**A Factor Endowment Theory of Endogenous
Growth and International Trade**

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Abstract

The Frankel-Romer-Lucas theory of endogenous growth rests on the assumption of knowledge-based externalities and price-taking representative households. It is argued that, in a context of long-run growth, these assumptions are mutually incompatible (that representative households will co-operate to internalize the externalities) and that therefore the relevance of the theory must be questioned.

In the present paper we offer a model of endogenous growth, based on representative households but recognizing that, in a context of long-run growth, the households must cooperate. Our economy consists of many countries, populated by households which are identical within countries but not necessarily across countries. In each country, government expenditure on a public intermediate good plays a crucial role in the realization of persistent growth. It is shown that the long-run pattern of international trade is determined in a Heckscher-Ohlin manner.

1 Introduction

The pioneering papers on endogenous growth [Frankel (1962), Romer (1986) and Lucas (1988)] rest on the assumption that useful technical information cannot be patented, kept secret or otherwise immobilized, with the implication that the production of information by any firm generates an externality which accrues to all other firms. The externality plays a prominent role in the analysis of each paper. In particular, it ensures that the economy possesses a perfectly competitive equilibrium path distinct from and inferior to the socially optimal path.

The papers rest also on the assumption of a representative household. This second assumption was introduced without fanfare and has passed virtually unnoticed in subsequent discussion; see, for example, Solow (1991), Long and Wong (1997) and Aghion and Howitt (1998).

However, in a context of long-run growth, externalities and representative households make poor bedfellows. For, in that context, households can hardly remain unaware that they are representative; and if all households are identical and know themselves to be so then, by a simple adaptation of an argument advanced by Kemp and Shimomura (1995; also 2000 and 2002), the households will cooperate to internalize the externalities, thus resolving the coordination problem and ensuring that the economy will settle not at an externalities-distorted competitive equilibrium but at the unique socially optimal equilibrium.

This finding runs counter to the belief that "the presence of externalities is essential for the existence of an equilibrium" [Romer (1986), p.1004]; and if, as has been argued [Romer (1986), p.1014], the socially-optimal outcome runs counter to the historical behaviour of per capita output then the relevance of the

basic models of endogenous growth must be questioned.

Of course, the modern theory of endogenous growth has already been heavily criticized. However the earlier criticism has been directed not at the basic structure of the theory but at the special functional forms needed for the more spectacular possibilities unearthed by the theory (for example, unbounded output per capita and increasing rates of growth). Our present sermon remains pertinent even if the earlier criticisms are met or rejected.

In the present paper, we construct a theory of endogenous growth that is immune to the criticisms directed to current theory. Immunity can be achieved in either of two ways: (a) one can simply abandon the representative-household assumption, acknowledging that households differ in their preferences and/or in their endowments; or (b) one can retain the representative-household assumption, while acknowledging some of the neglected implications of that assumption, notably that identical households which recognize that they are identical must behave cooperatively.

The task of building a theory of endogenous growth with heterogeneous households is not trivial. It is easy enough to ensure that the community's collective behaviour is like that of a single rational household even though households are not identical (and therefore will not recognize each other as identical and will not cooperate to internalize externalities). It suffices to assume that the indirect utility functions of all households are of Gorman polar form with a common coefficient-of-wealth function. However the aggregate utility function obtained in this way would have no compelling appeal as a focus of social optimization. We therefore embrace alternative (b), retaining the representative-household assumption but recognizing that identical households must cooperate.

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If literally *all* households are identical and know themselves to be so, they will cooperate to drive the economy along the socially optimal path. To avoid that trivial case, we consider a world economy in which many countries produce and trade in a pure consumption good and a pure investment good, each produced with the aid of homogeneous physical capital and homogeneous labour measured in efficiency units. However we assume that all countries are small so that, even if in each country all households cooperate, no households will believe that it possesses market power. In each country the quality of the labour force can be augmented by devoting the proceeds of a general income tax to that end. The effectiveness of such expenditure in raising the quality of each unit of labour is independent of the size of the labour force; in other words the government's expenditure creates a public intermediate good - public in the sense that its availability in any particular industry is independent of its use in the other industry. In each economy, all households are identical in their preferences, endowments and information. Moreover, households everywhere share the same preferences. However, any two households, not from the same country, may differ in their endowments of labour and capital; as a consequence, any two countries may differ in their choice of tax rates.

"Trade and growth" is a traditional topic in economic theory. However, while there are many studies of trade and *exogenous* growth, the literature on trade and *endogenous* growth in a *multi-country* framework seems to be still quite small. In this paper we study the simultaneous determination of the long-run growth rate and trade pattern in a multi-country world economy in which two factors (privately owned physical capital and labour) combine with a public intermediate good (optimally provided by the government) to produce two private goods (a consumption good and an investment good) in a dynamic general equilibrium (DGE) model. In that model, the public intermediate good is the engine of persistent growth, since it

rules out decreasing returns to scale.[1]

We are not the first to formulate a dynamic Heckscher-Ohlin (HO) model of international trade and endogenous growth. Doi, Nishimura and Shimomura (2002) and Bond, Trask and Wang (2003) formulate dynamic Heckscher-Ohlin models of trade and endogenous growth and characterize the balanced growth path. While Bond *et al.* introduce an education sector to "produce" new human capital, Doi *et al.* assume that human capital accumulation is achieved by each household's effort involving disutility. While each of these papers is a development of the Lucas-type endogenous growth theory in which the engine of persistent growth is human capital accumulation, the persistent growth in the present model is based on the accumulation of a public intermediate good.

