Discussion Paper Series

RIEB

Kobe University

DP2015-33

Geography and Demography: New Economic Geography with Endogenous Fertility

Hiroshi GOTO
Keiya MINAMIMURA

July 27, 2015

Research Institute for Economics and Business Administration
Kobe University
2-1 Rokkodai, Nada, Kobe 657-8501 JAPAN

Geography and Demography: New Economic Geography with Endogenous Fertility☆,☆☆

Hiroshi Goto*

Research Institute for Economics & Business Administration, Kobe University, 2-1, Rokkodai-cho, Nada-ku, Kobe, Hyogo, 657-8501, Japan

Keiya Minamimura**

Graduate School of Economics, Kobe University, 2-1, Rokkodai-cho, Nada-ku, Kobe, Hyogo, 657-8501, Japan

Abstract

To explain the links between population distribution and economic integration, we construct a spatial economics model with endogenous fertility. A higher population concentration increases real wages and child-raising costs, thus lowering the fertility rate. However, people migrate to more populated regions to obtain higher real wages. We show that mobility across regions results in more people flowing into highly populated regions, but lowers fertility rates there. The population growth path resembles a logistic curve in the early phase, but population decreases in the last phase. Additionally, economic integration leads to population concentration and decreases population size in the whole economy.

JEL classification: F15, R12, R23

Keywords: Population change, Agglomeration, Migration, Trade, Economic integration

☆This article is a revised edition of Goto and Minamimura [Goto, H., Minamimura, K. (2015) Fertility, regional demographics, and economic integration. *RIEB Discussion Paper Series*, 2015-17.].

☆☆We thank to many useful comments by Kazuhiro Yamamoto and Yasuhiro Sato in the 27th Annual Conference of ARSC at Kyoto University and the 8th Macro Economics Conference for Young Professionals. We also thank to Nobuaki Hamaguchi, Tamotsu Nakamura, Noritsugu Nakanishi, Yoichi Matsubayashi, Omer Moav, Daishin Yasui, Dao-Zhi ZENG. Needless to say, any errors remaining in this article are the responsibility of the authors.

*TEL: +81-78-803-7270, FAX: +81-78-803-7059, E-mail: hgoto@rieb.kobe-u.ac.jp (Corresponding)

**TEL: +81-78-803-7247, FAX: +81-78-803-6877, E-mail: keiya.minami@gmail.com

1. Introduction

Population change is a traditional issue in economics. As early as around the turn of the 19$^{\text{th}}$ century, Malthus (1798) pointed out that population growth is curbed by the power of land to provide human subsistence.[1] Since then, economists have supposed that fertility and migration depend on income, commodity prices, levels of economic development, and so on. For example, many theoretical macroeconomic studies focus on the relationship between economic growth and fertility. Most such studies find that a negative relationship exists.[2] Another example is New Economic Geography (NEG), which shows that real income tends to be high in regions with large markets, and hence, the population tends to concentrate in these regions because of this potential for higher real income.[3]

However, in many cases, they focused on only country-level population change (i.e., they ignored migration) or migration (i.e., they ignored fertility). In other words, thus far, natural population change induced by fertility has been analyzed independently of social population change induced by migration. However, nowadays, people as well as goods can move among regions much more easily; this is called economic integration. Nevertheless, the majority of economic studies that address population growth have neglected this aspect. On the other hand, approaches that address economic integration (e.g., NEG) have largely ignored population growth, although population size is not constant over time. To fill this gap, in this article, we propose a model that considers not only natural population change but also social population change. Then, we set out to explore the relationship between economic integration and population change.

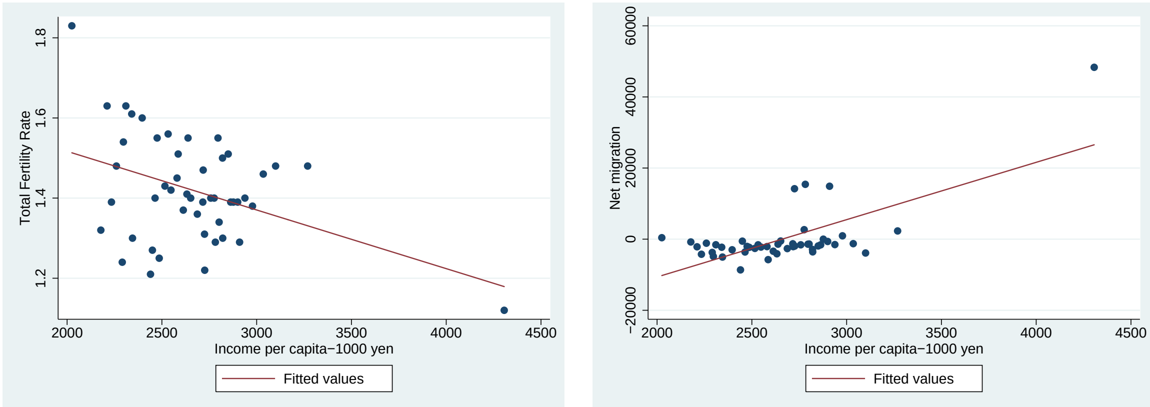
To construct the model, we must address three important facts. First, in Figure 1(a), we plot total fertility rates of Japan's 47 prefectures in 2010 according to per capita income.[4]

