On a Consumer-based Environmental Subsidy/Tax Policy in the Case of an Environmentally Differentiated Products Market

Tsuyoshi TOSHIMITSU☆
Kwansei Gakuin University

Abstract
This paper applies an environmentally differentiated duopoly model to the analysis of environmental policy in the form of a subsidy/tax on consumers based on emission levels of purchased products. More specifically, we consider environmental and welfare effects of subsidizing consumers who purchase environmentally friendly goods such as hybrid vehicles. Focusing on the cases of market coverage by heterogeneous consumers, we mainly examine the issue in the case of a Bertrand duopoly. In the case of full market coverage, a consumer-based environmental subsidy improves the environment and is optimal. However, in the case of partial market coverage, the optimal policy depends on the magnitude of the marginal social valuation of environmental damage. That is, if the marginal social valuation of environmental damage is sufficiently large (small), a consumer-based environmental tax (subsidy) is optimal.

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☆ School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, Japan, 662-8501
Tel: +81 798 54 6440, Fax: +81 798 51 0944, E-mail: ttsutomu@kwansei.ac.jp
1. Introduction

A growing demand for environmental care has been observed. That is, many people willingly purchase goods and services that are environmentally friendly or produced using environmentally friendly techniques. There is also sufficient evidence to suggest that firms are aware of consumers’ behavior in this regard, and that they invest funds in environmentally friendly goods, product lines, and facilities. Governments in many advanced countries also regulate polluting emissions, environmental waste, and global warming using various environmental policies, including taxes/subsidies, emission standards, tradable emission permits, and eco-labeling. Furthermore, even local governments in many advanced countries, e.g., the Tokyo Metropolitan Government, the City of London, and others, are currently addressing environmental problems such as air pollution.

Many seminal works in the field of environmental economics (e.g., Baumol and Oates, 1988, and others) have dealt with polluting wastes or environmental effluents that are the by-products of the production process, such as those found in chemical industries. Accordingly, they have usually considered environmental policies associated with producers such as private enterprises.

In this paper, however, we focus on products with environmental characteristics in a green market where effluents and noises are the by-products of consumption of heterogeneous ‘consumers’ who differ in terms of their willingness to pay for the products according to a product’s environmental quality. For example, from the point of view of the life cycle of manufactured products such as vehicles, the volume of CO₂ gas produced in the process of consumption is likely to be larger than that in the process of production. Furthermore, the environmental damage caused by polluting wastes and effluents associated with consumption of the products seems to be external for individual consumers, that is, they are environmental negative externalities. However, some consumers who are very conscious of environmental degradation may purchase an environmentally friendly product, even if its price is substantially higher than that of an environmentally unfriendly product, while other consumers who are not concerned about the environment may purchase a lower priced product, even if it is environmentally unfriendly. That is, consumers differ in their degree of consciousness about the

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1 ‘Consumers’ not only implies households driving cars but also companies using vehicles for transportation. In a sense, ‘consumers’ are ‘users’ of environmentally differentiated products. In addition, we should refer to the works of Choe and Fraser (1999) and others, who discuss the management of household waste. In this paper, however, we look at the behavior of heterogeneous consumers having various preferences for the environmental qualities associated with products. Thus, we do not deal with household waste management.
environment (Scherhorn, 1993). For example, in the context of car exhaust fumes, the emission level of a hybrid engine motor vehicle such as Toyota Prius is much lower than that of a wholly gasoline engine motor vehicle. Hence, consumers who care about the environment may prefer the hybrid vehicle, whereas others do not. In addition, more extreme environmentalists may not purchase any type of car, but instead would ride a bicycle or use public transport such as trains (Kahn, 2007).

A number of papers employ a model of vertically differentiated products to examine environmental subsidy/tax policies with heterogeneous consumers. Cremer et al. (1998, 2003) theoretically and empirically analyze the optimal tax design in the presence of environmental externalities. In reality, governments in many advanced countries allocate tax credits, i.e., a kind of subsidy, to consumers who purchase environmentally friendly goods. For example, in Japan the Ministry of the Environment enforces a taxation courtesy system for the introduction of low-emission vehicles. That is, owners of eco-cars receive tax credits, while owners of diesel vehicles incur heavier taxes. Furthermore, the Tokyo Metropolitan Government encourages owners of small companies to purchase low-emission vehicles through the provision of subsidies and financing.2

The purpose of this paper is to examine how emission regulations by subsidies/taxes levied on consumers purchasing an environmentally friendly good, i.e., a cleaner product, affect the environment and social welfare, by applying the model of environmentally differentiated products presented by Moraga-González and Padrón-Fumero (2002). We address this issue in the case of a Bertrand duopoly, looking at two cases of market coverage by heterogeneous consumers. That is, we deal with both cases of full market coverage, in which all consumers in the market necessarily purchase either product, and of partial market coverage, in which some consumers do not purchase any products in the market.

