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The Estimation of the New Keynesian Phillips Curve in Japan and Its Implication for the Inflation Response to a Monetary Policy Shock

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Abstract

The New Keynesian Phillips Curve (NKPC) is a key building block in many modern macroeconomic models. This study assesses the empirical fit of the NKPC in Japan by estimating a variety of its specifications. Some empirical results suggest that introducing nominal interest rates into the pure forward-looking NKPC, which implies the existence of a cost channel for monetary policy, helps improve their ability to explain Japanese inflation dynamics. In addition to the existence of the cost channel for monetary policy, these results show that the use of labor share (real unit labor costs) is an important factor in estimating the NKPC. As an implication of these findings, this study proposes that, in the context of the New Keynesian economics, both the existence of the cost channel for monetary policy and the sluggish adjustment of real unit labor costs can account for the fact that there is a long time lag between a monetary policy shock and its impact on inflation.

JEL Classification: C3; E3; E5

Keywords: New Keynesian Phillips Curve; Cost Channel of Monetary Policy; Inflation Responses to a Monetary Policy Shock

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1 Introduction

This study seeks to understand the dynamics of aggregate price inflation and the effects monetary policy has on it. It explores suitable structural inflation dynamics in the sense that the empirical specification provides a better fit, and then draws out some implications for the time lag in the effect of monetary policy on inflation. Specifically, it highlights the importance of the role of monetary policy effects on the supply-side as well as the demand-side.

It is well known that monetary policy affects the demand side of the economy by influencing the investment and savings decisions of households and firms through changing interest rates. On the other hand, some economists, such as Barth and Ramey (2001), have proposed that there may be important supply-side effects of monetary policy augmenting the conventional demand-side channel. To the extent that firms must pay the factors of production before they receive revenues from selling their products, they rely on borrowing from financial intermediaries. Higher interest rates translate into higher costs of working capital and then induce a rise in inflation. Such an effect is called the cost channel of monetary policy.

Recently, many macroeconomists have used a “New Keynesian” (NK) model, which can be derived from the optimizing decisions of economic agents and nominal rigidities, as a useful framework for the analysis of monetary transmission and monetary policy issues.\(^1\) The prominent feature of the NK model is that macroeconomic dynamics are critically dependent on private agents’ forward-looking expectations about the future state of the economy. And the presence of nominal rigidities yields a correlation between inflation dynamics and the real economy and the real impact of monetary policy.

Inflation dynamics are one of the important components of the monetary transmission mechanism. According to Woodford (2003), the New Keynesian Phillips Curve (NKPC) is used as a model of inflation dynamics in the context of the NK model. The NKPC relates current inflation to the present-value of current and future real marginal costs. The NKPC has emerged as the dominant theoretical attempt to explain the interaction between inflation and real aggregates, and gives a description of the supply-side of the

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\(^1\)See Woodford (2003) for a standard derivation of the NK model.
Within the NKPC, measures of real marginal costs are a crucial determinant of the evolution of the inflation rate. In estimating NKPC, many empirical studies, e.g., Gali and Gertler (1999), Sbordone (2002), Rudd and Whelan (2005, 2006, 2007), and Kurmann (2005), use real unit labor costs or detrended output as a proxy of real marginal costs.\(^2\) However, in the presence of the cost channel for monetary policy, we can consider nominal interest rates as one of the key driving variables for inflation, in addition to real unit labor costs or detrended output. Ravenna and Walsh (2006) and Chowdhury et al. (2006) introduce the cost channel of monetary transmission into an otherwise standard NK model. As shown by them, when firms' costs of working capital increase with interest rates, we can augment an NKPC by short-run nominal interest rates. And Tillmann (2008) shows that the cost channel adds significantly to the explanation of inflation dynamics in the NKPC for the U.S., U.K., and the Euro area. In particular, in Japan's financial system, where banks play a highly important role in financial intermediations, monetary policy through this channel may have significant effects on inflation.

This study examines the empirical performance of the NKPC in explaining Japanese inflation dynamics by estimating a variety of its specifications. To our knowledge, there are the following two empirical studies that estimate the NKPC in Japan. First, Muto (2009) estimates the NKPC in Japan by focusing on the measurement of real marginal costs. To obtain a better proxy for real marginal costs, he corrects the labor share of national income (real unit labor costs) by taking into account two kinds of labor market frictions: labor adjustment costs and real wage rigidity. He finds that if a corrected measure of real marginal costs is used, the NKPC provides a good explanation of Japan’s inflation dynamics, and that the inclusion of a lagged inflation term into the NKPC does not improve its fit. Although we also seek to obtain a better proxy for real marginal costs, we focus on the role of nominal interest rates for real marginal costs. Second, Chowdhury et al. (2006) estimate the NKPC including short-run nominal interest rates and lagged inflation, and report that there is less evidence for the existence of an effective cost channel.

\(^2\)Although the NKPC may hold a central place in the recent theoretical literature on monetary economics, the empirical fit of the NKPC is not always found to be successful. For example, Rudd and Whelan (2005, 2006, 2007) report that neither real unit labor costs or detrended output provide a good fit for the NKPC. Furthermore, by estimating the so-called “hybrid” class of the NKPC which allows partial dependence of inflation on its own lags, they find that the fit of the model is due only to the inclusion of lags of inflation.
in Japan. Although they employ the generalized method of moments (GMM) regression approach, the resulting GMM estimators often suffer from an econometric problem called “weak identification” as Ma (2002) and Mavroeidis (2005) point out. In order to avoid this problem, we estimate the NKPC by using an alternative approach based on the present value model, which is employed by Rudd and Whelan (2005, 2006, 2007), Muto (2009), Kurmann (2005), and Tillmann (2008).

Some empirical results in this study suggest that introducing nominal interest rates into the pure forward-looking NKPC model helps improve their ability to explain Japanese inflation dynamics within the NK economy. In addition to the existence of the cost channel for monetary policy, these results show that the use of labor share is an important factor in estimating the NKPC. As an implication of these findings, this study points out that both the existence of the cost channel and the sluggish adjustment of real unit labor costs account for the fact that there is a long time lag involved in the effect of monetary policy on inflation.

The remainder of the paper is structured as follows. Section 2 summarizes the transmission effects of monetary policy, focusing especially on the time lags with which the shock is transmitted. Section 3 describes the empirical model used to explore the fit of the NKPC, and reports some empirical results. Section 4 provides the implications of the empirical results obtained in Section 3 for the time lag of monetary policy effects on inflation. Some concluding remarks are offered in the final section.

2 Effects of Monetary Policy Shocks

In order to understand monetary policy effects on aggregate price inflation, it is important to distinguish between labor share (real unit labor cost) and detrended output (output gap), which is considered to be more closely associated with inflation. This section summarizes the transmission effects of monetary policy by using a vector autoregressive (VAR) approach, which is a standard framework used to study the effects of monetary policy shocks on macroeconomic variables. The VAR is useful in the sense that we can analyze the dynamic effects of economic variables without requiring a particular theoretical model.

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3See following subsection 3.5 for a further explanation of the weak identification problem.
This section provides our specification of the VAR model. We then identify a monetary policy shock and report estimates of how major macroeconomic variables, including labor share and detrended output, respond to a monetary policy shock, focusing especially on the time lags with which the shock is transmitted.

2.1 Monetary VAR Model and Identification of Monetary Policy Shocks

Consider a six-variable VAR model consisting of a general price inflation rate \( (\pi_t) \), sensitive price inflation rate \( (\pi^f_t) \), detrended output \( (y_t) \), labor share of national income \( (s_t) \), short-term interest rate \( (r_t) \) and money growth \( (\Delta m_t) \). Let \( X_t = [\pi_t, \pi^f_t, y_t, s_t, r_t, \Delta m_t]^\prime \) denote the vector of variables included in the analysis. Assume that the dynamics of \( X_t \) are given as follows:

\[
X_t = a_0 + A_1 X_{t-1} + \cdots + A_p X_{t-p} + \epsilon_t,
\]

where \( E(\epsilon_t) = 0, E(\epsilon_t \epsilon_t^\prime) = \Omega \) for \( t = \tau \) and 0 elsewhere. The lag order \( p \) is empirically determined by some appropriate statistical information criteria. Following Christiano et al. (1999), (1) is a reduced form VAR model, which may contain a block recursive restriction that the monetary policy variables be ordered fifth in the VAR on the assumption that the nonmonetary variables do not respond to the monetary policy shocks contemporaneously but the monetary variables do. A Choleski decomposition of the reduced form covariance matrix \( \Omega \) can be used to orthogonalize the reduced form innovations and to identify the structural model. The innovations in the equations of \( r_t \) are treated as monetary policy shocks. We estimate the parameters \( a_0, A_j (j = 1, \cdots, p) \) and \( \Omega \) using ordinary least squares (OLS). Using these estimates, we compute the impulse responses of \( X_t \) to a monetary policy shock.