The main results obtained in this paper are as follows. First, there exist world balanced growth paths such that along each path all quantities grow at a common rate equal to the rate of return on capital minus the rate of time preference. Second, along any balanced growth path factor prices are equalized not only internationally but also for the two factors of production, physical capital and efficiency labour. Third, along a balanced growth path the factor endowment ratio of an individual country is not necessarily equal to the world factor endowment ratio; in the Heckscher-Ohlin manner, each country's pattern of international trade is determined by the difference between its factor endowment ratio and the world factor endowment ratio. That is, if a country's capital/labour ratio is higher (resp. lower) than the world-wide ratio, it exports the capital-intensive (resp. labour-intensive) good.

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[1]Barro (1990) constructed a single sector, closed economy endogenous growth model based on productive public expenditure.

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It could be shown, by standard arguments, that the world economy described in the paper is saddlepoint stable, implying that, if initially the economy is not in a steady state, it will asymptotically converge to a steady state. However, for the economy described in this paper, much more can be said. As long as the initial world factor endowment ratio is sufficiently close to its ultimate steady-state level, the economy reaches a balanced growth path *within finite time* and then grows forever along the balanced growth path. Accordingly, we shall use the term *reachability* instead of *stability.*

Combining the local reachability property and the aforementioned trade-pattern result, we can say that, as long as the (historically given) initial world factor endowment ratio is close to its balanced growth level, the long-run trade pattern is determined by the initial international distribution of factor endowments such that the dynamic Heckscher-Ohlin Theorem is established.

Our multi-country DGE model is based on a strict interpretation of the representative household discussed in Kemp and Shimomura (1995). Specifically, we assume not only that the households in each country are identical but also that each household is aware of the fact.[2] In each country, therefore, households unanimously elect a government the sole duty of which is to choose a welfare-maximizing profile for the tax rate.

Section 2 sets up the DGE model. Section 3 characterizes the long-run growth rate and trade pattern along a balanced growth path. Section 4 discusses the reachability of a balanced growth path. Section 5 offers some concluding remarks.

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[2]Kemp and Shimomura (1995) discussed several interesting implications of the assumption.

2   The Model

We assume that there are N countries in the trading world. N could be a large number. The countries can be quite disparate in size. However we assume that there is no particularly large country among them so that each country regards the price of internationally tradable goods as given to it. Each country consists of identical households. In what follows, we describe the representative household and firms in one of the N countries.

2.1   The household

There are two types of produced goods, a pure consumption good (labeled 1) and a pure investment good (labeled 2). Each household derives utility by consuming the former good and maximizes the discounted sum of utility from the time-profile of consumption

$$\int_0^\infty \ln c \exp[-\rho t] dt$$

subject to the flow-budget constraint

$$\dot{K} = (1 - \tau)(rK + wGL) - pc \tag{1}$$

where r is the rental rate, τ is the rate at which income tax is imposed by the government, L is the time-invariant home labour supply which, without loss, is assumed to be unity, G is the state of labour efficiency per unit of labour (so that GL is the supply of efficiency labour) and w is the (efficiency) wage rate[3]. K is the capital stock owned by the household, and p is the price of the consumption good. The capital good serves as numeraire.

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[3] Ohdoi (2002) was the first to extend the Barro model to a small open two-good and two-factor framework by interpreting "infrastructure" as enhancing labour's efficiency.

Like Barro (1990), we assume that the government can augment G by spending the tax revenue on education. However, while Barro assumed that G is a flow variable, we follow Futagami, Morita and Shibata (1993) in assuming that it is a stock variable, the behaviour of which is described by the following dynamic equation

$$\dot{G} = \tau(rK + wG). \tag{2}$$

In what follows, we interpret (2) as a linear production function; the government (and only the government) can transform one unit of the capital good into one unit of "efficiency".

We deviate from Barro and Futagami *et al.* in the following point. While they assume that the tax rate is given to the households, we assume that the government determines the time profile of τ so as to maximize the discounted sum of utility of the representative household. This assumption is based on our strict interpretation of the representative household, already mentioned. Then, as long as they are rational, with perfect foresight, the households support a government tax policy that maximizes their welfare.

As already noted, it is assumed that each of the N countries is small enough that its government regards the international prices of tradable goods as given. All households and firms are small in the same sense.

The foregoing assumptions imply that the time profiles τ, c, K, and G are chosen to maximize the discounted sum of utility subject to (1) and (2), for historically given initial conditions of K and G and for given time profiles of w, r and p. Thus, the household's Hamiltonian is

$$H \equiv \ln c + \lambda[(1 - \tau)(rK + wG) - pc] + \mu\tau(rK + wG). \tag{3}$$

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We assume that there are no differences in preferences and production technologies among the N countries; however the initial factor endowments, $K(0)$ and $G(0)$, possibly differ.