Some previous studies are closely related to the analysis undertaken here. Bansal and Gangopadhyay (2003) examine ad valorem commodity subsidies/taxes on environmentally differentiated products in the case of a Bertrand duopoly with partial market coverage.3 They show that discriminating commodity subsidy policy is welfare-superior to discriminating

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2 On the environmental policy of the Tokyo Metropolitan Government, see http://www.metro.tokyo.jp/ENGLISH/POLICY/environment.htm

3 Bansal and Gangopadhyay’s (2003) model is related to Cremer and Thisse’s (1999) endogenous quality choice model used to analyze an ad valorem commodity tax/subsidy policy in oligopolistic price competition with full market coverage. However, Cremer and Thisse assume that a marginal cost of production increases with quality level.
commodity tax policy. In the context of the linkage between tariff policy and the environment, Toshimitsu (2008a) analyzes how ad valorem import tariffs levied on a cleaner and a dirtier product affect the environment and welfare in the cases of a Bertrand and a Cournot duopoly with partial market coverage. He finds that the effect of an ad valorem tariff policy on the environment and welfare depends on the mode of competition and the degree of social valuation of environmental damage. As shown below, we deal with an environmental subsidy/tax charged on consumers based on the difference in unit emission levels of products, but not with a commodity subsidy/tax policy, as in the case of a Bertrand duopoly.

Lombardini-Riipinen (2005) considers a mixed policy with a uniform ad valorem commodity tax and an emission (or environmental) subsidy/tax with full market coverage, assuming that a firm incurs marginal costs of production as an increasing function of its abatement efforts. He discusses the impact of optimal first-best policies in the context, including ad valorem commodity or emission taxes on firms, or ad valorem commodity taxes on firms and emission subsidies on consumers. In addition, he argues that the second-best subsidy for consumers and the second-best emission tax on a firm should be set equal to the marginal social value of the environmental damage.

Although our model is closely related to Lombardini-Riipinen’s (2005), we instead assume a fixed cost associated with a unit emission level, i.e., environmental quality, and analyze the cases both of full and of partial market coverage. In addition, we mainly address a second-best environmental subsidy/tax policy associated with consumers. Furthermore, if we are to discuss a mixed policy with a commodity tax/subsidy and an emission tax/subsidy, we should consider that the government authority choosing a commodity tax might not necessarily be identical to the authority choosing an emission (or environmental) tax. For example, in Japan, responsibility for the former falls under the control of the Ministry of Economy, Trade, and Industry, whereas the Ministry of the Environment administers the latter. Although the analysis of potentially conflicting choices of different authorities within a government is an interesting issue, it can also be complex. Therefore, in this paper, we deal with a consumer-based environmental subsidy/tax policy by a single government.

The remainder of this paper comprises the following three sections. Section 2 sets out the basic model. Section 3 examines environmental and welfare effects of a consumer-based environmental subsidy/tax policy in the case of full market coverage and shows that the subsidy is an optimal policy. Furthermore, in the case of partial market coverage, we show that the optimal policy depends on the magnitude of marginal social valuation of environmental damage. Finally, Section 4 summarizes the results and raises some issues.
2. The Model

We begin by describing a green market in which consumers have heterogeneous preferences for the quality of environmentally differentiated products. That is, a continuum of heterogeneous consumers exists who differ in their marginal valuations \( \theta \) of the green features of products. To simplify, we assume that the consumer-matching value is uniformly distributed with density one and falls in the range \( \theta \in [0, \theta] \). Accordingly, consumer \( \theta \) close to \( \theta(0) \) is sensitive (insensitive) to the environmental qualities of products.