As for the data on the general price inflation rate \( (\pi_t) \), we use two kinds of variables: the quarterly change of the log GDP deflator, and the consumer price index (CPI) in percentage points. The sensitive price inflation \( \pi^f_t \) is measured as the quarterly change \(^4\)In many monetary VAR studies, it is now standard practice to specify the VAR model that does not generate the “price puzzle”, the frequent finding in VAR literature that a contractionary monetary policy shock is followed by a permanent increase in the price level. Sims (1992) explains that the “price puzzle” can be interpreted as the result of imperfectly controlling for any information that the central bank may have about future inflation. In actuality, because specification with the sensitive price variable often solves the “price puzzle”, estimation with such a variable generalizes.

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of the log domestic Wholesale Price Index (WPI) in percentage points.\textsuperscript{5} The detrended output measure is the percentage deviation of the log of real GDP from an HP-filter trend. The labor share of income $s_t$ (in percentage point deviation) is $s_t = 100 \log \left( \frac{WL}{PY} \right)_t$, where $WL$ is the System of National Accounts' (SNA’s) “compensation of employees” and $PY$ is nominal GDP—“household’s operating surplus”.\textsuperscript{6} The short-term interest rate $r_t$ is the three-month Gensaki rate (average). The money growth is the quarterly change of the log $M2 + CD$ in percentage points.

In the estimation, we use quarterly data from 1975:Q1 to 1999:Q1.\textsuperscript{7} The lag length is set to 5 in the estimation of the reduced-form model (1).\textsuperscript{8}

### 2.2 Estimating Effects of Monetary Policy Shocks

Figures 1 and 2 show the estimated impulse responses of all variables in $X_t$ to a 1% monetary policy shock in the cases of GDP deflator inflation and CPI inflation measures, respectively. The solid line indicates the estimated response. The upper and lower dashed lines plotted in each graph represent 90% bootstrap confidence intervals, based on 10000 bootstrap samples.

Overall, estimated impulse responses look reasonable in both figures. The results suggest that after a contractionary monetary policy shock,

1. Detrended output immediately falls, peaking after a half year and returning to pre-shock levels after one year;

\textsuperscript{5}Hosono et al. (2001) examine four possible candidates for the sensitive price variable when estimating structural VAR models: the Nikkei commodity price index, the import price index, total and domestic WPI. The results suggest that, among the four possible candidates, domestic WPI shows the most favorable results.

\textsuperscript{6}We cannot use the definition, which is “compensation of employees” divided by nominal GDP, because the definition of “compensation of employees” does not include compensation of employees in self-employed firms. Following Muto (2009), we employ the definition as above.

\textsuperscript{7}We restrict the sample period to before the introduction of the period of zero interest rate policy. Ueda (2002) thinks of the zero interest rate policy as the combination, the zero rate and the commitment to maintain it until a certain set of conditions are met. Following Ueda (2002), we assume that the Bank of Japan implemented the zero interest rate policy in the period from April 1999, and adopt 1999:Q1 as the ending point.

\textsuperscript{8}We perform a modified likelihood ratio (LR) test proposed by Sims (1980) to check whether taking 5 lags is sufficient. Here, the null of 5 lags is tested against the alternative of 6, 7, or 8 lags. The LR statistics indicate that the null is not rejected by conventional significance levels. In addition, we perform the multivariate Lagrange Multiplier (LM) test statistics for residual serial correlation for up to the 1st and 4th orders. See Johansen (1995), p. 22, for the formula of the LM statistics. The LM statistics for each order indicate that the null hypothesis of no serial correlation is not rejected by asymptotically significant levels.
2. Labor share (real unit labor costs) slowly falls, peaking after one year;

3. Interest rate falls for about 2 years.

4. General price inflation falls at five quarters, but the decrease is small; on the other hand, sensitive price inflation falls slowly, peaking after one year, and the decrease is larger than that of general price inflation;

5. Money growth falls at five quarters.

In Figures 1 and 2, some important features of the effects of a monetary policy shock are worth emphasizing here. First, we can confirm the stylized fact that there is a long time lag between a monetary policy shock and its impact on price inflation. There is a considerable delay in the effect of monetary policy on price inflation, unlike detrended output.

Second, the detrended output and labor share responses are noticeably different in response to a monetary policy shock. We find that the responses for labor share follow those for price inflation very closely. As for the demand side effect of monetary policy, an increase in the nominal interest rate reduces the demand on consumption and investment because of the influence of the real interest rate, and then leads to a negative detrended output. Under monopolistic competition, the reduction of current demand then decreases real unit labor costs through a reduction in labor demand. However, as the model with wage rigidity implies, deflation pressures can impede the effective clearing of labor markets.

With regard to wage rigidity, a gap between detrended output and labor share can arise. This gap may lead on the demand side effects of monetary policy to a difference in the influence between real unit labor costs and detrended output.

3 Japanese Inflation Dynamics

This section examines the empirical performance of the NKPC in explaining Japanese inflation dynamics by estimating a variety of its specifications. In particular, it provides an empirical model to investigate whether the presence of the cost channel for the monetary

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9Shibamoto (2007) reports that with regard to regular workers, employment responses to a monetary policy shock in Japan follow those for the price indices very closely, using a factor-augmented VAR approach.
policy, in which monetary policy has an effect on the supply-side of the economy, and the presence of lagged inflation affects structural inflation dynamics. Then, it reports some empirical results.

### 3.1 Interest-Rate Augmented New Keynesian Phillips Curve

Following Woodford (2003) among others, the marginal-costs-based NKPC is described as follows:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{r} \hat{m} \hat{c}_t + \text{error}, \tag{2}
\]

with a subjective discount factor, \( \beta \). \( \hat{\pi}_t \) is a measure of the deviation of the inflation rate from the steady-state value, \( \hat{r} \hat{m} \hat{c}_t \) denotes a measure of the deviation of real marginal costs from the steady-state value, and \( E_t \) is the expectations operator. The composite parameter \( \kappa \) is a function of the structural model parameters. Under rational expectations, equation (2) can be re-written as

\[
\hat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t \hat{r} \hat{m} \hat{c}_{t+j} + \text{error}, \tag{3}
\]

Equation (3) states that the inflation rate at time \( t \) is a fraction of the present-value of the expected path of future real marginal costs.\(^{10}\)

We now consider the specification of the NKPC to allow an impact of the nominal interest rate on the supply-side of the economy by raising firms’ marginal costs and, thus, on the inflation rate.\(^{11}\) Following Chowdhury et al. (2006) and Tillmann (2008), the proxy for the measure of the deviation of real marginal costs from the steady-state value, \( \hat{r} \hat{m} \hat{c}_t \), is

\[
\hat{r} \hat{m} \hat{c}_t = \hat{s}_t + \psi R \hat{r}_t, \tag{4}
\]

where \( \hat{s}_t \) and \( \hat{r}_t \) are the deviations of the labor share and the gross nominal interest rate

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\(^{10}\)As Roberts (1995) has shown, the rational expectations sticky-price models of Taylor, Rotemberg, Calvo, and others imply an NKPC in which current inflation incorporates a forward-looking component.

\(^{11}\)Christiano and Eichenbaum (1992), Christiano et al. (2005) and Ravenna and Walsh (2006) assume that factors of production have to be paid before the proceeds from the sale of output are received, to introduce the costs of working capital into a general equilibrium model.
from their respective steady-state values. The parameter $\psi_R$ measures the strength of the cost channel. This means that its strength may depend on the share of bank-dependent firms and the degree of the pass-through from money market and capital market rates to corporate lending rates. If $\psi_R = 0$, the cost channel is absent.