2.2 Firms

Goods 1 and 2 are produced by means of capital and efficiency labour. The two goods are tradable but the productive services provided by the existing factors of production are non-tradable. The production function of each good is neo-classical and linearly homogenous in capital and efficiency labour:

$$Y_i = F^i(K_i, GL_i) \equiv GL_i f_i(K_i/GL_i).$$

The full employment conditions for the two factors of production at each point in time are

$$K = K_1 + K_2 \tag{4a}$$

$$1 = L_1 + L_2 \tag{4b}$$

The factor prices are derived from profit maximization. If production in a country is incompletely specialized, the following equations must be established.

$$p = \Lambda^1(r, w) \tag{5a}$$

$$1 = \Lambda^2(r, w) \tag{5b}$$

where $\Lambda^i(r, w)$, is the unit-cost function of good i, $i = 1, 2$. We assume that for any $p > 0$ (5) has a unique solution $(r(p), w(p))$, which is the case if, for example, technologies in both sectors are of Cobb-Douglas type.

3 Balanced growth paths and the long-run trade pattern

Let us identify the variables belonging to country j by attaching the superscript or subscript j to them. Under incomplete specialization, the Hamiltonian concerning the typical household in country j can be written as

$$H^j \equiv \ln c^j + \lambda^j [(1 - \tau^j)(r(p)K^j + w(p)G^j) - pc^j] + \mu^j \tau^j (r(p)K^j + w(p)G^j)$$

As we have already explained, not only c^j but also τ^j are control variables. Thus,

$$\dot{K}^j = (1 - \tau^j)(r(p)K^j + w(p)G^j) - pc^j \tag{6}$$

$$\dot{G}^j = \tau^j (r(p)K^j + w(p)G^j) \tag{7}$$

$$\frac{1}{c^j} = p\lambda^j \tag{8}$$

$$\tau^j = \begin{cases} 1, \text{ if } \mu^j > \lambda^j \\ \text{any value in } [0,1], \text{ if } \mu^j = \lambda^j \\ 0, \text{ if } \mu^j < \lambda^j \end{cases} \tag{9}$$

$$\dot{\lambda}^j = \rho\lambda^j - (1 - \tau^j)r(p)\lambda^j - \mu^j \tau^j r(p) \tag{10}$$

$$\dot{\mu}^j = \rho\mu^j - (1 - \tau^j)\lambda^j w(p) - \mu^j \tau^j w(p) \tag{11}$$

The world market-clearing condition is

$$r'(p)\sum_{i=1}^N K^i + w'(p)\sum_{i=1}^N G^i = \sum_{i=1}^N c^i \tag{12}$$

where $r'(p) \equiv dr(p)/dp$ and $w'(p) \equiv dw(p)/dp$. Thus, the left-hand side of (12) is the world supply of the consumption good.

Letting $k_j \equiv K^j/G^j$, $h_j \equiv 1/\lambda^j G^j$ and $z_j \equiv \mu^j/\lambda^j$ and integrating the above differential equations, we obtain

$$\dot{k}_j = (1 - \tau_j)(r(p)k_j + w(p)) - h_j - \tau_j k_j(r(p)k_j + w(p)) \tag{13a}$$

$$\dot{h}_j = h_j\{\tau_j r(p)z_j - [\rho - (1 - \tau_j)r(p) + \tau_j(r(p)k_j + w(p))]\} \tag{13b}$$

$$\dot{z}_j = -(1 - \tau_j)w(p) + [(1 - \tau_j)r(p) - \tau_j w(p)]z_j + \tau_j r(p)z_j^2 \tag{13c}$$

Definition 1: *A Balanced Growth Path (BGP) is defined as a world growth path such that all quantities, K^j, G^j and c^j, $j = 1, ..., N$, grow at a common, positive and constant rate.*

Considering (6), (7) and (9), we see that, along a *BGP*, τ^j, $j = 1, ..., N$, must satisfy $0 < \tau^j < 1$. It follows from (9) that, along a *BGP*, $z_j = 1$, $j = 1, ..., N$; hence (13c) reduces to

$$0 = \dot{z}_j = -w(p) + r(p)$$

Therefore, factor price equalization holds within a country, that is,

$$w(p) = r(p) \tag{14}$$

Assumption 1: (14) *has a unique and positive solution* $\bar{p}$.

Thus, along a *BGP*, the price is constant over time. From (13b), therefore,

$$r(\bar{p}) - \rho = \tau_j r(\bar{p})(k_j + 1)$$

or

$$\tau_j = \frac{r(\bar{p}) - \rho}{r(\bar{p})(k_j + 1)} \equiv \bar{\tau}_j \tag{15}$$

It follows that the *BGP* tax rate in country j, $j = 1, ..., N$, is between 0 and 1 and constant over time, if the following additional assumption is made.

Assumption 2: $r(\bar{p}) - \rho > 0$.