Let \( e \) denote an observable unit emission level associated with a product, \( e \in (0, \infty) \). Without losing generality, we assume that firm \( C \) (\( D \)) supplies a cleaner (dirtier) product with a per unit emission of \( e_C \) (\( e_D \)) at a price of \( p_C \) (\( p_D \)) and \( e_D \geq e_C > 0 \). A consumer purchases at most either one or zero units of the product. Hence, the utility of consumer \( \theta \) who acquires the variant \( e \) at a price of \( p \) is given by:

\[
    u = \max \{ v - e\theta - p + dS - \gamma E, -\gamma E \},
\]

where \( v \) is the intrinsic utility obtained from a single unit of the product, irrespective of the variant’s unit emission level, and \( d \) is a dummy variable with \( d = 1 \) if \( e = e_C \) and \( d = 0 \) if \( e = e_D \). \( S = s(e_D - e_C) \) is a subsidy paid to a consumer purchasing a cleaner product. If \( s < 0 \), then a government levies a tax on the consumer. As shown below, \( E \) denotes aggregate emissions that degrade the environment. Furthermore, \( \gamma(\geq 0) \) expresses the marginal social valuation of environmental damage, which is the same for all individual consumers.

The utility function as in (1) implies that individual consumers not only are sensitive to environmental qualities of products, but also consider environmental damage to the whole economy such as acid rain, global warming, and air pollution. Furthermore, a consumer, who does not purchase any products in the market, only suffers from environmental damage caused by aggregate emissions. Thus, the reservation utility is expressed as \( -\gamma E \).

Because we focus on how a consumer-based environmental subsidy/tax policy has an impact on the behavior of firm \( C \) producing a cleaner product, let us normalize a unit emission level of a dirtier product to unity: \( e_D = 1 \) and \( e_C = \varepsilon \leq 1 \).

Let us derive demand functions for environmentally differentiated products. The index of
the marginal consumer who is indifferent between the surplus given by purchasing a dirtier and a cleaner product is characterized by \( \tilde{\theta} = \frac{P_C - P_D}{1 - \varepsilon} - s \). Furthermore, the index of the marginal consumer who is indifferent between the surplus given by purchasing a cleaner product and nothing is characterized by \( \hat{\theta} = \frac{v - P_C + s(1 - \varepsilon)}{\varepsilon} \). Thus, consumer \( \theta \) falling into \( 0 \leq \theta < \tilde{\theta} \) (\( \tilde{\theta} < \theta < \hat{\theta} \)) purchases a dirtier (cleaner) product. Accordingly, there may be two states of the market if the following conditions hold.

Case FMC: Full market coverage, if \( \hat{\theta} \geq \tilde{\theta} \).

Case PMC: Partial market coverage, if \( \hat{\theta} < \tilde{\theta} \).

With respect to Case FMC, if \( v \) is sufficiently large, consumers are willing to pay, even for a dirtier product. This is because the direct intrinsic utility is large enough to compensate for the loss of utility caused by emissions of the product. Thus, the case of full market coverage holds. This is where all consumers necessarily purchase either product. Hence, let \( q_D \) represent the quantity demanded for a dirtier product. Given a uniform distribution, the demand is given by \( q_D = \tilde{\theta} \). Moreover, the quantity demanded for a cleaner product is given by \( q_C = \tilde{\theta} - \hat{\theta} \). Therefore, demand functions are given by:

\[
q_D = \frac{P_C - P_D}{1 - \varepsilon} - s \quad \text{and} \quad q_C = \tilde{\theta} + s - \frac{P_C - P_D}{1 - \varepsilon}. \tag{2}
\]

On the other hand, with regard to Case PMC, consumer \( \theta \) falling into \( \hat{\theta} < \theta \leq \tilde{\theta} \) does not purchase anything in the market. The condition holds if \( v \) is not large. That is, consumers are not willing to pay, even for a cleaner product, because the direct intrinsic utility is not large enough to compensate for the loss of utility caused by emissions of the product. Therefore, the demand functions are given by:

\[
q_D = \frac{P_C - P_D}{1 - \varepsilon} - s \quad \text{and} \quad q_C = \frac{(1 - \varepsilon)v - P_C + \varepsilon P_D + s}{\varepsilon(1 - \varepsilon)}. \tag{3}
\]

Here we should interpret the two cases from the viewpoint of the environment. In the case of FMC, an increase in the proportion of consumers purchasing a cleaner product improves the environmental. On the other hand, in the case of PMC, outside products such as public transport and bicycles are implicitly assumed. Accordingly, it is better for the environment that consumers do not purchase even a cleaner product.