Considering the cost channel by combining equations (2) and (4) thus leads to the following present-value relation for the inflation rate

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\hat{s}_t + \psi_R \hat{r}_t) + \text{error}, \quad (5)$$

and equivalently,

$$\pi_t = \mu_0 + \mu_1 \sum_{j=0}^{\infty} \beta^j E_t s_{t+j} + \mu_2 \sum_{j=0}^{\infty} \beta^j E_t r_{t+j} + \text{error}, \quad (6)$$

where $\mu_0$ is constant, which includes the constant steady-state values for inflation, labor share and nominal interest rate, $\mu_1 = \kappa$ and $\mu_2 = \kappa \psi_R$. Many previous studies, e.g., Gali and Gertler (1999), Sbordone (2002), Rudd and Whelan (2005, 2006, 2007), and Kurmann (2005), study only the present-value of the labor share as a driving variable of inflation, i.e., they assume $\mu_2 = 0$. The empirical analysis in this section examines whether the inflation rate is driven by the present-value of expected future interest rates in addition to the labor share.

The NKPC as derived above has frequently been criticized for a lack of inflation inertia.\(^{12}\) We now include $\pi_{t-1}$ to see whether there is an extra source in the inflation rate not present in the standard formulation of the NKPC as above.\(^{13}\) If there is some direct effect

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\(^{12}\)The notion of inertia refers to the slow adaptation of a variable to unexpected changes in economic conditions. By this definition, a variable is inertial if and only if it has past levels, or past expectations about its current level, have a direct influence on its current level. The notions of stickiness and inertia are used as synonyms. In contrast to inertia, the notion of persistence refers to the slow transition of a variable to its steady state after the initial impact of the unexpected shock. Note that a variable can theoretically be persistent even if it is not sticky.

\(^{13}\)Inflation inertia models assume some kind of backward looking behavior or bounded rationality. Christiano et al. (2005) model backward looking behavior on the assumption that all firms apply indexation to lagged inflation in periods when they cannot readjust their prices. Gali and Gertler (1999) specify backward looking behavior on the assumption that a fraction of firms are backward looking, and hence follow a rule of thumb in readjusting their prices. The two assumptions are equivalent in that they yield the same aggregate relationship, linking current inflation rate to lagged inflation, expected future inflation and some measure of real economic activity. On the other hand, as an alternative to bounded rationality, Mankiw and Reis (2002) model inflation inertia based on sticky information. The assumptions of some kind of backward looking behavior and bounded rationality yield similar implications for inflation and output dynamics.
of lagged inflation besides the one that occurs because lagged inflation helps to forecast the labor share and nominal interest rates, ignoring inflation inertia could lead to biased estimates that attribute too much weight to contemporaneous labor share and interest rates.

Following Christiano et al. (2005) and Woodford (2003), the NKPC in the presence of price indexation becomes

\[ \hat{\pi}_t - \rho \hat{\pi}_{t-1} = \beta E_t(\hat{\pi}_{t+1} - \rho \hat{\pi}_t) + \kappa \hat{m} c_t + \text{error}. \]  

(7)

Solving (7) forward yields

\[ \hat{\pi}_t - \rho \hat{\pi}_{t-1} = \kappa \sum_{j=0}^{\infty} \beta^j \hat{m} c_{t+j} + \text{error}. \]  

(8)

Using (4), (8) can be re-written as follows:

\[ \pi_t = \mu_0 + \mu_1 \sum_{j=0}^{\infty} \beta^j E_t s_{t+j} + \mu_2 \sum_{j=0}^{\infty} \beta^j E_t r_{t+j} + \rho \pi_{t-1} + \text{error}. \]  

(9)

Note that (9) nests the purely forward-looking model when \( \rho \) is set to zero. The empirical analysis in this section also examines whether the inflation rate is driven by the lagged inflation.

3.2 Forecasts from VAR projections

We estimate the Japanese NKPC, which is derived as (9). Following Rudd and Whelan (2005, 2006, 2007), Kurmann (2005), Muto (2009), and Tillmann (2008), we employ the estimation used originally by Campbell and Shiller (1987). In this approach, we first forecast the future path of labor share and interest rate based on a VAR, then use it to estimate (9).\(^{14}\)

Letting \( Z_t = [1, X_t, X_{t-1}, \ldots, X_{t-p+1}]' \), (1) can be represented as a VAR(1) process:

\[ Z_t = B Z_{t-1} + H \epsilon_t, \]  

(10)

\(^{14}\)Campbell and Shiller (1987) propose this framework to assess the fit of forward-looking present-value models.
where

$$
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
\alpha_0 & A_1 & \cdots & A_{p-1} & A_p \\
0 & I & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I & 0 \\
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
0 \\
I \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
0 \\
I \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
$$

Hence, \( Z_t \) is an approximation to the agents’ information set, which can be described by constant, current and past realizations of price inflation; WPI inflation; detrended output; labor share; interest rate, and money growth. Forecasts of the endogenous variables are given by the multi-period forecasting formula

$$
E_t[Z_{t+j}] = B^j Z_t.
$$

(11)

Using the matrix version of the summation formula for geometric series, the vector of the discounted future paths of the variables can be calculated as

$$
\sum_{j=0}^{\infty} \beta^j E_t Z_{t+j} = (I - \beta B)^{-1} Z_t,
$$

(12)

where \( I \) is the \( 6 \times 6 \) identity matrix. Let \( e_s' \) and \( e_R' \) be selection vectors that single out the present-value of future labor shares and interest rates, respectively. We map these forecasts into the present-value representation of the NKPC model (9) to obtain an expression for the model-consistent inflation rate \( \pi^m_t \) as follows,

$$
\pi^m_t = \mu_0 + \mu_1 e_s'(I - \beta B)^{-1} Z_t + \mu_2 e_R'(I - \beta B)^{-1} Z_t + \rho \pi_{t-1}.
$$

(13)

To calculate this series of fundamental inflation from the VAR coefficients and the data, we need to fix the discount factor \( \beta \). In accordance with the vast literature, we choose to calibrate \( \beta = 0.99 \). However, all results are robust to other choices of the discount factor as long as the discount factor is close to but below unity. Using the present-value of future labor shares and interest rates based on the VAR, we simply estimate (9) by OLS
and then calculate the adjusted coefficient of determination (adj-$R^2$) to assess the fit of the NKPC by comparing actual inflation $\pi_t$ with the series of fundamental inflation $\pi_t^m$ resulting from this estimation.\(^{15}\)

Note that using VAR projections disguises the degree of estimation uncertainty, as Kurmann (2005) points out. The VAR companion matrix $B$ that is needed to derive fundamental inflation according to (13) contains only point estimates. Following Tillmann (2008) to assess the accuracy of the model’s fit, we employ a bootstrap approach that infers the distribution of our measures of fit by estimating the model with artificial data. We obtain confidence intervals by drawing from the residuals of the estimated model and generating new observations for the data vector using the estimated companion matrix. The VAR model is estimated again and a new coefficient matrix $B$ is computed. From this, we compute the series of expected future labor shares and interest rates and regress actual inflation on the present-value of these two forcing variables to infer the slope coefficients. Finally, adj-$R^2$ is computed. Repeating this procedure 10000 times provides us with an empirical distribution for adj-$R^2$ from which an interval that includes 90% of the estimates can be calculated.

### 3.3 Estimation of the Discounted Present Value Model

We estimate the NKPC under four different scenarios. In the first scenario, the cost channel is absent and inflation is driven solely by the discounted present value of future labor share. This scenario corresponds to a model with $\mu_2 = 0$ and $\rho = 0$. In the second scenario, as well as in Rudd and Whelan (2005, 2006, 2007) and Kurmann (2005), the cost channel is absent, but lagged inflation is included. This scenario corresponds to a model with $\mu_2 = 0$. In the third scenario, the cost channel is present and inflation is driven by the discounted present value of future labor share and future interest rates, but lagged inflation is not included. This scenario corresponds to a model with $\rho = 0$. In the fourth scenario, the cost channel is present, and lagged inflation is included.