Next, let us check the *BGP* consumption of country j. Since $\dot{k}^j = 0$, (13a) implies that

$$h_j = \{1 - \bar{\tau}_j(k_j + 1)\}r(p)(k_j + 1) \tag{16}$$

The substitution of (15) into (16) yields

$$h_j = \rho(k_j + 1) \equiv \bar{h}_j \tag{17}$$

Finally, we must check the market-clearing condition (12). Letting

$$\theta_j \equiv \frac{G_j}{G_1 + \cdots + G_N}, \tag{18}$$

and considering (9), we can rewrite (12) as

$$pr'(p)\sum_{i=1}^{N}\theta_i k_i + pw'(p) = \sum_{i=1}^{N}\theta_i h_i \tag{19}$$

Note that $\sum_{i=1}^{N}\theta_i = 1$. Substituting $p = \bar{p}$ and $h_j = \bar{h}_j$ into (19), we obtain

$$\sum_{i=1}^{N}\theta_i k_i = \tilde{k} \equiv \frac{\bar{p}w'(\bar{p}) - \rho}{\rho - \bar{p}r'(\bar{p})} \tag{20}$$

Since $\sum_{i=1}^{N}\theta_i k_i = \sum_{i=1}^{N}K^i / \sum_{i=1}^{N}G^i$, (20) means that along a *BGP* the world factor endowment ratio takes on a specific value.

We are ready to prove the first main theorem.

PROPOSITION 1: *If the initial values of the state variables, $\theta_j(0)$ and $k_j(0)$, $j = 1, ..., N$, happen to satisfy (20) with $0 < \theta_j(0) < 1$ and $k_j(0) > 0$, then the world economy grows along a BGP right from the beginning, $t = 0$. All quantities grow at $r(\bar{p}) - \rho > 0$. The trade pattern of each country, say country j, is determined in the Heckscher-Ohlin manner, by the factor intensity ranking between the two goods and the sign of $(k_j(0) - k_{-j}(0))$, where*

$$
\begin{aligned}
k_{-j}(0) \ &\equiv \ \frac{\sum_{i=1,i\neq j}^{N} K^i(0)}{\sum_{i=1,i\neq j}^{N} G^i(0)} \\
&= \ \frac{\sum_{i=1,i\neq j}^{N} \theta_i(0)k_i(0)}{1 - \theta_j(0)}
\end{aligned}
$$

Proof: Consider (15), or

$$
r(\bar{p}) - \rho = \tau_j r(\bar{p})(k_j(0) + 1),
$$

where the righthand side is the growth rate of G. Thus the BGP growth rate must be $r(\bar{p}) - \rho$.

To help characterize the trade pattern of country j, let us define

$$
\begin{aligned}
h_{-j}(0) \ &\equiv \ \frac{\bar{p}\sum_{i=1,i\neq j}^{N} c^i(0)}{\sum_{i=1,i\neq j}^{N} G^i(0)} \\
&= \ \frac{\sum_{i=1,i\neq j}^{N} \theta_i(0)h_i(0)}{1 - \theta_j(0)} \quad (21)
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\sum_{i=1}^{N} \theta_i(0)h_i(0) \ &= \ \theta_j(0)h_j(0) + \sum_{i=1,i\neq j}^{N} \theta_i(0)h_i(0) \\
&= \ \theta_j(0)h_j(0) + (1 - \theta_j(0))h_{-j}(0) \quad (22)
\end{aligned}
$$

Similarly,

$$
\sum_{i=1}^{N} \theta_i(0)k_i(0) = \theta_j(0)k_j(0) + (1 - \theta_j(0))k_{-j}(0) \quad (23)
$$

Substituting (22) and (23) into (19), we obtain

$$\bar{p}r'(\bar{p})\{\theta_j(0)k_j(0) + (1 - \theta_j(0))k_{-j}(0)\} + \bar{p}w'(\bar{p})$$

$$= \theta_j(0)h_j(0) + (1 - \theta_j(0))h_{-j}(0) \qquad (24)$$

Since the world economy grows along a BGP from $t = 0$, the following equalities must be satisfied:

$$
\begin{aligned}
h_j(0) &= \rho(k_j(0) + 1) \\
h_{-j}(0) &= \frac{1}{1 - \theta_i(0)} \sum_{i=1, i \neq j}^{N} \theta_i(0)\rho(k_j(0) + 1) \\
&= \rho(k_{-1}(0) + 1)
\end{aligned}
$$

Hence (24) may be written as

$$\bar{p}r'(\bar{p})[\theta_j(0)k_j(0) + (1 - \theta_j(0))k_{-j}(0)] + [\theta_j(0) + (1 - \theta_j(0))]\bar{p}w'(\bar{p})$$

$$= \theta_j(0)h_j(0) + (1 - \theta_j(0))h_{-j}(0), \qquad (25a)$$

and

$$\bar{p}r'(\bar{p})\{\theta_j(0)k_j(0) + (1 - \theta_j(0))k_{-j}(0)\} + \bar{p}w'(\bar{p})$$

$$= \theta_j(0)\rho[k_j(0) + 1] + (1 - \theta_j(0))\rho[k_{-j}(0) + 1] \qquad (25b)$$