Next, we explain firms producing environmentally differentiated products in the market. The firms decide on a unit emission level, i.e., environmental quality, of the product before
competition in price. For example, they need to invest in environment-technology research and development or build a product line and plant associated with environmental qualities. As discussed earlier, to focus on the decision of a unit emission level by firm $C$, we assume that the cost function is given by: $F_C = F_C (\varepsilon), \quad F_C ' (\varepsilon) < 0, \quad F_C '' (\varepsilon) \geq 0$. We also assume that the cost of firm $D$ is constant with respect to the unit emission level: $F_D = F_D (1)$. Furthermore, for simplicity, the marginal costs of production are independent of a unit emission level and zero. Therefore, the profit functions of the firms are expressed by: $\Pi_D = p_D q_D - F_D$ and $\Pi_C = p_C q_C - F_C (\varepsilon)$.

Finally, we present the composition of social welfare. First, aggregate emissions are expressed as:

$$E = q_D + \varepsilon q_C. \quad (4)$$

Second, based on (1), in general, net consumer surplus included in the social valuation of environmental damage is given by:

$$NCS = \int_0^{\theta'} (v - \theta - p_D - \gamma E) d\theta + \int_{\theta}^{\theta'} (v - \varepsilon \theta - p_C + s(1 - \varepsilon) - \gamma E) d\theta - \gamma E (\theta' - \theta'), \quad (5)$$

where $\theta' = \overline{\theta} (\hat{\theta})$ in the case of full (partial) market coverage. Thus, (5) can be rewritten as $NCS = CS - \gamma E \overline{\theta}$, where $CS$ denotes gross consumer surplus expressed as:

$$CS = \int_0^{\theta} (v - \theta - p_D) d\theta + \int_{\theta}^{\theta'} (v - \varepsilon \theta - p_C + s(1 - \varepsilon)) d\theta.$$

Furthermore, producer surplus is:

$$PS = \Pi_D + \Pi_C. \quad (6)$$

Therefore, given (4), (5), and (6), social surplus is given by:

$$W \equiv NCS - \Omega + PS = CS - \gamma E \overline{\theta} - \Omega + PS,$$

where $\Omega = s(1 - \varepsilon) q_C$ is the incurred budget deficit of a government subsidizing consumers.

We present a three-stage game as follows. In the first stage, a government subsidizes/taxes consumers purchasing a cleaner product, according to the difference between unit emission levels of the products. In the second stage, firm $C$ decides on a unit emission level of the product, given the consumer-based environmental subsidy/tax. In the final stage, the firms compete in price in the market, given the subsidy/tax and the unit emission level. The solution of this game is the subgame perfect Nash equilibrium.

3.1 The Full Market Coverage Case

Because the derivation of the Bertrand–Nash equilibrium in the final stage is simple, we present the equilibrium outcomes only as follows.

\[
p_D = \frac{(1 - \varepsilon)(\tilde{\theta} - s)}{3} \quad \text{and} \quad p_C = \frac{(1 - \varepsilon)(2\tilde{\theta} + s)}{3}.
\]

Substituting (8) into (2), the equilibrium quantities are given by:

\[
q_D = \frac{\tilde{\theta} - s}{3} \quad \text{and} \quad q_C = \frac{2\tilde{\theta} + s}{3},
\]

Given (8) and (9), an increase in a unit emission level of a cleaner product reduces prices, but does not affect quantities. Furthermore, a consumer-based environmental subsidy has an impact on market share between the products and decreases (increases) the price of a dirtier (cleaner) product.

In the second stage, the first-order condition for profit maximization of firm \(C\) is given by:

\[
\frac{\partial \Pi_C}{\partial \varepsilon} = -\frac{(2\tilde{\theta} + s)^2}{9} - \frac{\partial F_C}{\partial \varepsilon} = 0,
\]

where the second-order condition always holds. With regard to the effect of a consumer-based environmental subsidy on a unit emission level, we derive:

\[
\frac{d\varepsilon}{ds} = -\frac{\partial^2 \Pi_C}{\partial \varepsilon^2}\frac{\partial \varepsilon\partial s}{\partial \varepsilon\partial s} \leq 0 \iff \frac{\partial^2 \Pi_C}{\partial \varepsilon^2} > 0.
\]

As to the right-hand side expression in (11), we have \(\frac{\partial^2 \Pi_C}{\partial \varepsilon^2} = -\frac{2(2\tilde{\theta} + s)}{9} < 0\). Thus, \(\frac{d\varepsilon}{ds} < 0\).

That is, a consumer-based environmental subsidy reduces a unit emission level of the product. Because a consumer-based environmental subsidy increases the market share as well as the price, and thereby increases the revenue of firm \(C\), it has an incentive to upgrade the level of environmental quality of the product.