Figures 3 and 4 compare the series of fundamental inflation with the actually observed inflation rates under four scenarios, in cases with GDP deflator inflation and CPI inflation.

\(^{15}\)To examine the possibility that $Z_t$ correlates with an error term, we have also estimated $\mu_0, \mu_1, \mu_2, \rho$ by the instrumental variables method. Using $Z_{t-1}$ as instruments, we find that the result is almost the same as with OLS.
measures, respectively. The solid lines in this figure are fundamental inflation series $\pi^m_t$ of the discounted present value generated using an econometric forecasting model. The dashed lines are actual Japanese inflation $\pi_t$. For convenience, we include shaded regions, which begin at an Economic and Social Research Institute (ESRI) business cycle peak, and end at a trough.

(a) in Figures 3 and 4 reveals a bad performance under the first scenario. (b) shows that, although the fit is better than the one under the first scenario, fundamental inflation $\pi^m_t$ is quite volatile. On the other hand, (c) and (d) show that fundamental inflation $\pi^m_t$ tracks the behavior of actual inflation $\pi_t$ quite well, and satisfactorily explain the historical pattern of inflation in the sense of the popular empirical evidence that inflation is procyclical. In particular, we find that in the recession phase (1975Q1, 1977Q1–1977Q4, 1980Q1–1983Q1, 1985Q2–1986Q4, 1991Q1–1993Q4, 1997Q2–1999Q1), their fundamental inflation tends to fall.

Below we can interpret the differences among four scenarios. Table 1 presents baseline estimation results under four scenarios. In the first scenario, we find that the present value of future real unit labor costs turns out to be negatively correlated with inflation. The table shows that its present value of labor share generated from a VAR that fits the data well has an adj-$R^2$ of only 1.5 and 4.2% in the cases with GDP deflator inflation and CPI inflation measures, respectively. In the second scenario, we also find that the present value of future real unit labor costs turns out to be negatively correlated with inflation. Although the adj-$R^2$ from the second scenario is relatively larger than that from the first, the fit of this model might be due only to the inclusion of a lagged value of $\pi_t$. Therefore, in the NKPC model without the cost channel, not only does inflation not appear to equal the present value of future real unit labor costs, it does not even have the correctly signed correlation. This implies that the use of labor share alone as a proxy of real marginal costs is not a suitable driving variable for inflation.

On the other hand, in the third scenario, we find that the present value of both future real unit labor costs and interest rates turns out to be positively correlated with inflation.\(^{17}\)

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\(^{16}\)As we show below, an adj-$R^2$ is not as high as (c) and (d).

\(^{17}\)The parameters $\mu_1$ and $\mu_2$ are quite small. However, this does not always imply that the fraction of firms that re-optimize prices within the quarter is small. For example, the strength of strategic complementarity depends on the value of the parameter $\kappa$, coefficient of the real marginal costs. See Woodford (2003), for details. In addition, Altig et al. (2005) present models in which capital is firm-specific and
In addition, the adj-$R^2$ is 45.4 and 73.7% in the cases with GDP deflator inflation and CPI inflation measures, respectively. Furthermore, the presence of the cost channel means a significant improvement in the model’s empirical performance, because the 90% confidence intervals do not include the value of the adj-$R^2$ for the case without the cost channel, i.e., $0.015 \notin [0.158,0.579]$ and $0.042 \notin [0.543,0.809]$. These results mean that both labor share and interest rate as a proxy for real marginal costs are suitable driving variables for inflation. Moreover, in the fourth scenario, we find that the estimated $\rho$ coefficient is statistically insignificant, which implies that lagged inflation has little influence on current inflation. In addition, the adj-$R^2$ from the fourth scenario is almost the same as that in the third scenario. These results mean that the fit of NKPC with the cost channel can hardly be attributed to the contribution made by the lagged value of $\pi_t$.

It should not be surprising that, in the case with the restriction $\mu_2 = 0$, the present value of future real unit labor costs is negatively correlated with inflation. Here we provide the possible explanation that underspecification, without considering the cost channel for monetary policy, leads to a severe problem for the estimation of the coefficient of the present value of future real unit labor costs $\mu_1$. The estimator $\mu^*_1$ without considering the cost channel is as follows:

$$
\mu^*_1 = \mu_1 + \frac{\partial (\sum_{j=0}^{\infty} E_t r_{t+j} - \bar{r})}{\partial (\sum_{j=0}^{\infty} E_t s_{t+j} - \bar{s})} \mu_2,
$$

(14)

where $\bar{r}$ and $\bar{s}$ denote the average of $\sum_{j=0}^{\infty} E_t r_{t+j}$ and $\sum_{j=0}^{\infty} E_t s_{t+j}$, respectively. (14) shows that the estimator $\mu^*_1$ is produced by the effects of future real unit labor costs on inflation mixing with those of future interest rates through the cost channel. Except in the cases that $(\partial (\sum_{j=0}^{\infty} E_t r_{t+j} - \bar{r}))/(\partial (\sum_{j=0}^{\infty} E_t s_{t+j} - \bar{s}))$ is equal to zero and/or $\mu_2$ is equal to zero, the estimator $\mu^*_1$ is biased and inconsistent.

Figure 5 displays a scatter plot of the present value of future labor share and future interest rate estimated by using the auxiliary VAR model (10). As is obvious in Figure 5, $(\partial (\sum_{j=0}^{\infty} E_t r_{t+j} - \bar{r}))/(\partial (\sum_{j=0}^{\infty} E_t s_{t+j} - \bar{s}))$ is strongly negative. Thus, if $\mu_2$ is not equal to
zero, then the estimator $\mu_1^*$ is biased and inconsistent. In addition, if $\mu_2$ is positive, then the estimator $\mu_1^*$ is possibly negative. This implies that the estimation without considering the cost channel spuriously causes the present value of future real unit labor costs to be negatively correlated with inflation, because of confusion with the effects of future interest rates on inflation.

Next, we use the detrended output, which some empirical researchers (for example, Rudd and Whelan (2005, 2006, 2007)) often employ as a proxy for real marginal cost in estimating NKPC, in place of the real unit labor costs. This is because of the assumption that real unit labor costs are proportional to the gap between actual and potential output (equilibrium output in the case of fully flexible prices), that is detrended output. With this assumption and (9), the detrended output-based NKPC with cost channel becomes

$$\pi_t = \lambda_0 + \lambda_1 \sum_{j=0}^{\infty} \beta^j E_t y_{t+j} + \lambda_2 \sum_{j=0}^{\infty} \beta^j E_t r_{t+j} + \rho_1 \pi_{t-1} + \text{error.} \quad (15)$$

Table 2 presents estimation results under four scenarios by using the detrended output in place of the labor share.\(^1\)\(^9\) When we use the detrended output instead of the labor share, the estimated coefficient on the detrended output is negative, even allowing for the cost channel.\(^2\)\(^0\) This suggests that the link between real unit labor costs and detrended output is the problem and not the link between marginal costs and inflation.\(^2\)\(^1\) To go from real unit labor costs to a detrended output measure, real wages are replaced by the marginal rate of substitution between leisure and consumption. As Woodford (2003) among others show, this procedure assumes that nominal wages are perfectly flexible, so that the real wages can adjust to maintain workers on their labor supply curve. If nominal wages are sticky, a gap can open between the real wage and the marginal rate of substitution between leisure and consumption. In fact, as shown in the previous section, the time lags, with which the monetary policy shock are transmitted, vary somewhat between detrended output and labor share. Our results imply a problem in the replacing of labor share with

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\(^1\)\(^9\) Even in using the present values of future detrended output and interest rate generated from a VAR without the labor share, almost the same results as the following empirical ones are obtained.

\(^2\)\(^0\) It is well known that the present value of future detrended output is negatively correlated with inflation, in spite of the fact that detrended output is procyclical as well as inflation is procyclical. For empirical evidence in the U.S., see e.g., Rudd and Whelan (2005, 2006, 2007).

\(^2\)\(^1\) In contrast to this interpretation, Gali and Gertler (1999) argue that the problem is that “potential output” is hard to measure, thus, empirical proxies for the detrended actual output may be poor proxies for the real output gap.
detrended output when the NKPC is estimated, in the sense that these two variables respond to different influences over time.\(^{22}\)

In summarizing some empirical results in this subsection, we suggest that both the nominal interest rate and the labor share be used as a proxy for real marginal costs.

