$ at the value described above in both the open economy and EMU cases.

made of fiscal policy for stabilisation purposes, so only monetary policy is used to stabilise the economy. Figure 1 details the responses of key endogenous variables to the technology shock, under discretion[5]. It is important to note that, in the absence of sticky wages, monetary policy could completely offset the welfare consequences of this shock by reducing interest rates in line with the increase in productivity. This would ensure that domestic and foreign demand rises for the additional products and that the full effects of the productivity gain are captured in real wages. However, when nominal wages are also sticky it is not possible for monetary policy alone to offset the effects of the shock. Wage stickiness means that real wages are slow to rise following the positive productivity shock and, as a result, marginal costs fall initially and this means that the initial jump in inflation is negative. This leads to a cut in nominal interest rates (greater than that implied by the productivity shock's affect on the natural interest rate) and a jump depreciation of the nominal exchange rate, although interest rates will be relatively lower after this initial jump as rising marginal costs increase inflation. The terms of trade depreciate initially, but this is far more modest than in the flexible wage case. As a result consumption rises in the home country relative to abroad, but not by as much as output since the depreciation of the terms of trade makes domestic goods attractive to foreign consumers. Implicitly IRS and the positive productivity shock imply that resources are being sent abroad to support foreign consumption, although this is not as pronounced as in the flexible wage case.

We know from our derivation of optimal policy above that when we utilise all fiscal instruments we can completely offset the impact of this shock on all welfare-relevant gap variables, implying that there is no welfare cost to the shock. Essentially, the monetary instrument eliminates the impact on the output gap of the shock by cutting interest rates. This creates demand for domestically produced goods by encouraging domestic consumption, which has a bias towards domestically produced goods, and depreciating the exchange rate leading to an increase in foreign demand. Income taxes are reduced to eliminate wage inflation, but simultaneously achieve the required increase in the post tax real wage. The sales tax is increased to eliminate the deflation that would otherwise emerge as a result of the reduction in marginal costs (due to falling income taxes and rising productivity). There is no need to adjust government spending when the government has access to the tax instruments without constraint.

We can also consider a number of intermediate cases where not all fiscal instruments are employed. The welfare benefits of various combinations of fiscal instrument are given in Table 1[6]. These suggest that the greatest gains to stabilisation in the open economy case come from the tax instruments, with

[5] The same shock under commitment introduces a slightly more inertial policy response, but is qualitatively very similar and can be found in Leith and Wren-Lewis (2005b). The numerical solution of optimal policy under commitment and discretion is based on Soderlind (2003).

[6] The figures in Tables 1-2 and 4 capture the costs of deviating from the efficient level of variables due to sticky-wages and prices in the face of the particular shock, expressed as a percentage of one-period's steady-state consumption. In Table 3 the figures are a percentage of every period's steady-state consumption.

only relatively minor benefits from varying government spending. Either tax instrument is highly effective in reducing the welfare costs of the technology shock.

Table 1 - Costs of Technology Shock in Small Open Economy with Alternative Fiscal Instruments.

| | No Taxes | Income Tax | Sales Tax | Both Taxes |
|---|---|---|---|---|
| Commitment Policy | | | | |
| Govt Spending | 0.5793 | 0.0673 | 0.0863 | 0 |
| No Govt Spending | 0.5804 | 0.0708 | 0.0915 | 0 |
| Discretionary Policy | | | | |
| Govt Spending | 0.5824 | 0.1051 | 0.1356 | 0 |
| No Govt Spending | 0.5835 | 0.1082 | 0.1412 | 0 |

In Leith and Wren-Lewis (2005b) we also consider the costs of wage and price mark-up shocks. There we show that, as we might expect from the analysis above, such shocks can be effectively dealt with with the appropriate tax instruments, but the inappropriate fiscal instrument does little to offset these mark-up shocks.

## 5.2 EMU Simulations

We now consider the response to an idiosyncratic technology shock for a country operating under EMU (see Figure 2). We begin by considering the case where there is no fiscal response to the shock. In this case the equilibriating mechanism is the need to restore competitiveness following the shock. Relative to the small open economy case, there is now no monetary policy response to either the local productivity shock or its inflationary repercussions. As a result there is no attempt to boost consumption and output with a fall in interest rates in response to the shock (in an attempt to replicate the flex price outcome). There is an initial fall in marginal costs and inflation which induces a depreciation in the terms of trade, although this is far smaller than in the open economy case above. This shifts demand towards domestic goods such that prices and wages rise until the competitiveness gain has been reversed. In the presence of nominal inertia and with no monetary policy/exchange rate instrument, it is difficult to induce the necessary movements in the terms of trade/real exchange rate to create a market for the extra goods that can be produced as a result of the productivity shock. This failure is reflected in the large negative output gap and real wage gap.

We then contrast this to the case where country i employs all the fiscal instruments at its disposal in Figure 3[7]. We find that optimal policy attempts to reduce the impact of the technology shock on competitiveness. Therefore,

[7] Figure 3 also considers the use of fiscal instruments when there are no lump-sum taxes available to balance the budget following shocks. For a discussion of this case, see Section 6 below.

following the technology shock, sales and income taxes are increased. The latter completely offsets the impact of the shock on wage inflation, while the former allows for only a very limited reduction in prices following the productivity shock. As a result of this attempt to avoid price adjustment, there is a substantial negative output gap, although this is partially offset by a rise in government spending. This has the advantage of creating a market for the additional goods, which given complete home bias in government spending, boosts real wages and moderates the fall in inflation. There is now a smaller depreciation of the terms of trade due to the changes in taxation and the increase in government spending. As we note below, the welfare gain from fiscal stabilisation to this degree is an approximate halving of the costs of a technology shock when part of a monetary union.