It follows from (25a) that

$$[h_{-j}(0) - (\bar{p}r'(\bar{p})k_{-j}(0) + \bar{p}w'(\bar{p}))]$$

$$= \frac{\theta_j(0)}{(1 - \theta_j(0))}[(\bar{p}r'(\bar{p})k_{-j}(0) + \bar{p}w'(\bar{p})) - h_j(0)],$$

and from (25b) that

$$\theta_j(0)[\bar{p}r'(\bar{p}) - \rho][k_j(0) - k_{-j}(0)]$$

$$= h_{-j}(0) - (\bar{p}w'(\bar{p}) + \bar{p}r'(\bar{p})k_{-j}(0))$$

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Combining these two equations, we have

$$\theta_j(0)[\bar{p}r'(\bar{p}) - \rho][k_j(0) - k_{-j}(0)]$$
$$= \frac{\theta_j(0)[(\bar{p}w'(\bar{p}) + \bar{p}r'(\bar{p})k_j(0)) - h_j(0)]}{1 - \theta_j(0)} \tag{26}$$

Thus, if the consumption good is capital-intensive, so that $\bar{p}r'(\bar{p})/r(\bar{p}) > 1 > \rho/r(\bar{p})$, and if country j is relatively capital-abundant, so that $k_j(0) > k_{-j}(0)$, then the country exports the consumption good. On the other hand, if the consumption good is labour-intensive, $r'(\bar{p}) < 0$, and if country j capital-abundant, then the consumption good is imported. (QED)

4   Transitional dynamics

In the previous section we studied the case in which the initial values of the state variables satisfy equality (20),

$$\sum_{i=1}^{N} \theta_i(0)k_i(0) = \tilde{k}$$

In this section, we derive an equilibrium growth path along which this equality does not hold.

To this end, let us first consider the following tax policy.

[1] The government of country j sets $\tau_j = 1$ as long as

$$\frac{\sum_{i=1}^{N} K^i(t)}{\sum_{i=1}^{N} G^i(t)} > \tilde{k}$$

holds.

[2] It jumps the tax rate to $\bar{\tau}_j$ defined in (15) when

$$\frac{\sum_{i=1}^{N} K^i(T)}{\sum_{i=1}^{N} G^i(T)} = \tilde{k},$$

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first holds, and thereafter maintains the tax rate at that level.

[3] For any j, $j = 1, ..., N$, the government of country j implements the tax policy [1]-[2].

Let us call [1]-[3] *the 100% tax rate policy.* If $\tau_j = 1$ for any $j = 1, ..., N$, (13) can be rewritten as

$$\dot{k}_j = -h_j - k_j(r(p)k_j + w(p)) \tag{13$'$a}$$

$$\dot{h}_j = h_j[r(p)z_j - \rho - (r(p)k_j + w(p))] \tag{13$'$b}$$

$$\dot{z}_j = z_j[r(p)z_j - w(p)] \tag{13$'$c}$$

On the other hand,

$$\dot{\theta}_s = \theta_s[(1 - \theta_s)(r(p)k_s + w(p)) - \sum_{i=1, i \neq s}^{N} \theta_i(r(p)k_i + w(p))] \tag{27}$$

for $s = 1, ..., N$. Moreover, we have the market-clearing condition

$$pr'(p) \sum_{i=1}^{N} \theta_i k_i + pw'(p) = \sum_{i=1}^{N} \theta_i h_i. \tag{19}$$

(13$'$a)-(13$'$c), (19) and (27) express the dynamic system in which all governments adopt the 100% tax rate policy.

Let us define the *BGP* set as

$$\Omega \equiv \left\{ \omega : \sum_{i=1}^{N} \theta_i = 1, \ (14), (17), (19), \theta_j \in (0, 1), k_j > 0, \ z_j = 1, \ j = 1, ..., N \right\}$$

where $\omega \equiv (\theta_j, k_j, h_j, z_j)_{j=1}^{N}$.

Take an arbitrary ω in Ω and denote by

$$\theta_j(t, \omega), \ k_j(t, \omega), \ h_j(t, \omega), \ z_j(t, \omega), \ p(t, \omega), \ j = 1, ..., N,$$

the solution to the above system of differential equations which passes through ω at $t = 0$.[4] Differentiating $\sum_{i=1}^{N} \theta_i(t, \omega) k_i((t, \omega))$ with respect to t at $t = 0$, we see that

$$
\begin{aligned}
\frac{d}{dt} \sum_{i=1}^{N} \theta_i(t, \omega) k_i((t, \omega)) \bigg|_{t=0} \\
= \sum_{i=1}^{N} [\dot{\theta}_i(0, \omega) k_i + \theta_i \dot{k}_i(0, \omega)] \\
= - \sum_{i=1}^{N} [k_i(r(\bar{p})\tilde{k} + \rho) + (r(\bar{p})k_i + \rho)] \\
< 0,
\end{aligned}
$$

where we use (13′a) and (27) evaluated at $t = 0$ and the fact that $\tilde{k} = \sum_{i=1}^{N} \theta_i k_i$. This result implies that

$$
\tilde{k} < \sum_{i=1}^{N} \theta_i(t, \omega) k_i((t, \omega)) \tag{28}
$$

for $t \doteqdot 0$ and < 0.

Next, let us consider the following tax policy.

[1a] The government of country j sets $\tau_j = 0$ as long as

$$
\frac{\sum_{i=1}^{N} K^i(t)}{\sum_{i=1}^{N} G^i(t)} < \tilde{k}
$$

holds.