### 3.4 Comparison with the GMM Estimation

As an alternative method for estimating the NKPC, some researchers employ the GMM regression approach, which is often proposed in the literature for estimating the forward-looking rational expectation model. Chowdhury et al. (2006) estimate the hybrid NKPC, including the short-run nominal interest rate in G7 countries, by using the GMM estimation approach. They report that there is less evidence for the existence of an effective cost channel in Japan, relatively in Canada, France, Italy, the U.K., and the U.S. This subsection reports the GMM estimation results similar to those obtained by following Chowdhury et al. (2006), and discusses the difference between these results and the results obtained before the previous subsection.

Consider the cost channel by combining equations (7) and (4), thus leads to the equation for the inflation rate

\[
\hat{\pi}_t = \frac{\beta}{1+\beta \rho} E_t \hat{\pi}_{t+1} + \frac{\kappa}{1+\beta \rho} (\hat{s}_t + \psi R \hat{r}_t) + \frac{\rho}{1+\beta \rho} \hat{\pi}_{t-1} + \text{error}
\]

(16)

From (16), we obtain

\[
\pi_t = \zeta_0 + \zeta_1 E_t \hat{\pi}_{t+1} + \zeta_2 s_t + \zeta_3 r_t + \zeta_4 \pi_{t-1} + \text{error},
\]

(17)

where \(\zeta_0\) is a constant that includes the constant steady-state values for inflation, the labor share and nominal interest rate, \(\zeta_1 = \frac{\beta}{1+\beta \rho}, \zeta_2 = \frac{\kappa}{1+\beta \rho}, \zeta_3 = \frac{\kappa \psi R}{1+\beta \rho}\) and \(\zeta_4 = \frac{\rho}{1+\beta \rho}\). (17) is a linear GMM regression, with moment conditions of the following form:

\[
E_t [Q_t(\pi_t - \zeta_0 - \zeta_1 \pi_{t+1} - \zeta_2 s_t - \zeta_3 r_t - \zeta_4 \pi_{t-1})] = 0,
\]

(18)

\(^{22}\)Blanchard and Gali (2007) show that, under the assumption of the real wage rigidities, the NKPC becomes the relation between inflation, expected inflation and both the level and change of the detrended output.
where $Q_t$ is a $q \times 1$ vector of instruments. According to moment conditions (18) with and without the restriction $\rho = 0$ (i.e. $\zeta_4 = 0$), GMM estimations are carried out for the quarterly time-series data. The sample period spans the time interval 1975–1999 as above. Following Chowdhury et al. (2006), the vector of instruments $Q_t$ includes four lags of general price inflation, sensitive price inflation, labor share, and short-run interest rate. To account for possible correlation in the moment conditions and to control for autocorrelation and heteroskedasticity in unknown form in the weighting matrix, we allow for the Newey and West (1987) correction up to order 4.

Table 3 reports the estimates of the interest rate augmented NKPC as specified in (17) with and without the restriction $\zeta_4 = 0$ in the cases with two inflation measures using the GDP-deflator and the CPI. In all cases, as reported in Chowdhury et al. (2006), the estimated coefficients of the labor share and the nominal interest rate are not statistically significant. This is inconsistent with the empirical results obtained before the previous subsection. Furthermore, as is obvious from the results shown in Table 3, the J-statistic fails to reject the null that the moment conditions (18) are asymptotically satisfied at the estimated values $\hat{\zeta}_j$ for $j = 0, \cdots, 4$. Conventionally, empirical researchers have considered that this specification works well in the sense that the regression model (17) is correctly specified and the instruments $Q_t$ are asymptotically uncorrelated with the error term.

However, some applications of GMM have what is known as “weak instruments” or “weak identification”; that is, instruments that are only weakly correlated with the included endogenous variables. Ma (2002) and Mavroeidis (2005) point out that we suffer from the weak identification problem of the GMM in the context of NKPC. If instruments are weak, then the sampling distributions of GMM are in general nonnormal, and standard GMM point estimates, hypothesis tests, and confidence intervals are unreliable. Therefore, empirical researchers need to check whether or not the instruments are weak. In practice, there are methods more robust to weak instruments than conventional GMM. One central finding of Stock and Wright (2000) is that the usual limiting distribution theory does not apply, and this motivates the use of alternative inference methods. Furthermore, Kleibergen (2005) proposed test statistics (K statistics) that do not depend only on whether or not the parameter is weakly identified but also on the number of moment conditions. If the instruments are weak, the values of the K statistics evaluated at certain parameter
values, other than the true values, are very close to the minimized value, and therefore the confidence sets contain very large sets of parameters. A detailed test method based on Kleibergen (2005) is presented in Appendix.

Figures 6 and 7 show 90% confidence sets for $\zeta_1, \zeta_2, \text{ and } \zeta_3$ based on the K statistics concentrated with respect to $\zeta_0, \zeta_4$ (shaded) in the GDP deflator inflation measure with and without the restriction $\rho = 0$ (i.e. $\zeta_4 = 0$), respectively. Figures 8 and 9 show 90% confidence sets for $\zeta_1, \zeta_2, \text{ and } \zeta_3$ based on the K statistics concentrated with respect to $\zeta_0, \zeta_4$ (shaded) in CPI inflation measure with and without the restriction $\rho = 0$ (i.e. $\zeta_4 = 0$), respectively.

In Figures 6, 7, 8, and 9, we find that the 90% confidence sets for $\zeta_1, \zeta_2, \text{ and } \zeta_3$ based on the K statistics contain a much larger set of parameters than the conventional asymptotic confidence sets. This implies that it is possible that the instruments are weak. When the instruments are weak, conventional asymptotic inferences fail even if the sample size is large. This casts doubt on the results using GMM estimation.

4 Inflation Responses to a Monetary Policy Shock

From our empirical results in the previous section, we find that (i) the existence of the cost channel for monetary policy and (ii) the use of labor share are key points to estimate the NKPC. So, can the pure forward-looking NK model with the cost channel explain the observed inflation responses to the monetary policy shock, which are hump-shaped as shown in section 2? This section provides one plausible answer to this question by considering some characteristics that influence the demand-side and supply-side effects of monetary policy on real marginal costs over time. We set up four simple endogenous variable dynamic systems to quantitatively investigate the role of cost channels for monetary policy and the sluggish adjustment of real unit labor costs.

We use the simple NK dynamic general equilibrium model to calibrate the effects of a monetary policy shock. Without the cost channel of the monetary policy, the pure forward-looking NK model always generates an immediate decline of inflation after a contractionary monetary policy shock. This is because current inflation depends on the current and future real unit labor costs, which always decline after a monetary policy contraction. Then, inflation declines sharply back to its original level. Thus, as shown from the following results in Figure 11, inflation responses to a monetary policy shock are not hump-shaped. Note that, in the hybrid NKPC case, the inflation responses are hump-shaped, because inflation has an inertia effect. See Woodford (2003) for details.

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tightening of monetary policy. In the model, the NKPC (16) as introduced in the previous Section corresponds to the supply side. From (16) with \( \rho = 0 \), the aggregate supply side is characterized by a pure forward-looking NKPC:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{s}_t + \kappa \psi R \hat{r}_t.
\]  

(19)

We set \( \beta = 0.99 \). And, from the empirical results in section 3, we set \( \kappa = 0.01 \) and \( \psi_R = 1 \) as the baseline. In the case without the cost channel, we set \( \psi_R = 0 \).

The aggregate demand representation is derived from the Euler condition for the representative agent’s decision problem. Following Woodford (2003), we assume that the aggregate demand side is characterized by the following forward-looking IS (NKIS) curve:

\[
y_t = E_t(y_{t+1}) + \sigma_r (\hat{r}_t - E_t \hat{\pi}_{t+1});
\]  

(20)

and the empirical equation is expressed in the following form:

\[
y_t = \sigma_0 + E_t(y_{t+1}) + \sigma_r (\hat{r}_t - E_t \hat{\pi}_{t+1}) + \text{error}.
\]  

(21)

Many empirical studies add some forms of habit formation to the model in order to match the hump-shaped output responses that are apparent in the data. See Woodford (2003) for details. Therefore, we replace \( E_t(y_{t+1}) \) with a term such as \( (1 - \sigma_y)E_t(y_{t+1}) + \sigma_y y_{t-1} \). Although empirical estimates of \( \sigma_y \) with quarterly data are hard to find in the literature, we set \( \sigma_y = 0.3 \). Under this parameter value, we can replicate the observed hump-shaped output responses to a monetary policy shock as shown in the following empirical results.