We again consider a number of intermediate cases where not all fiscal instruments are employed. The welfare benefits of various combinations of fiscal instrument are given in Table 2. This suggests that the greatest gains to stabilisation, when part of monetary union, come from utilising government spending as a stabilisation instrument. This is due to the assumed home-bias in government spending which allows policy makers to purchase the additional goods produced as a result of the productivity shock without requiring any competiveness changes which subsequently have to be undone once the shock has passed. It is also interesting to note that even with all fiscal instruments in place the costs of the shock under EMU are still greater than in the small open economy case with just monetary policy as the only available policy instrument.

Table 2- Costs of Technology Shock Under EMU with Alternative Fiscal Instruments[8].

| Commitment Policy | No Taxes | Income Tax | Sales Tax | Both Taxes |
|---|---|---|---|---|
| Govt Spending | 1.6707 | 1.6050 | 1.2089 | 1.1486 |
| No Govt Spending | 2.3121 | 2.1495 | 1.9988 | 1.8487 |
| Discretionary Policy | No Taxes | Income Tax | Sales Tax | Both Taxes |
| Govt Spending | 1.6755 | 1.6115 | 1.2131 | 1.1486 |
| No Govt Spending | 2.3121 | 2.1537 | 2.0073 | 1.8487 |

The above Table shows the costs of our 1% autocorrelated technology shock, expressed as a percentage of one-period's steady-state consumption.What would be the equivalent numbers for an historically representative set of shocks, rather than a 1% technology shock? Smets and Wouters (2005) have estimated the stochastic properties of shocks hitting the complete Euro area. We focus on three of these shocks: namely price and wage mark-up shocks which are taken to be iid shocks, and an autocorrelated productivity shock. If we subject our model of a small open economy to these shocks, we find that the gains from

[8] The figures in Table 2 capture the costs of deviating from the efficient level of variables due to sticky-wages and prices in the face of the particular shock, expressed as a percentage of one-period's steady-state consumption.

optimal fiscal stabilisation (compared to no fiscal action) are 2.4% of steady state consumption. Making the even more heroic assumption that these shocks can be applied to our model of an individual union member, we find that optimal fiscal stabilisation reduces their costs from 2.9% to 1.9% of steady-state consumption (for details of these calculations see Leith and Wren-Lewis (2005b))- see Table 3.

Table 3 - Benefits of Fiscal Stabilisation[9]

| Benefits of Fiscal Stabilisation | No Fiscal Response | Full Fiscal Response |
|---|---|---|
| Small Open Economy | 2.37% | 0% |
| Monetary Union | 3.91% | 1.90% |

## 5.3 Implementation Lags

A frequently cited argument against employing fiscal instruments in a stabilisation role is that it often takes long periods to implement the tax changes and government spending changes suggested by optimal policy. In this subsection we assess the extent to which implementation lags affect the welfare gains from fiscal stabilisation. We assume that it takes n-periods to change policy instruments following a change in the information set. This can be modelled by conditioning policy instruments on information sets of n-periods ago, such that our structural model can be written as follows, with our NKPC for wage inflation,

$$\pi^w_{i,t} = \beta E_t \pi^w_{i,t+1} + \widetilde{\lambda}_w ((1+\varphi) y^{i,g}_t - E_{t-n} g^{i,g}_t - (w^{i,g}_t - p^g_{i,t}) - E_{t-n} \ln(1-\tau^i_t)^g + u^{i,w}_t) \tag{130}$$

the similar expression for price inflation,

$$\pi_{i,t} = \beta E_t \{\pi_{i,t+1}\} + \lambda[(w^{i,g}_t - p^g_{i,t}) - E_{t-n} \ln(1-\tau^{i,s}_t)^g + u^{i,p}_t] \tag{131}$$

and the euler equation for consumption,

$$y^{i,g}_t = E_{t-n} g^{i,g}_t + E_t \{y^i_{t+1} - E_{t-n} g^i_{t+1} + \pi_{i,t+1}\} - (r_t - r^{i,n}_t) \tag{132}$$

The equation describing the evolution of the 'gapped' real wage is unaffected. This implies that it will take n-periods following the shock for the fiscal authorities to be able to implement a fiscal policy plan. In assessing the impact on such implementation lags on welfare we consider four cases: (1) There are no lags in adjusting fiscal instruments; (2) there is a one period lag in adjusting tax in struments and 2 periods in adjusting government spending; (3) there is a two period lag in adjusting tax instruments and a one year lag in adjusting

[9] The figures in Table 3 capture the expected costs of deviating from the efficient level of variables due to sticky-wages and prices in the face of ongoing shocks, expressed as a percentage of steady-state consumption.

government spending; and (4) fiscal instruments are not changed over the course of the business cycle.

In Table 4 below we look at these four cases for a currency union member. It is clear that implementation lags do reduce the effectiveness of fiscal instruments as stabilisation devices. However, there are still non-trivial benefits from fiscal stabilisation even under the 'slow response' scenario. This is in part because expectations that instruments will change in the future will impact on private sector decisions today in a forward looking model.

Table 4: Implementation Lags[10]

| Inertia | (1) No Delay | (2) Moderate Response | (3) Slow Response | (4) No Response |
|---|---|---|---|---|
| $\rho_a = 0.6$ | 1.1485 | 1.8770 | 2.0451 | 2.3121 |
| $\rho_a = 0.9$ | 2.6735 | 3.5055 | 4.0023 | 5.3955 |

Of course these results are highly dependent upon the amount of inertia in the economy. For example, the table shows that increasing the degree of presistence in the technology shock from 0.6 to 0.9 such that the impacts of shocks are felt for longer, implies that even with implementation lags fiscal policy has a valuable role to play in stabilising the economy. Overall, these results show the potential value of any measures that can be taken to reduce implementation lags for fiscal instruments.[11]

# 6 Introducing Debt

In this section we consider the impact of introducing government debt into our analysis of policy within a small open economy or within EMU[12]. Until now we have assumed that there was a lump-sum tax instrument which was utilised to balance the budget whenever other fiscal instruments were used in a stabilisation role. In this section we assume that any variations in government spending or our sales or income tax instruments are not paid for in this way. Instead, any inconsistency between government tax revenues and spending will affect government debt. Policy must then ensure that the relevant intertemporal government budget constraint is satisfied.