[2a] It jumps the tax rate to $\bar{\tau}_j$ defined in (15) when

$$
\frac{\sum_{i=1}^{N} K^i(T)}{\sum_{i=1}^{N} G^i(T)} = \tilde{k},
$$

first holds, and thereafter maintains the tax rate at that level.

---

[4]We can prove from that for any $\omega \in \Omega$ and t

$$
\sum_{j=1}^{N} \theta_j(t, \omega) = 1
$$

[3a] For any j, $j = 1, ..., N$, country j adopts the tax policy [1a]-[2a].

We call [1a]-[3a] *the 0% tax rate policy.* Under this policy, (13) can be rewritten as

$$\dot{k}_j = (r(p)k_j + w(p)) - h_j \qquad (13''a)$$

$$\dot{h}_j = h_j(r(p) - \rho) \qquad (13''b)$$

$$\dot{z}_j = z_j r(p) - w(p) \qquad (13''c)$$

Therefore, $(13''a)$-$(13''c)$, (19) and (27) express the dynamic system in which all governments adopt the 0% tax rate policy. By a now-familiar argument, for any $\omega \in \Omega$

$$\tilde{k} > \sum_{i=1}^{N} \theta_i(t, \omega) k_i((t, \omega)) \qquad (29)$$

for $t \doteq 0$ and < 0.

Summarizing the foregoing argument, we arrive at the following lemma.

LEMMA 1: *For any point ω in the $4N$-dimensional set Ω, the solution path of the dynamic system,* $(13')$ *(resp.* $(13'')$*), (19) and (27), passing through ω at $t = 0$, say $\{(\theta_j(t, \omega), k_j(t, \omega), h_j(t, \omega), z_j(t, \omega))_{j=1}^{N},$* $p(t, \omega)\}$, *satisfies the following inequality*

$$\left. \frac{d}{dt} \sum_{j=1}^{N} \theta_j(t, \omega) k_j(t, \omega) \right|_{t=0} < (resp. >)0,$$

which implies that (28) (resp. (29) holds when t is negative but close to 0.

Based on this lemma, we can prove the second main result in this paper.

PROPOSITION 2: *Suppose that the initial world factor endowment ratio*

$$k(0) \equiv \frac{\sum_{i=1}^{N} K^i(0)}{\sum_{i=1}^{N} G^i(0)}$$

slightly differs from $\tilde{k}$. If $k(0)$ is greater (smaller) than $\tilde{k}$, the government of each country makes its tax rate one (zero) until $k(t)$ equals $\tilde{k}$. Once $k(T) = \tilde{k}$ at some T, then the tax rate of each country jumps to the level in (15) and, from $t = T$ on, the world economy grows along a BGP.

Proof: What we have to prove is that Lemma 1 implies the proposition. See Appendix.

5   A concluding remark

We have assumed in this paper that all countries are so small that each of them regards itself as a "price taker" in the international market place. We hope to extend this study to accommodate large countries which recognize their market power and take it into account when they implement their tax policies.

Appendix: A proof of Proposition 2

Local reachability of an equilibrium path to the *BGP* set

Let us focus on the case $\tilde{k} < k(0)$, under which all governments implement the 100% tax policy. We first prove the following lemma.

LEMMA A1: *Define*

$$V(\eta) \equiv \left\{ (k_j, \theta_j)_{j=1}^{N} > (0,0)_{j=1}^{N} : \tilde{k} \le \sum_{j=1}^{N} \theta_j k_j \le \tilde{k} + \eta, \ \sum_{j=1}^{N} \theta_j = 1 \right\} \tag{a1}$$

Then, there is a sufficiently small $\eta > 0$ such that for any $v \in V(\eta)$ there exists a 2N-dimensional vector $\gamma \equiv (h_j, z_j)_{j=1}^{N}$ such that the solution path of the dynamic system, (13$'$), (19) and (27), passing through

(v, γ) at $t = 0$ *arrives at the BGP set* Ω *in some finite time, say* T. *That is, the following equality holds*

$$\tilde{k} = \sum_{j=1}^{N} k_j(T, (v, \gamma)) \theta_j(T, (v, \gamma)) \tag{a2}$$

Proof: Suppose the lemma is not true. Then, for any $\eta > 0$, there is $v \in V(\eta)$ such that for any γ

and $t \geq 0$ the solution path of the dynamic system (13′), (19) and (27), passing through (v, γ) at $t = 0$,

satisfies

$$\sum_{j=1}^{N} k_j(t, (v, \gamma)) \theta_j(t, (v, \gamma)) > \tilde{k}. \tag{a3}$$

Since γ can be chosen arbitrarily, we assume that

$$\gamma = (\rho(k_j + 1), \breve{z}_j)_{j=1}^{N}, \tag{a4}$$

where k_j, $j = 1, ..., N$, are the same as in the vector $v = (k_j, \theta_j)_{j=1}^{N}$ and $\breve{z}_j \equiv 1$, $j = 1, ..., N$.