For the NKIS function, the coefficient of the real interest rate is expected to be negative. The first line in Table 4 shows that the estimate of \( \sigma_r \) in (21) obtained by using GMM is \( \hat{\sigma}_r = -0.175 \), which means that the detrended output has the correctly-signed correlation with the real interest rate.\(^{24}\) Thus, in (20), we set \( \sigma_r = -0.175 \).

We can show the simple linear relationship between the real unit labor costs and the detrended output in the standard model with no labor market rigidities.\(^{25}\) However, we

\(^{24}\)The set of instruments used include a constant and lags 1–5 of \( y_t, \pi_t, s_t, \Delta m_t \) and \( r_t \).

\(^{25}\)Strictly speaking, in the presence of habit formation, the real unit labor costs relate expected detrended output at current, future and past periods (see Woodford, 2003).
want to consider the case in which the real unit labor costs respond sluggishly to labor market conditions as a result of some unmodeled imperfection or friction in labor markets. To this end, we assume the partial adjustment model as follows:

\[
\dot{s}_t = g_1 \dot{s}_{t-1} + (1 - g_1) g_2 y_t, \quad (22)
\]

and the empirical equation is expressed in the following form:

\[
s_t = g_0 + g_1 s_{t-1} + (1 - g_1) g_2 y_t + \text{error}, \quad (23)
\]

where \(g_1\) can be interpreted as an index of the persistent effects of the real unit labor costs. We view (22) as an admittedly ad hoc but simple way of modeling the sluggish adjustment of real unit labor costs without taking a stand on what the right model is. Assume that we cannot obtain accurate information about the current values of the output gap \(y_t\). Therefore, we estimate (22) by using the GMM.\(^{26}\)

The second line in Table 4 shows the GMM estimates of the partial adjustment equation of real unit labor costs as specified in (23). The estimates of the coefficients for the lagged real unit labor costs and the detrended output are \(\hat{g}_1 = 0.761\) and \((1 - \hat{g}_1) \hat{g}_2 = 0.124\), respectively. Thus, as in the baseline case, we set \(g_1 = 0.761\) and \(g_2 = 0.124/(1 - 0.761) = 0.519\). If there is no persistency of real unit labor costs, we set \(g_1 = 0\).

We close the model with a simple monetary policy rule, in our case, the Taylor rule (Taylor, 1993), an interest rate rule that reacts to deviations of inflation and output:

\[
\dot{r}_t = \theta \dot{r}_{t-1} + (1 - \theta) (\eta_y y_t + \eta_{\pi} \dot{\pi}_t) + v_t \quad (24)
\]

where \(v_t\) is a monetary policy shock. In (24), we set \(\theta = 0.8\), \(\eta_y = 0.5\) and \(\eta_{\pi} = 1.5\).

It is instructive to compare the dynamics of the NK structural model (19), (20), (22), and (24) with given parameter values in the baseline to those of a VAR model. Figure 10 shows the responses of these two systems to a monetary policy shock. The VAR impulse responses from Figure 1 are shown as solid lines and their 90% confidence intervals as dashed lines. The impulse responses of the structural system are shown as solid lines with

\(^{26}\)The set of instruments used include a constant and lags 1–5 of \(y_t\), \(s_t\), \(\pi_t\), \(\Delta m_t\), and \(r_t\).
an (○). Although the responses of output are much larger in the model than appears to be the case in practice, the empirical structural model and VAR impulse responses appear to be quite similar overall. Thus, the empirical structural model appears to be very useful as a benchmark, at least, to investigate the roles of cost channel for monetary policy and the sluggish adjustment of real unit labor costs quantitatively as we will next discuss.

Now we examine the inflation responses to a monetary policy shock focusing on the roles of the presence of the cost channel and the sluggish adjustment of real unit labor costs. Figure 11 shows the inflation responses to a monetary policy shock for some alternative values of the parameters, \(g_1, \psi_R\). The solid lines with (○) represent the simulated impulse responses in the baseline case \((g_1 = 0.76, \psi_R = 1)\). The dashed lines with (△) represent the simulated impulse responses with the restriction, \(\psi_R = 0\). The solid lines with (□) represent the simulated impulse responses with the restriction, \(g_1 = 0\). The dashed lines with (★) represent the simulated impulse responses with the restrictions, \(g_1 = 0\) and \(\psi_R = 0\).

We see from Figure 11 that the model fails to replicate the observed hump-shaped responses for inflation when \(g_1 = 0\) and/or \(\psi_R = 0\). The maximum or minimum response of inflation occurs in the period of the monetary shock. This suggests that, within the pure forward-looking NKPC framework, the existence of the cost channel alone or the sluggish adjustment of real unit labor costs alone cannot generate the observed hump-shaped behavior of inflation.

In addition, we see that the structural model can replicate the observed hump-shaped responses for inflation only when \(g_1 \neq 0\) and \(\psi_R \neq 0\). Here, we consider the roles of the existence of the cost channel and the sluggish adjustment of real unit labor costs within the pure forward-looking NKPC for the inflation responses to a monetary policy shock. First, by considering the cost channel for monetary policy, monetary policy has significant effects on the supply-side as well as the demand-side. As the demand-side effect of monetary policy, an increase in the monetary policy target rate reduces the demand on output, because of the influence on the real interest rate through the investment and savings decisions of households and firms. Under monopolistic competition, the reduction of the current demand then decreases real unit labor costs through a labor demand reduction. This leads to the exertion of downward pressure on inflation. On the other
hand, with a cost channel of monetary policy, an increase in the monetary policy target rate will have a direct effect on all firms that rely on bank borrowing or loans of any kind linked to short-term money market interest rates. A rise in interest rates, which increases borrowing costs, puts upward pressure on inflation. Thus, changes in the interest rate have an opposite influence on inflation, at least in the short-run. Second, by considering the sluggish adjustment of the real unit labor costs, a monetary policy contraction has persistent effects on inflation due to the persistent reduction of the real unit labor costs.

By taking into account the monetary policy effects on inflation as above, we can provide a plausible explanation for the hump-shaped responses of inflation to a monetary policy shock. An increase in the monetary policy target rate exerts downward pressure on current inflation because of the reduction of current and future aggregate demand through the investment and saving decisions of households and firms. On the other hand, an increase in the interest rate directly causes current and future borrowing costs to increase through the cost channel of the monetary policy. This effect acts as a cost-push shock but endogenously, and then removes the downward pressure on the current inflation on the demand-side. Over time, the upward pressure on inflation through the cost channel of the monetary policy gradually disappears as the interest rate moves back to its baseline level. Although detrended output also moves back to its baseline level, real unit labor costs persistently decrease, and inflation falls because of the reduction of the current and future real unit labor costs. This implies that the demand-side effects eventually dominate the supply-side effects.

In this way, by considering the existence of the cost channel and the sluggish adjustment of real unit labor costs within the pure forward-looking NKPC, we can explain the empirical fact that there is a long time lag between a monetary policy shock and its impact on inflation.

\[27\] It is well known that a contractionary monetary policy shock identified by the VAR model is followed by a small and temporary rise in the price level. See Christiano et al. (1999) and Walsh (2003) for details. We see from Figure 11 that our structural model even replicates inflation increases for 2 quarters. This implies that the supply-side effects initially dominate the demand-side effects after a monetary policy tightening. Rabanal (2007) provides some calibration results in the NK model with nominal and real rigidities following Christiano et al. (2005), and points out that high real-wage stickiness and variable capital utilization rates (which lead to the sluggish adjustment of the production costs) and the presence of the cost channel are needed to generate a positive response of inflation after a monetary policy contraction.
5 Concluding Remarks

This study examines the empirical performance of the NKPC in explaining Japanese inflation dynamics by estimating a variety of its specifications. It then provides the implications of the empirical results for the time lags of monetary policy effects on inflation. The main message of this study is that both (i) the cost channel of the monetary policy and (ii) the sluggish adjustment of real unit labor costs play a critical role in understanding the dynamics of inflation and the monetary transmission on it.