To analyze environmental and welfare effects of a consumer-based environmental subsidy/tax policy, we first examine the effect on aggregate emissions. Substituting (9) into (4), aggregate emissions are given by:
\[
E = \frac{(1+2\varepsilon)\bar{\theta} - (1-\varepsilon)s}{3} > 0,
\]

where \(s < \bar{\theta} < \frac{(1+2\varepsilon)\bar{\theta}}{1-\varepsilon}\).

In view of (12), we have
\[
\frac{\partial E}{\partial s} = -\frac{1-\varepsilon}{3} < 0, \quad \frac{\partial E}{\partial \varepsilon} = \frac{2\bar{\theta} + s}{3} = q_C > 0, \quad \text{and} \quad \frac{\partial E}{\partial s} < 0.
\]

Hence, it holds that \(\frac{dE}{ds} = \frac{\partial E}{\partial s} + \frac{\partial E}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial s} < 0\). Thus, we summarize the result as follows.

Proposition 1

*In the case of full market coverage, a consumer-based environmental subsidy improves the environment.*

Given a unit emission level of a cleaner product in the short run, some consumers change from purchasing a dirtier product to purchasing a cleaner product because of the subsidy policy. This change reduces aggregate emissions. In addition, because firm \(C\) reduces a unit emission level of the product in the long run, a decrease in the unit emission level reduces aggregate emissions.

Second, by substituting \(\theta' = \bar{\theta}\) into (5), we examine the effect on gross consumer surplus. The direct effect of a consumer-based environmental subsidy is
\[
\frac{\partial CS}{\partial s} = \frac{1-\varepsilon}{3} (q_D + 2q_C) > 0.
\]

However, because the indirect effect is
\[
\frac{\partial CS}{\partial \varepsilon} = q_D^2 + q_Dq_C - \frac{1}{2} q_C^2 \propto 2\bar{\theta}^2 - 10\bar{\theta}s - s^2,
\]

the sign is not unidirectional. For example, we have \(\frac{\partial CS}{\partial \varepsilon}|_{s=0} > 0\). In this case, paradoxically, the degradation of environmental quality of a cleaner product increases gross consumer surplus without a subsidy or with a sufficiently small subsidy. Although the change of a unit emission level does not have an impact on quantities of the product, it reduces prices of them. However, if \((\bar{\theta} >)s \geq (3\sqrt{3} - 5)\bar{\theta} \approx 0.2\bar{\theta}\), it holds that \(\frac{\partial CS}{\partial \varepsilon} \leq 0\). Thus, unless a consumer-based environmental subsidy is sufficiently small, the effect on gross consumer surplus is positive, i.e., \(\frac{dCS}{ds} > 0\). We can also understand that net consumer surplus included in the social valuation of environmental damage, \(NCS\), increases because of a consumer-based environmental subsidy.
Third, we have the partial effects on producer surplus:
\[
\frac{\partial PS}{\partial s} = \frac{2(1-\varepsilon)(\theta + 2s)}{3} > 0 \quad \text{and} \quad \frac{\partial PS}{\partial \varepsilon} = -\frac{(\theta - s)^2}{9} < 0.
\]
Given \(\frac{\partial \varepsilon}{\partial s} < 0\), the sign of the effect on producer surplus is positive, \(dPS > 0\). Thus, a consumer-based environmental subsidy increases producer surplus.

Here we note that the sign of the sum of indirect effects on gross consumer surplus and producer surplus is negative, i.e.,
\[
\frac{\partial CS}{\partial \varepsilon} + \frac{\partial PS}{\partial \varepsilon} = q_C \left( q_D - \frac{1}{2} q_C \right) < 0.
\]
Thus, a consumer-based environmental subsidy increases the sum of net consumer surplus and producer surplus.

Fourth, we should consider the effect on the government’s budget deficit. We derive
\[
\frac{\partial \Omega}{\partial s} = \frac{2(1-\varepsilon)(\theta + s)}{3} > 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial \varepsilon} = -sq_C < 0.
\]
Thus, given \(\frac{\partial \varepsilon}{\partial s} < 0\), a consumer-based environmental subsidy increases budget deficit, i.e., \(d\Omega > 0\).