In the case of EMU, Leith and Wren-Lewis (2005b) derive the intertemporal

[10] These are expressed as percentages of one period's steady-state consumption.

[11] This may be easier when lags are operational rather than political. However, in the UK in the 1960s, when fiscal policy was actively used in demand management, the so called 'regulator' set aside sales taxes that could be changed by the government without the need to obtain parliamentary approval, so as to reduce these lags.

[12] In Leith and Wren-Lewis (2005c), we consider more fully the significance of adding debt to New Keynesian models of monetary policy.

budget constraint for the union as a whole,

$$\int D_t^i di = R_{t-1}B_{t-1} = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(\int_0^1 (P_{i,T}G_T^i - W_T^i N_T^i(\tau_T^i - \varkappa_i) - \tau_T^{i,s}P_{i,T}Y_T^i - T_T^i)di)] \tag{133}$$

where $B_t$ is the aggregate level of the national debt stocks. With global market clearing in asset markets the series of national budget constraints imply that the only public-sector intertemporal budget constraint in our model is a union-wide constraint. What is the intuition for this? Given complete capital markets and our assumed initial conditions (zero net foreign assets and identical *ex ante* structures in each economy) this means that initially consumers expect similar fiscal policy regimes in their respective economies. To the extent that *ex post* this is not the case, there will be state contingent payments under IRS that ensure marginal utilities are equated throughout the union (after controlling for real exchange rate differences)[13]. This would seem to suggest that fiscal sustainability questions within this framework are a union-wide rather than a national concern. Given that a national government's contribution to union-wide finances is negligible then this could be taken to imply that debt is not an issue in utilising fiscal instruments at the national level within EMU.

However, given the fiscal institutions which have been constructed as part of EMU, it seems unlikely that without such constraints each member state would expect to operate under *ex ante* similar fiscal regimes. Therefore it may be reasonable to assume that each member state operates a budget constraint of this form at the national level, such that there is no need for the only institution with a union-wide instrument, the ECB, to be concerned with issues of fiscal solvency. Therefore we impose, as an external constraint created within the institutions of EMU, a national government budget constraint of the same form. We also need to transform this 'national' budget constraint into a log-linearised 'gap' equation to allow it to be integrated into our policy problem. Additionally, in order to support the assumption that the steady-state level of output was efficient (which was implicit in the welfare functions we developed) an obvious assumption to make is that lump-sum taxation is used to finance the steady-state subsidy (which offsets, in steady-state, the distortions caused by distortionary taxation and imperfect competition in wage and price setting). We shall then assume that lump-sum taxation cannot be used to alter this subsidy or to finance any other government activities, including the kind of spending and distortionary tax adjustments as stabilisation measures we are interested in. This implies that $W_T^i N_T^i \varkappa_i = T_T^i$ in all our economies at all points in time, allowing us to simplify the budget constraint to,

$$R_{t-1}B_{t-1}^i = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_{i,T}G_T^i - W_T^i N_T^i \tau_T^i - \tau_T^{i,s}P_{i,T}Y_T^i)] \tag{134}$$

[13] For the purposes of illustration, suppose taxes were lump-sum and one economy unexpectedly cut all taxes to zero. There would be transfers from this economy to the other economies to ensure that the consumers in the other economies were not disadvantaged by the higher taxes they had to pay to ensure union-wide solvency.

i.e. distortionary taxation and spending adjustments are required to service government debt as well as stabilise the economy. This defines the basic trade-off facing policy makers in utilising these instruments.

Leith and Wren-Lewis (2005b) show that this intertemporal budget constraint can give rise to log-linearised flow dynamics of,

$$
\begin{aligned}
b_t^i \quad = \quad & \overline{R} b_{t-1}^i + \overline{R}(r_{t-1} - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{B}^i} \ln G_t^i + \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i} \ln(1-\tau_t^{i,s}) \\
& - \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} y_t^i + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i} \ln(1-\tau_t^i) - \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B/P}^i}(rw_t^i + n_t^i) \\
& - \overline{R} \ln \overline{B}^i - \overline{R}(\overline{r}) - \frac{\overline{G}^i}{\overline{B}^i} \ln \overline{G}^i - \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{b}^i} \ln(1-\overline{\tau}_t^i) \\
& - \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} \overline{Y}^i + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i} \ln(1-\overline{\tau}^i) - \frac{\overline{\tau}^i\overline{rw}^i\overline{N}^i}{\overline{B}^i}(\overline{rw}^i + \overline{n}^i)
\end{aligned} \tag{135}
$$

where $b_t^i = \ln(\frac{B_t^i}{P_{i,t}})$ and $\overline{B}^i = (\overline{B^i/P_i})$. Which can be re-written in gap form,

$$
\begin{aligned}
b_t^{i,g} \quad = \quad & \overline{R} b_{t-1}^{i,g} + \overline{R}(r_{t-1}^g - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{B}^i} \frac{1-\gamma^{i,n}}{\gamma^{i,n}} g_t^{i,g} + \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i} \ln(1-\tau_t^{i,s})^g \\
& -(\overline{R}-1) y_t^{i,g} + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i} \ln(1-\tau_t^i)^g - \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i}(rw_t^{i,g})
\end{aligned} \tag{136}
$$

This is our national government budget constraint, which must remain stationary as an additional constraint on policy makers.