Some empirical results in this paper suggest that introducing nominal interest rates into the pure forward-looking NKPC model, which implies the existence of the cost channel for monetary policy, helps improve their ability to explain Japanese inflation dynamics within the NK economy. In addition to the existence of the cost channel for monetary policy, they show that the use of labor share is an important factor in estimating the NKPC. As an implication of these findings, we point out that both the existence of the cost channel for monetary policy and the sluggish adjustment of real unit labor costs account for the empirical fact that there is a long time lag between a monetary policy shock and its impact on inflation.

Some avenues related to this study’s empirical findings are worth pursuing. Although this study regards the cost channel of the monetary policy as a direct effect of interest rates on firms’ marginal costs and thus on inflation, we might be able to consider the cost-side effects of monetary policy through some alternative channels.

As Barth and Ramey (2001) point out, the cost channel may also run through the broad credit channel due to the existence of financial frictions as proposed by the financial-accelerator and bank lending channel literature. Given that financial intermediaries play an important role in monetary policy transmission, when the firms are forced to turn to external funds, an increase in interest rates may affect their marginal financial costs and the short-run productive capacity of the economy. Furthermore, the interest rate effect may reflect other factors leading to cost pressure and, thus, to higher inflation.\footnote{Tsuruga (2007) proposes an NK general equilibrium model with a dynamic externality into the production function of firms, and shows that this model explains the hump-shaped response of inflation to a monetary policy shock. In his model, in addition to the demand-side effects on inflation, contractionary monetary policy shocks act as cost push shocks through a dynamic externality. Firms decrease their production and lose knowledge in response to an expansionary monetary policy shock. The decreased stock of knowledge causes productivity to decrease and, thus, increases marginal costs. In addition, Aoki (2001)
detailed discussion about these issues is beyond the scope of this study, but is worthy of future research.

**Appendix: Inference in the presence of weak identification**

Letting the $2 \times 1$ identified parameters vector $\psi = (\zeta_0, \zeta_4)'$ and the $3 \times 1$ possibly weakly identified parameters vector $\phi = (\zeta_1, \zeta_2, \zeta_3)'$, we rewrite (17) as follows:

$$\pi_t = \psi' \Upsilon_{1t} + \phi' \Upsilon_{2t} + \text{error},$$  \hfill (25)

where $\Upsilon_{1t} = (1, \pi_{t-1})'$ and $\Upsilon_{2t} = (E_t \pi_{t-1}, s_t, r_t)'$. In such circumstances, valid confidence sets can be based on the minimand of the restricted GMM estimation in which the minimization is performed over the identified parameter, $\psi$, conditional on a value for the weakly identified parameter, $\phi$.

Let $\hat{\psi}_T(\hat{\phi})$ denote the GMM estimator of $\psi$ conditional on $\phi = \hat{\phi}$; that is:

$$\hat{\psi}_T(\hat{\phi}) = \arg \min_{\psi \in \Psi} S_T(\psi, \hat{\phi}; \bar{\psi}_T(\hat{\phi}))|_{\phi=\hat{\phi}},$$  \hfill (26)

where $S_T(\psi, \hat{\phi}; \bar{\psi}_T(\hat{\phi})) = T g_T(\psi, \hat{\phi})' \Sigma_T(\bar{\psi}_T(\hat{\phi})) g_T(\psi, \hat{\phi})$, $\bar{\psi}_T(\hat{\phi})$ is a preliminary estimator of $\psi$, $g_T(\psi, \hat{\phi}) = T^{-1} \sum_{t=1}^T g_t(\psi, \hat{\phi})$, $g_t(\psi, \hat{\phi}) = Q_t(\pi_t - \psi' \Upsilon_{1t} - \hat{\phi}' \Upsilon_{2t})$, and $\Sigma_T(\bar{\psi}_T(\hat{\phi}))$ is a positive definite weighting matrix that is computed using $\bar{\psi}_T(\hat{\phi})$. Stock and Wright (2000) show that:

$$S_T(\hat{\phi}, \bar{\psi}_T(\hat{\phi})) \overset{d}{\rightarrow} \chi^2_{q-2},$$  \hfill (27)

where $\chi^2_{q-2}$ denotes a chi-square distribution with $q - 2$ degrees of freedom. Therefore, they propose the following asymptotically valid $100(1 - \alpha)$% confidence sets for $\hat{\phi}$:

$$\{\hat{\phi} : S_T(\hat{\phi}, \bar{\psi}_T(\hat{\phi})) < c_{q-2}(\alpha)\},$$  \hfill (28)

and Carvalho (2006) model heterogeneity in the frequency of price changes and show that, in the presence of strategic complementarities in price setting, sectors with lower frequencies of price adjustment have a disproportionate effect on the aggregate price level. In their model, contractionary monetary policy shocks act as cost-push shocks through a relative price change.
where $c_{q-2}(a)$ is the $100(1 - a)$ percentile of the $\chi^2_{q-2}$ distribution. Stock and Wright (2000) refers to such sets as concentrated S-sets.

Although the S-sets are robust to weak identification, even if the number of the parameters is fixed, the power of the statistics $S_T(\hat{\phi}, \hat{\psi}_T(\hat{\phi}))$ is reduced as the number of moment conditions $q$ increases, so that the statistics $S_T(\hat{\phi}, \hat{\psi}_T(\hat{\phi}))$ have a $\chi^2_{q-2}$ limiting distribution, and the critical value $c_{q-2}(a)$ then becomes large. Concerning this problem with regard to the low power of the test statistic, Kleibergen (2005) proposed the K statistic, which does not depend only on whether or not the parameter is weakly identified but also on the number of the moment conditions. Under the null hypothesis $H_0 : \phi = \tilde{\phi}$, K statistic $K_T(\hat{\phi}, \hat{\psi}_T(\hat{\phi}))$ is as follows:

$$
\frac{1}{4T} \left( \frac{\partial S((\phi', \psi')')}{\partial (\phi', \psi_T(\phi)')'} \right) \left[ \hat{D}_T((\phi', \hat{\psi}_T(\phi)')') \Sigma_T((\phi', \hat{\psi}_T(\phi)')')^{-1} \hat{D}_T((\phi', \hat{\psi}_T(\phi)')') \right]^{-1} \left( \frac{\partial S((\phi', \psi')')}{\partial (\phi', \psi_T(\phi)')'} \right)',
$$

(29)

where $\hat{D}_T((\phi', \hat{\psi}_T(\phi)')') = (\hat{D}_{0,T}((\phi', \hat{\psi}_T(\phi)')') \cdots \hat{D}_{4,T}((\phi', \hat{\psi}_T(\phi)')'))$ and

$$
\hat{D}_{j,T}((\phi', \hat{\psi}_T(\phi)')') = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{\partial g_t((\phi', \hat{\psi}_T(\phi)')')}{\partial \zeta_j} \right) - \left( \frac{1}{T} \sum_{t=1}^{T} \left( \frac{\partial g_t((\phi', \hat{\psi}_T(\phi)')')}{\partial \zeta_j} \right) g_t((\phi', \hat{\psi}_T(\phi)')') \right) \Sigma_T((\phi', \hat{\psi}_T(\phi)')')^{-1} g_t((\phi', \hat{\psi}_T(\phi)')')
$$

for $j = 0, \cdots, 4$. Kleibergen (2005) propose the following asymptotically valid $100(1 - a)\%$ confidence sets for $\phi$.

$$
\{ \hat{\phi} : K_T(\hat{\phi}, \hat{\psi}_T(\hat{\phi})) < c_3(a) \}
$$

(30)

where $c_3(a)$ is the $100(1 - a)$ percentile of the $\chi^2_3$ distribution. If the instruments are weak, the values of the K statistics $K_T(\hat{\phi}, \hat{\psi}_T(\hat{\phi}))$ evaluated at certain parameter values $\tilde{\phi}$, other than the true values, are very close to the minimized value, and so the confidence sets defined by (30) contain very large sets of parameters.
References