[1]Malthus was the first to point out that the process of population growth takes the form of a logistic curve. Even though our model does not consider land, the model shows that the population growth path of the whole economy takes the form of a logistic curve.

[2]Galor and Weil (1996) argue that wages for women are increasing as economic growth progresses, which raises the opportunity cost for child-rearing and decreases the fertility rate. Becker et al. (1990) propose that people invest more in human capital and have fewer children with advancing economic growth.

[3]For details on NEG, see Fujita et al. (1999).

[4]We use the Ministry of Health, Labour and Welfare (2014) and Statistics Bureau (2013a, 2013b) as the



(a) Total Fertility Rate (b) Net migration

Figure 1: Total Fertility Rate and Net Migration in Japan in 2010

Regions with higher per capita income tend to have lower total fertility rates.[5] On the other hand, net migration tends to be higher in regions with higher per capita income (see Figure 1(b)). Thus, higher per capita income may have two opposite effects on regional population changes: lower fertility and higher net migration.

The second important fact is that the population distribution across regions has been changing over time. The dash line of Figure 2 describes the Gini concentration ratio for Japan for 1947–2010, which shows an upward trend in this period (we can see the same trend in other countries).[6] This trend means that unequal population distribution across regions becomes larger over time and that people have congregated in particular regions (e.g., Tokyo).

Finally, even though the total fertility rate differs among regions, its change has a certain

data sources for Figure 1, 2, and 3.

[5]Someone may concern that some outliers determine the relationship between income per capita and total fertility rate. However we confirmed that this relationship holds even if we exclude these outliers. We can say similar things for Figure 1(b) and Figure 3.

[6]The Gini concentration ratio is derived using the Lorenz curve, which plots the proportion of the total population (on the vertical axis) that is cumulatively held in the total inhabitable area of regions (horizontal axis). Note that the area share is measured by ordering regions according to population density. Here, we use Japan's 47 prefectures as regions, but Okinawa is excluded before 1972. Before 1975, inhabitable area data are not available, so we use 1975 data for inhabitable area before 1975.

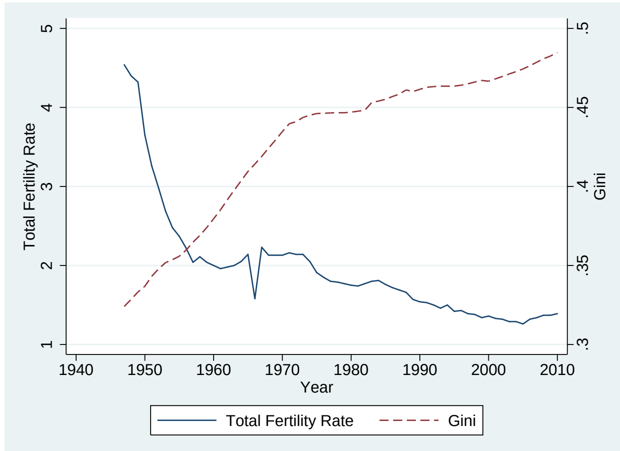


Figure 2: Gini Concentration Ratio and Total Fertility Rate in Japan from 1947 to 2010

tendency at the country level: it has been declining over time. The solid line of Figure 2 shows the total fertility rate of Japan in 1947–2010. It clearly indicates a negative trend of the total fertility rate in this period.[7] In particular, we should note that this decline in the total fertility rate seems to be associated with population concentration. Schultz (1985) shows that progress of urbanization reduces the fertility rate. Kondo (2015) examines how agglomeration affects fertility and finds that higher population density discourages child birth.

The model that we construct in this article should be able to explain these facts. It must consist of multiple regions (at least two) and take into account migration, fertility, and trade. Only a few studies address this issue. For example, Sato and Yamamoto (2005) explore how demographic and urbanization transition interact with each other considering agglomeration economies and congestion diseconomies as externalities. They show that in association with declines in the child mortality rate, there are advances in urbanization, declines in fertility, and an inverted U-shaped demographic transition. Sato (2007) develops a two-period overlapping generations model, which consists of multiple regions showing that the existence of

[7] The total fertility rate declined sharply in 1966. This is because 1966 was a *bingwu* year according to the Chinese calendar; many East Asian people believe that children born in such years will have a bad personality.