## 6.1 Optimal Precommitment Policy with Government Debt

### 6.1.1 Small Open Economy Case

The Lagrangian associated with the open economy case in the presence of a national government budget constraint is given by,

$$
\begin{aligned}
L_t \quad = \quad & \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda} \pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w} (\pi_{i,t}^w)^2 + (y_t^{i,g})^2 (1+\varphi) + \frac{1}{\chi} (g_t^{i,g})^2 \\
& + \lambda_t^{\pi^w,i} (\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w - \widetilde{\lambda}_w ((1+\varphi) y_t^{i,g} - g_t^{i,g} - (rw_t^{i,g}) - \ln(1-\tau_t^i)^g)) \\
& + \lambda_t^{\pi,i} (\pi_{i,t} - \beta E_t \{\pi_{i,t+1}\} - \lambda [rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\
& + \lambda_t^{y,i} (y_t^{i,g} - g_t^{i,g} - E_t \{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t^i - r_t^{i,n})) \\
& + \lambda_t^{rw,i} (rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t) \\
& + \lambda_t^{b,i} (b_t^{i,g} - \overline{R} b_{t-1}^{i,g} - \overline{R}(r_{t-1}^{i,g} - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^s} \ln(1-\tau_t^{i,s})^g \\
& + b_y y_t^{i,g} - b_\tau \ln(1-\tau_t^i)^g + b_{rw} rw_t^{i,g})]
\end{aligned}
$$

where $b_g = \frac{\overline{G}^i}{\overline{B}^i}\frac{1-\gamma^{i,n}}{\gamma^{i,n}}$, $b_{\tau^s} = \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}$,$b_y = \overline{R}-1$,$b_\tau = \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}$, and $b_{rw} = \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i}$. The foc for the national interest rate is given by,

$$\lambda_t^{y,i} - E_t\lambda_{t+1}^{b,i} = 0 \tag{137}$$

Here monetary policy must now take account of its impact on the government's finances.

In terms of national focs, we begin with the foc for the sales tax gap, $\ln(1-\tau^{i,s})^g$,

$$\lambda\lambda_t^{\pi,i} - b_{\tau^s}\lambda_t^{b,i} = 0 \tag{138}$$

Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w\lambda_t^{\pi^w,i} - b_\tau\lambda_t^{b,i} = 0 \tag{139}$$

and for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta\lambda_{t+1}^{rw,i} + b_{rw}\lambda_t^{b,i} = 0 \tag{140}$$

The remaining first-order conditions are for debt,

$$\lambda_t^{b,i} - \beta\overline{R}\lambda_{t+1}^{b,i} = 0 \tag{141}$$

which implies that, $E_0\lambda_t^{b,i} = \lambda^{b,i}\ \forall t$ . In other words policy must ensure that the 'cost' of the government's budget constraint is constant following a shock which is the basis of the random walk result of Schmitt-Grohe and Uribe (2004). This also implies that the lagrange multipliers for the wage and price phillips curves are constant over time too. The remaining focs are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} + \overline{R}\lambda_t^{b,i} = 0 \tag{142}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{143}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} - b_g\lambda_t^{b,i} = 0 \tag{144}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} + b_y\lambda_t^{b,i} = 0 \tag{145}$$

Combinations of these first order conditions define the national target criteria for a variety of cases. In the open economy case the optimal combination of wage and price inflation is given by,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi^w_{i,t} = 0 \tag{146}$$

This essentially describes the balance between wage and price adjustment in achieving the new steady-state real wage consistent with the new steady-state tax rates required to stabilise the debt stock following the shock. Taking the foc for the output gap, we have,

$$2(1+\varphi)y_t^{i,g} + \lambda^{b,i}(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y) = 0 \tag{147}$$

which defines the value of the Lagrange multiplier associated with the government's budget constraint which implies that the output gap is constant, but non-zero. The sales and income tax rules for the open economy case are given by, respectively,

$$-2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g) + (b_{rw} + b_\tau - b_{\tau^s})\lambda^{b,i} = 0 \tag{148}$$

and,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)) + (b_{rw} + b_\tau - b_{\tau^s})\lambda^{b,i} = 0 \tag{149}$$

Finally the government spending rule is given by,

$$\frac{2}{\chi}g_t^{i,g} + (b_\tau - (1-\beta^{-1}) - b_g)\lambda^{b,i} = 0 \tag{150}$$

which is again constant given the lagrange multiplier $\lambda^{b,i}$ . Leith and Wren-Lewis (2005c) show that this lagrange multiplier, associated with the budget constraint, can be solved as a function of the size of the initial debt stock and the expected fiscal repercussions of any modelled shock. They also investigate the nature of the time inconsistency problem inherent in adding debt to the model, which is discussed in the simulation section below.

Taken together these target criteria imply that optimal policy ensures that output and government spending adjust instantaneously to their new steady-state levels, while gradual price and wage adjustment implies that it is optimal, under commitment, to gradually reach the new steady-state tax rates consistent with debt sustainability.