Finally, using the results derived above, we should investigate an optimal consumer-based environmental subsidy/tax policy. Differentiating social surplus as in (7) with respect to a consumer-based environmental subsidy and arranging it, an optimal consumer-based environmental subsidy is given by:
\[
s^F = \frac{\left\{ \frac{dCS}{ds} + \frac{dPS}{ds} \right\} - \gamma \frac{dE}{ds}}{1 - \varepsilon} - \frac{(1-\varepsilon)q_C}{(1-\varepsilon)\frac{\partial q_C}{\partial s} - q_C \frac{\partial \varepsilon}{\partial s}} > 0,
\]
where the denominator is positive, and the nominator is also positive unless the budget deficit per subsidy is large. Therefore, we derive as follows.

Proposition 2

In the case of full market coverage, a consumer-based environmental subsidy policy is optimal.

3.2 The Partial Market Coverage Case

With regard to the case of partial market coverage, in which some consumers do not purchase any products, we need to confirm the results in the full market coverage case shown just before, i.e., Propositions 1 and 2.

We can easily derive the equilibrium prices in the final stage as follows:
\[ p_D = \frac{(1 - \epsilon)(v - s)}{4 - \epsilon} \quad \text{and} \quad p_C = \frac{(1 - \epsilon)(2v + (2 - \epsilon)s)}{4 - \epsilon}. \tag{14} \]

Substituting (14) into (4), the equilibrium quantities are given by:

\[ q_D = \frac{v - s}{4 - \epsilon} \quad \text{and} \quad q_C = \frac{2v + (2 - \epsilon)s}{\epsilon(4 - \epsilon)}. \tag{15} \]

As in (15), the upgrade of environmental quality of a cleaner product increases (reduces) the quantity demanded of a cleaner (dirtier) product.

In the second stage, firm \( C \) chooses a unit emission level of the product to maximize profits. The first-order condition for profit maximization of firm \( C \) is given by:

\[ \frac{\partial \Pi_C}{\partial \epsilon} = q_C \left\{ (1 - 2\epsilon)q_C + 2\epsilon(1 - \epsilon) \frac{\partial q_C}{\partial \epsilon} \right\} - \frac{\partial F_C}{\partial \epsilon} = 0, \tag{16} \]

where \( \{\bullet\} < 0 \) and we assume that the second-order condition is satisfied. Based on (16), the effect of a consumer-based environmental subsidy on the unit emission level is expressed as:

\[ \frac{d\epsilon}{ds} = -\frac{\partial^2 \Pi_C / \partial \epsilon \partial s}{\partial^2 \Pi_C / \partial \epsilon^2} > (\leq) 0 \iff \frac{\partial^2 \Pi_C}{\partial \epsilon \partial s} > (\leq) 0. \tag{17} \]

We calculate the right-hand side expression in (17) as follows.

\[ \frac{\partial^2 \Pi_C}{\partial \epsilon \partial s} = -\frac{2(2 - \epsilon)}{\epsilon(4 - \epsilon)^3} \left\{ 6v + (10 - 8\epsilon + \epsilon^2)s \right\} - \frac{2(1 - \epsilon)(8 - 4\epsilon + \epsilon^2)}{\epsilon(4 - \epsilon)^2} q_C < 0. \]

Thus, we have \( \frac{d\epsilon}{ds} < 0 \). That is, a consumer-based environmental subsidy reduces the unit emission level of a cleaner product.

We are now in a position to examine the effect of a consumer-based environmental subsidy policy on the environment and on social welfare. First, substituting (15) into (4), we obtain aggregate emissions in the case of partial market coverage as follows.

\[ E = \frac{3v + (1 - \epsilon)s}{4 - \epsilon}. \tag{18} \]

From (18), the direct effect on aggregate emissions is \( \frac{\partial E}{\partial s} = \frac{1 - \epsilon}{4 - \epsilon} > 0 \). That is, a consumer-based environmental subsidy increases aggregate emissions. Put differently, given the unit emission level of a cleaner product in the short run, the subsidy increases the quantity demanded of a cleaner product more than it reduces that of a dirtier product. This implies that some consumers, who would not buy any products in the market without the subsidy, newly purchase a cleaner product because of the upgrade of environmental quality brought about by the subsidy.

For example, it seems that a consumer who rode a bicycle until the subsidy policy purchases a...
Prius because of that policy. Thus, a consumer-based environmental subsidy directly degrades the environment. This result is different from that in the case of full market coverage, because, as mentioned earlier, it is not environmentally friendly to consume even a cleaner product in the case of partial market coverage.