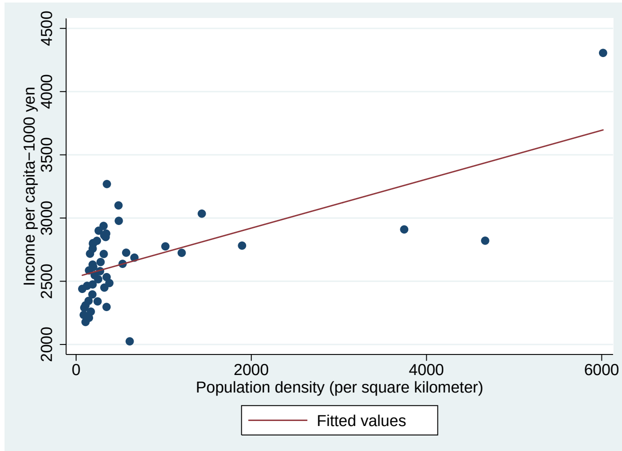


Figure 3: Population Density and Income per Capita (in Thousands of Yen) in Japan in 2010

agglomeration economies and congestion diseconomies leads to a negative relationship between fertility and regional population density. In particular, Sato (2007) shows population inflows to regions with higher population density in a steady state. Although these results are consistent with real-world evidence, they do not address economic integration because of the absence of spatial friction, like transportation costs. To address economic integration, in this article, we construct a basic NEG model with endogenous fertility.

The NEG framework has clear advantages. It can expressly consider spatial friction and effects of economic integration. In addition, it can explain why the distribution of population and economic activities among regions is radically uneven, without handling the agglomeration economies and congestion diseconomies as a black box like externalities. As already mentioned, in the NEG model, real income tends to be high in highly populated regions and people migrate to these regions to earn higher real income. Figure 3 describes the relationship between population density and per capita income in Japan in 2010. This shows that highly populated regions tend to offer higher incomes. Moreover, as illustrated in Figure 1(b), net migration tends to be higher in regions with higher real income. These facts justify use of the NEG model to describe regional population change.

To endogenize fertility, we employ a framework introduced by Becker (1965) that considers a time allocation problem between working and child-rearing in which parents obtain utility from the number of children. When substitutability between consumption goods and children is strong, a rise in the real wage reduces the fertility rate by increasing the opportunity cost of rearing children relative to the price of consumption goods.[8] In fact, Docquier (2004) and Jones and Tertilt (2008) show a negative relationship between income and the fertility rate in the United States. Borg (1989) finds the same relationship in Korea. Note that the NEG model employs monopolistic competition of the Dixit and Stiglitz (1977) type. Then, population growth expands the variety of consumption goods, which raises the real wage, and thus, reduces the fertility rate. This mechanism is proposed originally by Maruyama and Yamamoto (2010), but they do not address regions. We expand this model to the NEG framework.

Then, using this model, we analyze the regional population change, focusing on the effects of economic integration (i.e., higher migration and trade freeness among regions). We show that if people cannot migrate between regions, regional differences related to the initial population disappear in the long run. This result is contrary to the aforementioned facts. On the other hand, if migration is permitted, we obtain quite different results. Even though there are only subtle differences between regions, these differences become sufficiently large through migration with a snowball effect as the population concentrates in a region with an initially larger population share. Moreover, the region in which the population is concentrated has a higher real income, which results in a decreased fertility rate and increased net migration compared to less concentrated regions. Thus, in the long run, regions exhibit differences in population change and real income. Typically, higher migration and trade freeness bring about a more concentrated population, which leads to more regional

[8] Becker and Lewis (1973) consider the trade-off between quantity and quality of children. They argue that observed correlation between income and fertility may be negative in general. Moav (2005) shows that more educated parents have higher income and fewer children, and invest more in their children's education. The key reason why high-income families have fewer children is that they have high costs for each child. This point is not different to our model, while it excludes the quality of children for simplicity. The other specification of endogenous fertility is studied by Becker and Barro (1988) and Barro and Becker (1989), in which altruistic parents maximize dynastic utility. Shoven (2008) undertakes a survey for endogenous fertility.

differences. Additionally, as population concentration lowers the fertility rate in large regions, the population in the whole economy is suppressed. These results are consistent with the facts and imply that economic integration has a huge impact on population change in regional economies as well as the whole economy.

The remainder of this article is organized as follows. First, in the following Section 2, we construct a basic model without time and generations. Then, we discuss agglomeration force and spatial equilibrium in Section 3. In Section 4, we present an extension of the model that introduces time and generations for demographic analysis. Numerical simulations are conducted in Section 5 with several examples. Finally, Section 6 concludes.