#### 6.1.2 EMU Case

If we formulate the corresponding problem for the EMU case we have,

$$\begin{aligned}
L_t &= \int_0^1 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda}\pi^2_{i,t} + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi^w_{i,t})^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\
&\quad + \lambda_t^{\pi^w,i}(\pi^w_{i,t} - \beta E_t \pi^w_{i,t+1} - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (rw_t^{i,g}) - \ln(1-\tau_t^i)^g) \\
&\quad + \lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g])
\end{aligned}$$

$$+\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t - r_t^{i,n}))$$
$$+\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^{w} + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t)$$
$$+\lambda_t^{b,i}(b_t^{i,g} - \overline{R}b_{t-1}^{i,g} - \overline{R}(r_{t-1}^{g} - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^s}\ln(1-\tau_t^{i,s})^g$$
$$+b_y y_t^{i,g} - b_\tau \ln(1-\tau_t^i)^g + b_{rw} rw_t^{i,g})]di$$

In order to obtain intuition for optimal policy in this case it is helpful to relate the (constant) value of the lagrange multiplier associated with the national government budget constraint to national output and government spending gaps,

$$2(1+\varphi)y_t^{i,g} + \frac{2}{\chi}g_t^{i,g} + (b_y - \varphi b_\tau - b_g)\lambda_t^{b,i} = 0 \qquad (151)$$

which also implies a constant relationship between the output and government spending gaps following a shock.

There is an income tax rule,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)) + (b_{rw} + b_\tau - b_{\tau^s})\lambda^{b,i} = 0 \qquad (152)$$

and a sales-tax rule,

$$\begin{aligned} 0 &= 2(1+\varphi)y_t^{i,g} + (b_y - \varphi b_\tau + 1 - \beta^{-1} + b_{rw} - b_{\tau^s})\lambda^{b,i} \\ &\quad -2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g) \end{aligned} \qquad (153)$$

and a government spending rule,

$$\begin{aligned} 0 &= \frac{2}{\chi}g_t^{i,g} - 2(1+\varphi)\frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)}y_t^{i,g} \\ &\quad +2\epsilon(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})(rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g) \\ &\quad +2\epsilon_w(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g) \end{aligned} \qquad (154)$$

which in conjuction with the tax rules, will achieve the constant relationship between government spending and the output gap given above. Here we can see that the presence of the national government budget constraint essentially introduces a constant wedge into the target criteria outlined above for the EMU case without debt which reflects the needs to adjust fiscal instruments and steady-state output and real wages to be consistent with the new steady-state level of government debt which follows a random walk.

While the ECB will set the union-wide interest rate consistently with the following first-order condition,

$$\int_0^1 (\lambda_t^{y,i} - E_t\lambda_{t+1}^{b,i})di = 0$$

Assuming that the national fiscal authorities will follow these fiscal rules, this will ensure that union-wide monetary policy achieves the following balance between wage and price inflation,

$$\frac{\epsilon}{\lambda}\pi_t + \frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_t^w = 0 \tag{155}$$

with other union wide variables following paths consistent with the target criteria outlined for the small open economy case above.

## 6.2 Simulations with Debt

In this section we consider using numerical simulation the ability of an small open economy operating inside and outside of EMU to stabilise the economy following a productivity shock through the use of fiscal instruments when it must also ensure sustainability of the government's finances. Figure 3 details the paths of key endogenous variables following the same technology shock considered above when the economy is a member of monetary union and policy is conducted under commitment, with and without government debt. When we introduce debt, the results are very similar to the case where there was a lump-sum tax instrument balancing the national fiscal budget. The main difference is that there is a gradual reduction in government debt (this is shown in Figure 4) in response to the higher tax revenues generated by the positive productivity shock, until it reaches its new lower steady-state with reduced sales and income taxes and higher government spending to satisfy the national fiscal constraint. This is essentially a generalisation of the random walk result of Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004), which also has echoes of tax smoothing (Barro (1979)), but with additional inertia caused by the various sources of inertia in the model. Therefore, following the shock we have a random-walk in the steady-state debt and tax levels. However, these differences have little impact on welfare, with the costs of the shock rising from 1.150% to 1.154% of one period's steady-state consumption.

A more substantial difference occurs when we consider the discretionary solution (see Figure 4). Under discretion the national fiscal authorities taking future inflationary expectations as given, are tempted to use inflation rather than their fiscal instruments to stabilise national government debt. As a result, the larger initial fall in inflation and the initial fall in income taxes serves to increase rather than reduce debt initially. This temptation, which is a form of inflationary bias, remains unless the debt stock returns close to its initial value[14] (this is demonstrated formerly in Leith and Wren-Lewis (2005c)). Therefore, even though there is no explicit debt target, optimal discretionary policy eliminates the effects of the productivity shock on the debt stock. Even in this case,

[14] Ellison and Rankin (2005) find, in the context of a flex price closed economy model that there is an unique level of debt which eliminates the time-inconsistency problem under discretion. Our assumption that the steady-state is efficient ensures that this coincides with our initial steady-state level of debt.

however, the welfare consequences of the shock are not dramatically affected by the introduction of government debt and welfare costs rise from 1.150% to 1.193% of one period's steady-state consumption.

We can also consider the same experiment in the case of a small open economy operating outside of monetary union. Without the need to utilise distortionary instruments to ensure fiscal solvency we have already seen that the combination of monetary and fiscal instruments can perfectly offset the impact of technology shocks in a sticky wage/price economy. However, when the government must also ensure fiscal sustainability by varying distortionary fiscal instruments this first-best solution will no longer be attainable. The welfare costs of our technology shock gives the welfare costs of having to stabilise debt of only 0.0012% of one-period's steady-state consumption under discretion, and an insignficant $1.23 \text{x} 10^{-4}$% under commitment.

# 7 Conclusions

We have considered the potential role of various fiscal instruments in dealing with technology and cost-push shocks in a microfounded open economy model which contains both wage and price inertia. We looked at two policy regimes: flexible exchange rates and the case where the economy is a member of a 'large' monetary union. The three fiscal instruments we consider are government spending, income taxes and sales taxes.