However, the indirect effect of the change in a unit emission level is negative, because it holds that $\frac{\partial E}{\partial \varepsilon} = \frac{3}{4-\varepsilon} q_D > 0$ and $\frac{\partial E}{\partial s} < 0$. In other words, a consumer-based environmental subsidy reduces aggregate emissions in the long run. If the magnitude of the indirect (or long run) effect is larger than that of the direct (or short run) effect, the subsidy policy decreases aggregate emissions. Otherwise, it increases them. Therefore, we summarize this result as follows.

Proposition 3

*In the case of partial market coverage, the effect of a consumer-based environmental subsidy on the environment is unidirectional.*

Second, the effect of a consumer-based environmental subsidy on gross consumer surplus is expressed by

$$\frac{dCS}{ds} = \frac{\partial CS}{\partial s} + \frac{\partial CS}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial s}.$$  

With regard to the first term on the right-hand side, the sign is positive: $\frac{\partial CS}{\partial s} = \frac{1-\varepsilon}{4-\varepsilon} (q_D + 2q_C) > 0$. However, as to the indirect effect of the second term, we obtain

$$\frac{\partial CS}{\partial \varepsilon} = \frac{q_D (3q_D + (2 + \varepsilon)q_C)}{4-\varepsilon} - \frac{q_C^2}{2}.$$  

The sign is not always negative. For example, it holds that $\lim_{\varepsilon \to 1} \frac{\partial CS}{\partial \varepsilon} > 0$. However, if it holds that $\varepsilon \leq \frac{4}{5}$, then we have $\frac{\partial CS}{\partial \varepsilon} < 0$. It may not be intuitively unnatural to assume that the sign of the effect on gross consumer surplus is negative when the level of environmental quality of a cleaner product deteriorates. Accordingly, even though a consumer-based environmental subsidy increases gross consumer surplus, since its effect on the environment is ambiguous, the effect on net consumer surplus is not unidirectional.

Third, with regard to the effect on producer surplus, the direct effect is given by:

$$\frac{\partial PS}{\partial s} = \frac{2(1-\varepsilon)}{4-\varepsilon} \{(2-\varepsilon)q_C - q_D\} + \frac{8-4\varepsilon + \varepsilon^2}{\varepsilon^2(4-\varepsilon)^2} > 0.$$
Furthermore, we obtain \( \frac{\partial PS}{\partial \epsilon} = \frac{\partial \Pi_D}{\partial \epsilon} = -\frac{2 + \epsilon}{4 - \epsilon} q_D^2 < 0 \). That is, a decrease in the unit emission level of a cleaner product extends the degree of differentiation between environmental qualities of the products. This, in turn, mitigates price competition. Thus, a consumer-based environmental subsidy increases the profit of firm \( D \) because it reduces the unit emission level of a cleaner product. Thus, taking \( \frac{\partial \epsilon}{\partial s} < 0 \) into account, we have \( \frac{dPS}{ds} = \frac{\partial PS}{\partial \epsilon} + \frac{\partial PS}{\partial \epsilon} \frac{\partial \epsilon}{\partial s} > 0 \).

As a result, a consumer-based environmental subsidy increases producer surplus.

Fourth, we should consider the effect on the government’s budget deficit. We derive the followings: \( \frac{\partial \Omega}{\partial s} = (1 - \epsilon) \left\{ q_C + \frac{(2 - \epsilon)s}{\epsilon(4 - \epsilon)} \right\} > 0 \), and

\[
\frac{\partial \Omega}{\partial \epsilon} = -s \left\{ q_C + \frac{(1 - \epsilon)[4(2 - \epsilon)v + (8 - 4\epsilon + \epsilon^2)s]}{\epsilon^2(4 - \epsilon)^2} \right\} < 0.
\]

Thus, the subsidy policy increases the budget deficit, i.e., \( \frac{d\Omega}{ds} = \frac{\partial \Omega}{\partial s} + \frac{\partial \Omega}{\partial \epsilon} \frac{\partial \epsilon}{\partial s} > 0 \).

Using the results addressed above, we consider an optimal consumer-based environmental subsidy/tax policy in the case of partial market coverage. Differentiating social surplus with respect to a consumer-based environmental subsidy and arranging it, we have

\[
s^{\star} = \left\{ \frac{dCS}{ds} + \frac{dPS}{ds} - \gamma \frac{dE}{ds} \right\} - (1 - \epsilon)q_C
\]

\[
(1 - \epsilon) \frac{\partial q_C}{\partial s} - \left[ q_C - (1 - \epsilon) \frac{\partial q_C}{\partial \epsilon} \frac{\partial \epsilon}{\partial s} \right] \frac{\partial E}{\partial s}
\]

where the denominator is positive. The sign of the sum of the effect on gross consumer surplus and that on producer surplus in the nominator are positive, however, the sign of the nominator itself is ambiguous, even though the budget deficit from a subsidy is sufficiently small.