Consider a monotone decreasing sequence

$$\eta_1 > \eta_2 > \cdots > \eta_n > \cdots \geq 0,$$

such that $\lim_{n \to \infty} \eta_n = 0$. If the lemma is not true, there is a sequence,

$$(v^1, \gamma^1), \ (v^2, \gamma^2), \ \cdots, \ (v^n, \gamma^n), \ \cdots$$

in which v^n corresponds to η_n and γ^n is defined by (a4), such that

$$\sum_{j=1}^{N} k_j(t, (v^n, \gamma^n)) \theta_j(t, (v^n, \gamma^n)) > \tilde{k} \tag{a6}$$

for any $t \geq 0$ and $n = 1, 2, \cdots$, and

$$\sum_{j=1}^{N} k_j(t, (v^n, \gamma^n)) \theta_j(t, (v^n, \gamma^n)) \to \tilde{k} \quad (n \to \infty), \tag{a7}$$

for any given $t > 0$. Notice that (a7) implies that, for $n \to \infty$, the point (v^n, γ^n) approaches the *BGP*

set Ω.

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Since

$$\lim_{t \to 0} \frac{\sum_{j=1}^{N} k_j(t, (v^n, \gamma^n))\theta_j(t, (v^n, \gamma^n)) - \tilde{k}}{t} \geq 0, \tag{a8}$$

we have, letting $n = [1/t]$,

$$\frac{d}{dt} \sum_{j=1}^{N} k_j(t, (v^\infty, \gamma^\infty))\theta_j(t, (v^\infty, \gamma^\infty))|_{t=0} \geq 0. \tag{a9}$$

This inequality is incompatible with

$$\frac{d}{dt} \sum_{j=1}^{N} \theta_j(t, \omega)k_j(t, \omega)\bigg|_{t=0} < 0,$$

which must hold when $k(0)$ is greater than $\tilde{k}$, as stated in Lemma 1. Therefore, a contradiction emerges, which means that the lemma is true.

Applying a parallel argument to the other case, in which $k(0)$ is smaller than $\tilde{k}$, the reachability of BGP can again be demonstrated. We omit the proof. (QED)

Justifying the tax policy in the transition process

Case 1. $k(0) > \tilde{k}$

Under the 100% tax rate policy and at an arbitrary point in the BGP set Ω, we have

$$\dot{k}_j = -(k_j + 1)[\rho + k_j r(\bar{p})],$$

$$\dot{h}_j = -\rho(k_j + 1)[\rho + k_j r(\bar{p})],$$

$$\dot{z}_j = 0,$$

from $(13')$ for $j = 1, 2, \cdots, N$. On the other hand, from (7) and (18),

$$\dot{\theta}_j = \frac{1}{(\sum G_i)^2}[(\sum G_i)\dot{G}_j - G_j(\sum \dot{G}_i)]$$

$$= \theta_j \frac{\dot{G}_i}{G_i} - \theta_j \sum_i \theta_i \frac{\dot{G}_i}{G_i}$$

$$= \theta_j [(rk_j + w) - \sum_i \theta_i (rk_i + w)]$$

$$= \theta_j r(\bar{p})(k_j - \tilde{k}).$$

As we showed in the previous section, this dynamic system under the 100% tax rate policy reaches the BGP set within finite time. However, we have not yet proved that for any $j = 1, ..., N$, and for any $t \in [0, T)$, $z_j(t)$ is not less than one. We must prove it; otherwise the dynamic system would not satisfy the necessary condition (9). In what follows we shall prove that, for any $j = 1, ..., N$ and for any $t \in [0, T)$, $z_j(t) > 1$.

First of all, we obtain a preliminary result concerning the sign of $\dot{p}(t)$. Differentiating (19) with respect to time, we obtain

$$\Delta \dot{p} + pr' \sum \theta_j \dot{k}_j - \sum \theta_j \dot{h}_j + pr' \sum k_j \dot{\theta}_j - \sum h_j \dot{\theta}_j = 0, \tag{a10}$$

where

$$\Delta \equiv \sum_{j=1}^{N} \theta_j [r'(p)k_j + w'(p)] + p \sum_{j=1}^{N} \theta_j [r''(p)k_j + w''(p)]$$

$$> 0.^5$$

Evaluating (a10) at the point in Ω, we see that, along the solution path of (13'),

$$\Delta \dot{p} = -pr'(p) \sum \theta_j \dot{k}_j + \sum \theta_j \dot{h}_j - [pr'(p) - \rho] \sum k_j \dot{\theta}_j + \rho \sum \dot{\theta}_j$$

$$= pr' \sum \theta_j [\rho + k_j r(p)](1 + k_j) - \sum \theta_j \rho (k_j + 1)[\rho + k_j r(p)]$$

$$- [pr'(p) - \rho] \sum k_j \theta_j r(p)[k_j - \tilde{k}]$$

---

[5] Note that $[r'(p)k_j + w'(p)]G$ is the supply of the consumption good in country j and $[r''(p)k_j + w''(p)]G$ is the slope of the supply curve. Following the standard duality approach, we know that both terms in Δ are positive under incomplete specialization.