In the case of a small open economy, when all three fiscal instruments are freely available, then the impact of the technology shock on gap variables can be completely eliminated, whether policy acts with discretion or commitment. However, once any one of these fiscal instruments is excluded as a stabilisation tool, significant costs emerge. Using simulations, we find that the useful fiscal instrument in this case (in the sense of reducing the welfare costs of the shock) is either income taxes or sales taxes. In constrast, having government spending as an instrument contributes very little.

The results for an individual member of a monetary union facing an idiosyncratic technology shock (where monetary policy in the union does not respond) are very different. First, even with all fiscal instruments freely available, the technology shock will incur welfare costs. Government spending is potentially useful as a stabilisation device, because it can act as a partial substitute for monetary policy. Finally, sales taxes are more effective than income taxes in reducing the costs of a technology shock under monetary union. If all three instruments are freely available, then the costs of the shock can be reduced by around a half, compared to the case where there is no fiscal stabilisation. We also found that implementation lags could significantly affect (but not eliminate) the ability of fiscal instruments to deal with shocks, but that the need to ensure fiscal solvency when utilising tax instruments in a stabilisation role had negligible welfare consequences.

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## Appendix I - Target Criterion for Alternative Instrument Sets for Small Open Economy

| Instruments | Target Criterion |
| --- | --- |
| Government Spending | $g_t^{i,g} = 0$ |
| Sales Tax | $\ln(1-\tau_t^{i,s})^g = rw_t^{i,g} + u_t^{i,p}$ |
| Income Tax | $\ln(1-\tau_t^{i})^g = -rw_t^{i,g} + u_t^{i,w}$ |
| Monetary Policy | $y_t^{i,g} = 0$ |
| | |
| Government Spending | $y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0$ |
| Sales Tax | $y_t^{i,g} - \epsilon r w_t^{i,g} + \epsilon \ln(1-\tau_t^{i,s})^g - \epsilon u_t^{i,p} = 0$ |
| Monetary Policy | $\frac{\epsilon_w}{\tilde{\lambda}_w}\pi_{i,t}^w + \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{1}{\tilde{\lambda}_w}\Delta y_t^{i,g} = 0$ |
| | |
| Government Spending | $y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0$ |
| Income Tax | $\frac{1}{\chi} g_t^{i,g} + \epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^{i})^g - u_t^{i,p}) = 0$ |
| Monetary Policy | $\frac{\epsilon_w}{\tilde{\lambda}_w}\pi_{i,t}^w + \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{1}{\lambda}(\Delta y_t^{i,g}) = 0$ |
| | |
| Government Spending | $y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0$ |
| Monetary Policy | $0 = \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{\epsilon_w}{\tilde{\lambda}_w}\pi_{i,t}^w + \frac{1}{\tilde{\lambda}_w}\Delta y_t^{i,g} + \frac{1}{\lambda}\left(\Delta y_t^{i,g} + \epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} + u_t^{i,w}) + \frac{1}{\tilde{\lambda}_w}(\Delta^2 y_t^{i,g} - \beta\Delta^2 E_t y_{t+1}^{i,g})\right)$ |
| | |
| Monetary Policy | $0 = \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{\epsilon_w}{\tilde{\lambda}_w}\pi_{i,t}^w + \frac{1}{\tilde{\lambda}_w}\Delta y_t^{i,g} + \frac{1}{\lambda}\left(\Delta y_t^{i,g} + \epsilon_w((1+\varphi)y_t^{i,g} - rw_t^{i,g} + u_t^{i,w}) + \frac{1}{\tilde{\lambda}_w}(\Delta^2 y_t^{i,g} - \beta\Delta^2 E_t y_{t+1}^{i,g})\right)$ |

## Appendix II - Target Criterion for Alternative Instrument Sets for an EMU Member

| Instruments | Target Criterion |
|---|---|
| Government Spending | $(1+\varphi)y_t^{i,g} + \frac{1}{\chi}g_t^{i,g} = 0$ |
| Sales Tax | $(1+\varphi)y_t^{i,g} + \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g} + u_t^{i,p}) = 0$ |
| Income Tax | $(1+\varphi)y^{i,g} - g^{i,g} - rw^{i,g} - \ln(1-\tau_i^i)^g + u_t^{i,w} = 0$ |
| ECB's Monetary Policy | $y_t^g = 0$ |
| | |
| Government Spending | $-\frac{2}{\varphi\chi}g_t^{i,g} = 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} + u_t^{i,w})$ $+\frac{2}{\varphi\chi\tilde{\lambda}_w}(\Delta g_t^{i,g} - \beta\Delta E_t g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\tilde{\lambda}_w}(\Delta y_t^{i,g} - \beta\Delta E_t y_{t+1}^{i,g})$ |
| Sales Tax | $\frac{1}{\chi}g_t^{i,g} = \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g} + u_t^{i,p})$ |
| ECB's Monetary Policy | $\frac{\epsilon_w}{\tilde{\lambda}_w}\pi_t^w + \frac{\epsilon}{\lambda}\pi_t + \frac{1}{\tilde{\lambda}_w}\Delta y_t^g = 0$ |
| | |
| Government Spending | $\frac{1}{\chi}g_t^{i,g} + (1+\varphi)y_t^{i,g} = 0$ |
| Income Tax | $(1+\varphi)y_t^{i,g} - \epsilon(rw_t^{i,g} + u_t^{i,p})$ $-\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g + u_t^{i,w})$ $+\frac{\epsilon_w}{\lambda}((1+\varphi)(\beta E_t y_{t+1}^{i,g} - y_t^{i,g}) - (\beta E_t g_{t+1}^{i,g} - g_t^{i,g}) - (\beta E_t u_{t+1}^{i,w} - u_t^{i,w})$ $-(\beta E_t rw_{t+1}^{i,g} - rw_t^{i,g}) - (\beta E_t \ln(1-\tau_{t+1}^i)^g - \ln(1-\tau_t^i)^g) = 0$ |
| ECB's Monetary Policy | $\frac{\epsilon_w}{\tilde{\lambda}_w}\pi_t^w + \frac{\epsilon}{\lambda}\pi_t + \frac{1}{\lambda}(\Delta y_t^g) = 0$ |