The key point is the effect on aggregate emissions. When a consumer-based environmental subsidy reduces aggregate emissions, i.e., \( \frac{dE}{ds} < 0 \), we can conclude that a consumer-based environmental subsidy is the optimal policy. However, when the magnitude of the direct effect on aggregate emissions is larger than that of the indirect effect, it holds that \( \frac{dE}{ds} > 0 \). In this case, we derive the relationships as follows.
\[s^p \times <(>)0 \Leftrightarrow \gamma >(<) \frac{dE}{ds} \frac{dPS}{ds} \equiv \gamma^*. \tag{20}\]

That is, if the marginal social valuation of consumers is larger than a certain value, \(\gamma^*\), then a consumer-based environmental tax policy is optimal. Put differently, when consumers care about the environment significantly, the government should levy a tax on consumers purchasing a cleaner product to reduce aggregate emissions and thus to decrease aggregate consumption.

**Proposition 4**

*In the case of partial market coverage, (i) when a consumer-based environmental subsidy reduces aggregate emissions, it is an optimal policy; however, (ii) when a consumer-based environmental subsidy increases aggregate emissions, an optimal policy depends on the magnitude of marginal social valuation of environmental damage.*

**4. Concluding Remarks**

We have analyzed the environmental and welfare effects of a consumer-based environmental subsidy/tax policy, and examined an optimal policy, looking at the cases of market coverage.

Our results are as follows.

(i) In the case of full market coverage, a consumer-based environmental subsidy reduces aggregate emissions, whereas in the case of partial market coverage, the effect of the subsidy on aggregate emissions is ambiguous.

(ii) A consumer-based environmental subsidy policy is socially optimal in the case of full market coverage.

(iii-a) When a consumer-based environmental subsidy reduces aggregate emissions, it is an optimal policy in the case of partial market coverage.

(iii-b) When a consumer-based environmental subsidy reduces aggregate emissions, an optimal policy also depends on the marginal social valuation of environmental damage. That is, if the marginal social valuation of environmental damages is larger (smaller) than a certain value, a consumer-based environmental tax (subsidy) is an optimal policy in the case of partial market coverage.
Here we should discuss the same issue in the case of a Cournot duopoly. Toshimitsu (2008b) has shown that a consumer-based environmental subsidy damages the environment while it increases the sum of gross consumer and producer surplus. Thus, with regard to the optimal policy, he finds the same result as in Proposition 4, that is, whether a consumer-based environmental subsidy is optimal or not depends on the degree of the marginal social valuation of environmental damage.

Furthermore, Toshimitsu (2008b) also examines the case of an emission subsidy (tax) policy charged on a firm producing a cleaner product. In this case, he shows that environmental and welfare effects of subsidizing the firm are equivalent to those of subsidizing consumers purchasing a cleaner product.

Let us now discuss some outstanding issues. First, as mentioned in the Introduction, an interesting issue arises in the form of an environmental subsidy/tax policy game between local and central governments. For example, a local government decides on an environmental subsidy/tax on consumers by taking into account the welfare improvements for residential consumers included in the valuation of the environment in the local area. However, a central government chooses an environmental subsidy/tax on firms to maximize social welfare included in aggregate industry profits. In this case, as discussed by Lombardini-Riipinen (2005) regarding commodity tax policy, we can analyze the policy mix, e.g., an environmental subsidy/tax and a commodity tax.

Second, we should discuss other environmental policies using direct pollution controls such as emission standards and quotas as well as indirect pollution controls such as tradable emission permits and emission and/or commodity taxes. For example, in future work we intend to address how emission taxes on an environmentally unfriendly polluting good, i.e., a dirtier product, affect the environment and social welfare by employing a similar model to that presented in this paper.

Finally, in this paper, we have assumed the cases of market coverage exogenously given. However, which case of market coverage holds may be endogenously decided by the strategies of firms such as prices and environmental qualities. In this case, an optimal consumer-based environmental subsidy/tax policy may be contingent on the type of endogenously decided market coverage. We hope to analyze this issue shortly.
References