$$= \quad [pr'(p) - \rho]\{\sum \theta_j(k_j + 1)[\rho + k_j r(p)] - \sum k_j \theta_j r(p)(k_j - \tilde{k})\}$$

$$= \quad [pr'(p) - \rho](\rho \sum \theta_j(k_j + 1) + r(p)\{\sum \theta_j(k_j + 1)k_j$$

$$- \sum \theta_j k_j[k_j - \tilde{k}]\})$$

$$= \quad [pr'(p) - \rho]\{\rho \sum \theta_j(k_j + 1) + r(p)(\sum \theta_j k_j + \sum \theta_j k_j \tilde{k})\}.$$

Thus we may record the preliminary result

$$\text{sign}[\dot{p}] = \text{sign}[pr'(p) - \rho]. \tag{a11}$$

We can now provide the required proof. Suppose $r'(p) > 0$. Then, by virtue of (a11) and the Stolper-Samuelson Theorem that $\dot{p} > 0$, $p < \bar{p}$ before the solution to the above dynamic system, starting from $(k_j(0), \theta_j(0))_{j=1}^N$ in a neighborhood of Ω with $k(0) > \tilde{k}$, strikes the BGP set. It follows that, for any $t \in [0, T)$, $w(p)/r(p) > 1 = w(\bar{p})/r(\bar{p})$.

Having this result in mind, let us consider

$$\dot{z}_j(t) = z_j(t)r(p(t))[z_j(t) - w(p(t))/r(p(t))] \tag{13'c}$$

Depict the phase diagram of this differential equation for any $t \in [0, T)$. As shown in Figure A, the above result means that the intersection E is always larger than one for any $t \in [0, T)$. Then, (13'c) implies that $z_j(t) > 1$ also for any $t \in [0, T)$. For, if not, that is, if there exists $t^* \in [0, T)$ such that $z_j(t^*) \leq 1$, then $z_j(t)$ would not be able to reach its BGP value 1 at any finite time $T > 0$, which violates the reachability to Ω. Therefore, $z_j(t) > 1$ also for any $t \in [0, T)$.

On the other hand, if $r'(p) < 0$, (a11) implies that $\dot{p} < 0$. Then, by an argument parallel to that of the preceding paragraph, we see that $p > \bar{p}$ before the solution, starting from the initial condition in a

neighborhood of Ω with $k(0) > \tilde{k}$, strikes the BGP set. Thus, we again have $w(p)/r(p) > 1$, which again implies $z_j(t) > 1$ for any $t \in [0, T)$ and for any $j = 1, ..., N$.

Case 2. $k(0) < \tilde{k}$

Under the 0% tax rate policy and at an arbitrary point in the BGP set Ω, we have

$$\dot{k}_j = (r(\bar{p}) - \rho)(k_j + 1)$$

$$\dot{h}_j = \rho[r(p) - \rho](k_j + 1)$$

$$\dot{z}_j = 0$$

$$\dot{\theta}_j = r(p)\theta_j(1 - \theta_j)(k_j - k_{-j}),$$

Thus, along a solution path of $(13'')$ starting from a point in the BGP set, we have

$$
\begin{aligned}
\Delta\dot{p} &= -pr'(p)\sum\theta_j\dot{k}_j + \sum\theta_j\dot{h}_j - [pr'(p) - \rho]\sum k_j\dot{\theta}_j + \rho\sum\dot{\theta}_j \\
&= -pr'\sum(\theta_j + 1)[r(p) - \rho] - \sum\theta_j\rho(k_j + 1)[r(p) - \rho] \\
&\quad -[pr'(p) - \rho]\sum k_j\theta_j r(p)[k_j - \tilde{k}] \\
&= [\rho - pr'(p)][(r(p) - \rho)\sum\theta_j(k_j + 1) + r(p)\sum\theta_j k_j(k_j - \tilde{k})],
\end{aligned}
$$

from (19) and the BGP conditions (14), (17) and (20). Since

$$
\begin{aligned}
\sum\theta_j k_j(k_j - \tilde{k}) &= \sum[\theta_j k_j^2 - \theta_j k_j\tilde{k}] = \sum\theta_j k_j^2 - \tilde{k}^2 \\
&= \sum\theta_j[k_j^2 - 2k_j\tilde{k} + \tilde{k}^2] \\
&= \sum\theta_j[k_j - \tilde{k}]^2 \geq 0,
\end{aligned}
$$

we have

$$\text{sign}[\dot{p}] = -\text{sign}[pr'(p) - \rho]. \tag{a12}$$

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Since for an initial point $(k_j(0), \theta_j(0))_{j=1}^N$ in a neighborhood of the BGP set Ω with $k(0) < \tilde{k}$, the reachability to the BGP set ensures that there is $T' < \infty$ such that the solution path of $(13'')$ starting from the initial point will strike the BGP set at T'. Then, by an argument similar to that provided for Case 1, we have $w(p)/r(p) < 1$ for any $t \in [0, T')$ whether $r'(p)$ is positive or negative. Inspecting the moving equation $(13''c)$, we can prove that $z_j(t) < 1$, for any $j = 1, 2, \cdots, N$ and any $t \in [0, T')$. It follows that, when the initial point satisfies $k(0) < \tilde{k}$, the 0% tax rate policy is an equilibrium tax policy for each country during the transition process.

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