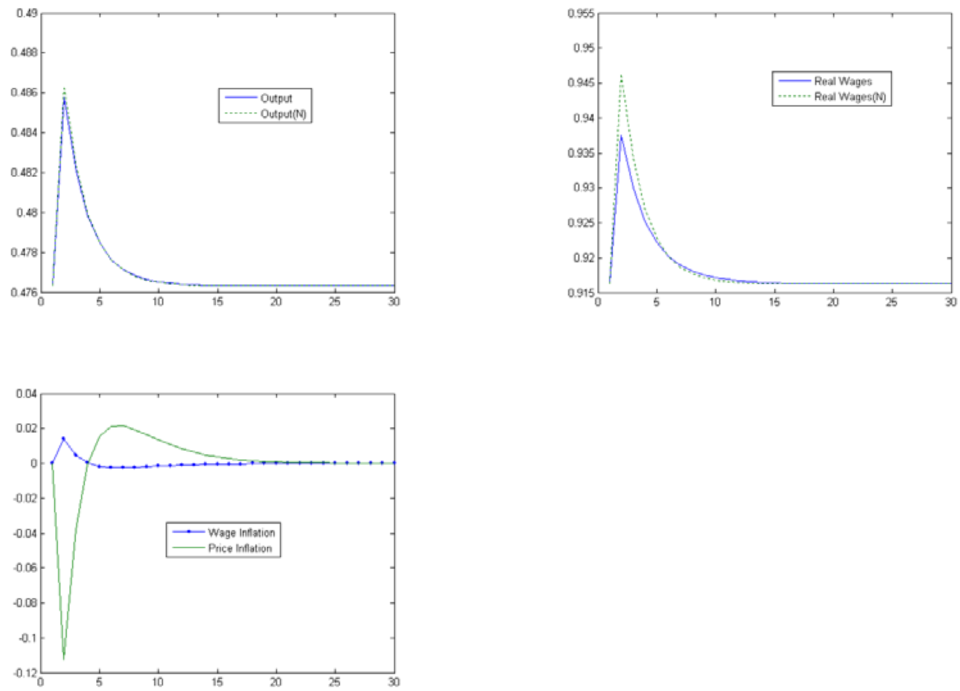



Figure 1: Response to a 1% technology shock in an open economy with only discretionary monetary policy as an instrument.

Note to Figure: An (N) suffix denotes natural (logarithmic) level of a variable, otherwise all variables, other than inflation rates, are logarithms of actual values.

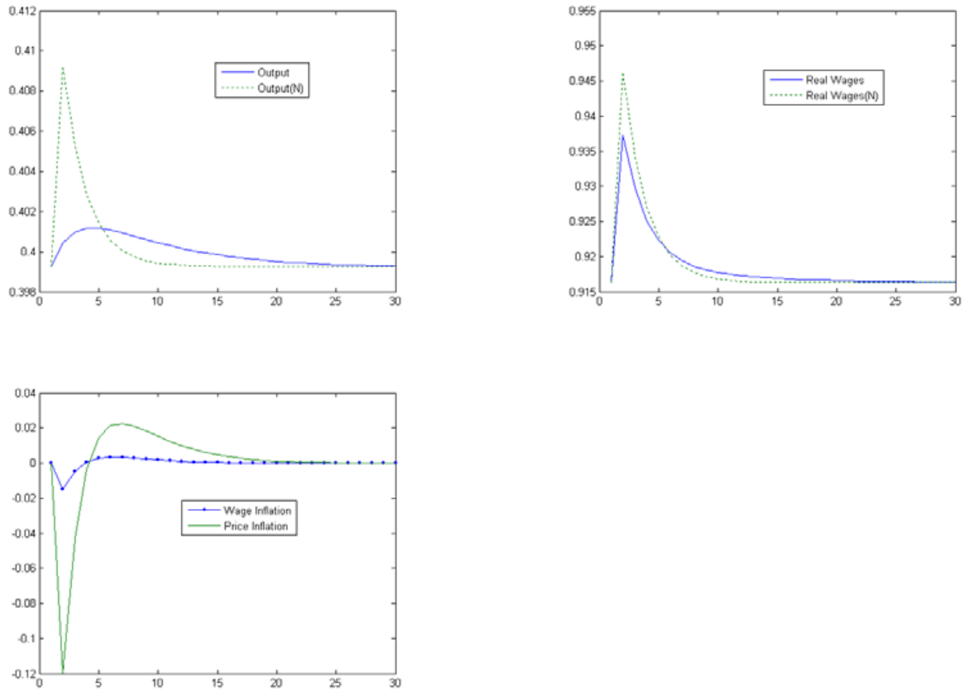
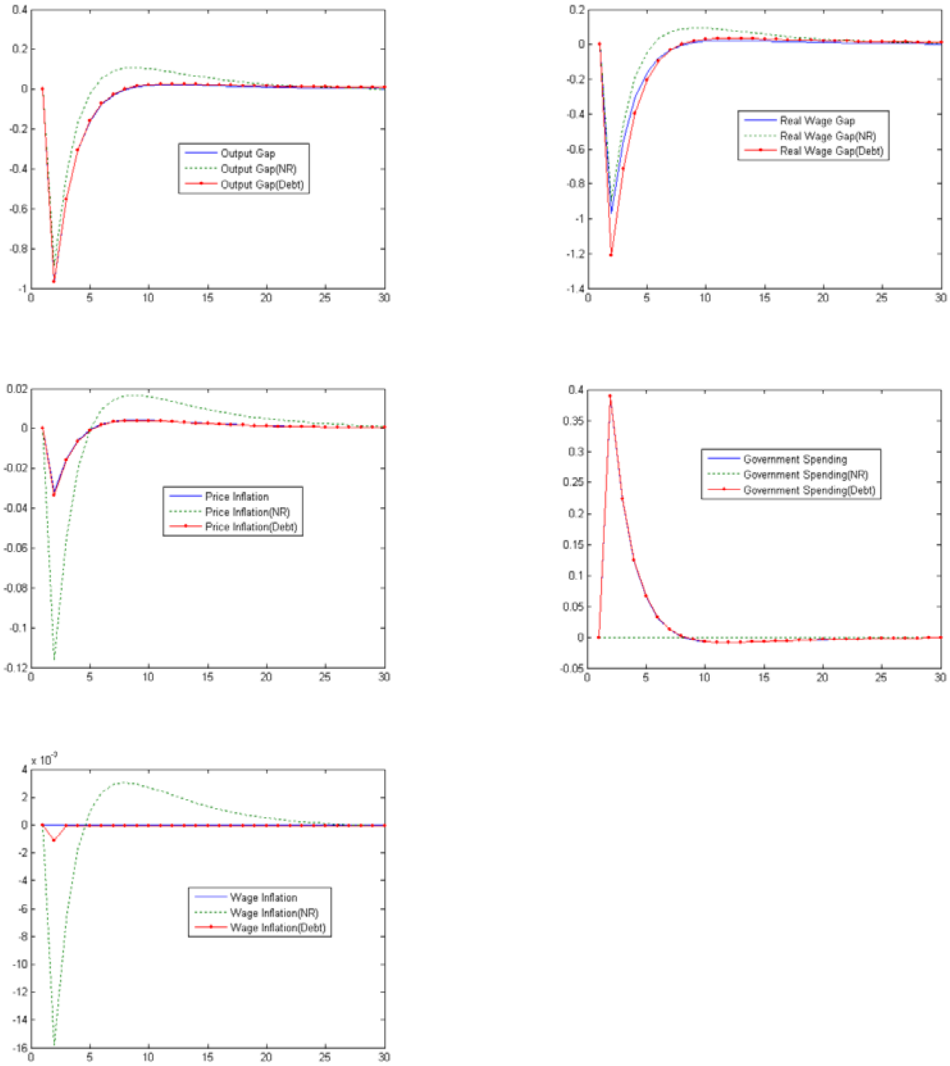