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<table>
<thead>
<tr>
<th>$\pi_t$</th>
<th>Model</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\rho$</th>
<th>adj-$R^2$</th>
</tr>
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<td>-0.002</td>
<td></td>
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</tr>
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<td></td>
<td></td>
<td>(0.001)</td>
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<td></td>
<td>[-0.007, 0.524]</td>
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<td></td>
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<td>(0.083)</td>
<td></td>
<td>[0.051, 0.538]</td>
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<td>(0.001)</td>
<td></td>
<td>[0.158, 0.579]</td>
</tr>
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<td>0.009</td>
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</tr>
<tr>
<td></td>
<td>Interest Rate</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.096)</td>
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<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td></td>
<td></td>
<td>[-0.006, 0.676]</td>
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<tr>
<td></td>
<td>Hybrid:</td>
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<td>0.682</td>
<td></td>
<td>0.579</td>
</tr>
<tr>
<td></td>
<td>No Interest Rate</td>
<td>(0.001)</td>
<td>(0.062)</td>
<td></td>
<td>[0.377, 0.740]</td>
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<tr>
<td></td>
<td>Including</td>
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<td>0.014</td>
<td></td>
<td>0.737</td>
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<td>(0.001)</td>
<td></td>
<td>[0.543, 0.809]</td>
</tr>
<tr>
<td></td>
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<td>0.013</td>
<td>0.013</td>
<td>0.075</td>
<td>0.736</td>
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<tr>
<td></td>
<td>Interest Rate</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.094)</td>
<td>[0.550, 0.817]</td>
</tr>
</tbody>
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Notes: Figures in parentheses are standard errors. Figures in brackets are 90% confidence intervals from bootstrapping.
<table>
<thead>
<tr>
<th>$\pi_t$</th>
<th>Model</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\rho_1$</th>
<th>adj-$R^2$</th>
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<td>-0.007</td>
<td>-0.007</td>
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<td>-0.007</td>
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<td>0.438</td>
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<td>0.006</td>
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<td></td>
<td></td>
<td></td>
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<td>No Interest Rate</td>
<td>-0.032</td>
<td>0.006</td>
<td>0.389</td>
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<td>Including Interest Rate</td>
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<td>0.006</td>
<td>0.101</td>
<td>0.389</td>
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<td>0.003</td>
<td>0.493</td>
<td>0.624</td>
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<tr>
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<td>No Interest Rate</td>
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<td>0.031</td>
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<td>Inflation Rate</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No Interest Rate</td>
<td>-0.040</td>
<td>0.006</td>
<td></td>
<td>0.483</td>
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<tr>
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<td>Including Interest Rate</td>
<td>-0.012</td>
<td>0.003</td>
<td>0.493</td>
<td>0.624</td>
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Notes: Figures in parentheses are standard errors. Figures in brackets are 90% confidence intervals from bootstrapping.
Table 3: GMM Estimates of NKPC Based on the Labor Share

<table>
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<tr>
<th>$\pi_t$</th>
<th>Model</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\zeta_4$</th>
<th>J</th>
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<td>GDP</td>
<td>Including</td>
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<td>0.026</td>
<td>0.032</td>
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<td>7.396</td>
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<tr>
<td></td>
<td>Interest Rate</td>
<td>(0.158)</td>
<td>(0.025)</td>
<td>(0.033)</td>
<td></td>
<td>[0.881]</td>
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<tr>
<td></td>
<td>Hybrid: Including</td>
<td>0.974</td>
<td>0.021</td>
<td>0.019</td>
<td>-0.113</td>
<td>6.677</td>
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<td></td>
<td>Interest Rate</td>
<td>(0.229)</td>
<td>(0.028)</td>
<td>(0.040)</td>
<td>(0.090)</td>
<td>[0.878]</td>
</tr>
<tr>
<td>CPI</td>
<td>Including</td>
<td>0.943</td>
<td>0.015</td>
<td>0.019</td>
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<td>10.576</td>
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<tr>
<td></td>
<td>Interest Rate</td>
<td>(0.089)</td>
<td>(0.021)</td>
<td>(0.018)</td>
<td></td>
<td>[0.646]</td>
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<tr>
<td>Rate</td>
<td>Hybrid: Including</td>
<td>0.815</td>
<td>-0.003</td>
<td>0.015</td>
<td>0.226</td>
<td>11.500</td>
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<tr>
<td></td>
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<td>(0.117)</td>
<td>(0.020)</td>
<td>(0.020)</td>
<td>(0.100)</td>
<td>[0.487]</td>
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Notes: Figures in parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with four lags. J denotes the test statistic of the model’s overidentifying restrictions, with p-value in brackets. The set of instruments used included a constant and lags 1–4 of $\pi_t$, $\pi_t^f$, $s_t$, and $r_t$. 
Table 4: Estimates of NKIS and Partial Adjustment Equation of Real Unit Labor Costs

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_r$</th>
<th>$J$</th>
</tr>
</thead>
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<tr>
<td>(21)</td>
<td>-0.175</td>
<td>15.953</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>[0.890]</td>
</tr>
<tr>
<td>Model</td>
<td>$g_1$ $g_1(1 - g_2)$</td>
<td>$J$</td>
</tr>
<tr>
<td>(23)</td>
<td>0.761</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.039)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.935]</td>
</tr>
</tbody>
</table>

Notes: Figures in parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with four lags. $J$ denotes the test statistic of the model’s overidentifying restrictions, with $p$-value in brackets. The set of instruments used included a constant and lags 1–5 of $y_t$, $s_t$, $\pi_t$, $\Delta m_t$, and $r_t$. 

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Figure 1: Impulse Response to a Monetary Policy Shock (GDP deflator). Note: The solid lines are the estimated impulse responses. The dashed lines are 90% confidence intervals about the impulse responses based on bootstrapping.
Figure 2: Impulse Response to a Monetary Policy Shock (CPI). Note: The solid lines are the estimated impulse responses. The dashed lines are 90% confidence intervals about the impulse responses based on bootstrapping.
Figure 3: Japanese fundamental (solid line) and actual (dotted line) inflation (GDP deflator)
Figure 4: Japanese fundamental (solid line) and actual (dotted line) inflation (CPI)
Figure 5: Scatter Plot: Present Value of Future Labor Share (deviation from the average, horizontal axis) and Future Interest Rate (deviation from the average, vertical axis) estimated by using the auxiliary VAR model (10)
Figure 6: 90% Confidence Sets for $\zeta_1$, $\zeta_2$ and $\zeta_3$ based on the K statistics, sliced in $\zeta_1$ dimension: NKPC Including Interest Rate (GDP deflator Inflation)
Figure 7: 90% Confidence Sets for $\zeta_1, \zeta_2$ and $\zeta_3$ based on the K statistics, sliced in $\zeta_1$ dimension (shaded): Hybrid NKPC Including Interest Rate (GDP deflator Inflation)
Figure 8: 90% Confidence Sets for $\zeta_1$, $\zeta_2$ and $\zeta_3$ based on the K statistics, sliced in $\zeta_1$ dimension: NKPC Including Interest Rate (CPI Inflation)
Figure 9: 90% Confidence Sets for $\zeta_1, \zeta_2$ and $\zeta_3$ based on the K statistics, sliced in $\zeta_1$ dimension: Hybrid NKPC Including Interest Rate (CPI Inflation)
Figure 10: Impulse Responses for VAR and Structural Model of the inflation, detrended output, real unit labor costs and interest rates. Note: The solid lines with (o) represent the simulated impulse responses in the baseline. The solid lines and dashed lines are from Figure 1.
Figure 11: Comparison of the Simulated Inflation Responses to a Monetary Policy Shock in the estimated NK Model: Note. The solid lines with (○) represent the simulated impulse responses in the baseline. The dashed lines with (△) represent the simulated impulse responses with the restriction, $\psi_R = 0$. The solid lines with (□) represent the simulated impulse responses with the restriction, $g_1 = 0$. The dashed lines with (*) represent the simulated impulse responses with the restrictions, $g_1 = 0$ and $\psi_R = 0$. 