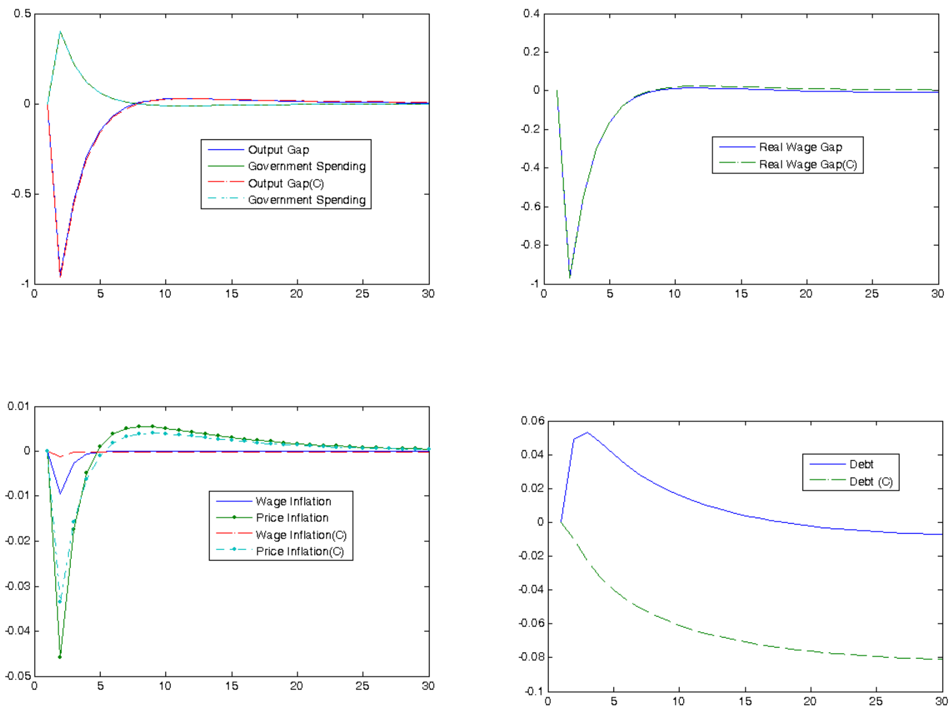

Figure 2: Response to a 1% technology shock under EMU with no policy response.

Note to Figure: An (N) suffix denotes natural (logarithmic) level of a variable, otherwise all variables, other than inflation rates are logarithms of actual values.




Response to a 1% technology shock under EMU, with and without fiscal policy.

Note to Figure: All variables are 'gaps'. An (NR) suffix denotes no fiscal response, no suffix denotes all fiscal instruments employed and the (Debt) suffix denotes the case where all fiscal instruments are employed, but there are no-lump sum taxes available to balance the budget. Policy is assumed to operate under commitment.




Response to 1% technology shock under EMU with all fiscal instruments and government debt.

Note to Figure: All variables are 'gaps'. An (C) suffix denotes commitment policy, while no suffix denotes policy under discretion.