2. The Basic Model

We start out by constructing a basic model without time and generations. Consider an economy with a finite set of regions, R (the number of regions is r). The economy consists of one differentiated goods sector characterized by monopolistic competition following Dixit and Stiglitz (1977).

2.1. Preference and Demand

All individuals gain utility from the consumption of a composite of differentiated goods, X, and their number of children, n. They share the same preference represented by the following utility function:

$$U = \left[\alpha X^{\rho} + (1-\alpha) n^{\rho}\right]^{\frac{1}{\rho}}, \qquad 0 < \alpha < 1, \tag{1}$$

where ρ is the substitution parameter, and σ ($\equiv 1/(1-\rho)$) represents the elasticity of substitution between the composite differentiated goods and the number of children. α represents the intensity of the preference for the consumption of differentiated goods. When ρ is close to zero, the utility function is close to the Cobb–Douglas form and α becomes the expenditure share of differentiated goods.

The composite index X takes the form of a CES function defined over a continuum of varieties of differentiated goods. Taking $x(\gamma)$ and Γ as the consumption of each available variety γ and the set of available varieties respectively, X is given by

$$X \equiv \left[\int_{\Gamma} x(\gamma)^{\rho_X} \mathrm{d}\gamma\right]^{\frac{1}{\rho_X}}, \quad 0 < \rho_X < 1,$$

where ρ_X is the substitution parameter for variety in differentiated goods and σ_X ($\equiv 1/(1-\rho_X)$) is the elasticity of substitution between any two varieties. A smaller ρ_X (i.e., a smaller σ_X) means that differentiated goods are more highly differentiated or that individuals have a stronger preference for variety. We assume $\sigma < \sigma_X$, which simply requires that substitutability between varieties of consumption goods is stronger than substitutability between consumption goods and children.

Individuals have one unit of time. They allocate this time to working and rearing children, while a positive constant time b must be spent to rear a child. Then, given the wage rate w_i in region i and price $p_{ji}(\gamma)$ for each variety that is produced in region j and sold in region i, the budget constraint of individuals in region i becomes

$$\sum_{j \in R} \left(\int_{\Gamma_j} p_{ji}(\gamma) x(\gamma) \mathrm{d}\gamma\right) \leqq w_i(1 - bn), \quad i \in R,$$

where Γ_j is the set of varieties produced in region j. The measure of Γ_j denoted by N_j is interpreted as the number of varieties produced in region j.

Solving the utility maximization problem, individual demand for both the composite index and for children in region i are given by

$$\begin{aligned} X_i = X(w_i, P_i) &\equiv \frac{\mu(w_i, P_i) w_i}{P_i}, \quad i \in R, \\ n_i = n(w_i, P_i) &\equiv \frac{1 - \mu(w_i, P_i)}{b}, \quad i \in R, \end{aligned} \tag{2}$$

where P_i is the price index for differentiated goods in region i, which is defined by

$$P_i \equiv \left[\sum_{j \in R} \left(\int_{\Gamma_j} p_{ji}(\gamma)^{1-\sigma_X} \mathrm{d}\gamma\right)\right]^{\frac{1}{1-\sigma_X}}, \quad i \in R, \tag{3}$$

and $\mu(\cdot)$ ($\equiv PX/w$) is the working time, which is given by

$$\mu(w_i, P_i) \equiv \frac{\beta P_i^{1-\sigma}}{\beta P_i^{1-\sigma} + (1-\beta)(b w_i)^{1-\sigma}} = \frac{\beta}{\beta + (1-\beta)(b w_i / P_i)^{1-\sigma}}, \quad i \in R,$$

where $\beta = \alpha^\sigma / (\alpha^\sigma + (1-\alpha)^\sigma)$. Then, individual demand in region i for variety γ produced in region j can be written as

$$x_{ji}(\gamma) = x(w_i, p_{ji}(\gamma), P_i) \equiv \frac{\mu(w_i, P_i)w_i}{P_i}\left(\frac{P_i}{p_{ji}(\gamma)}\right)^{\sigma_X}, \quad \gamma \in \varGamma_j,\ j, i \in R. \tag{4}$$

Let us denote the real wage in region i as ω_i ($\equiv w_i/P_i$). A rise in the real wage has two opposite effects on the number of children per individual. Clearly, because children are superior goods, the rise in the real wage increases the number of children (income effect). However, it also increases the opportunity costs of having them, which reduces the number of children (substitution effect). If the elasticity of substitution between the composite differentiated goods and the number of children, σ, is larger than one, this latter effect outweighs the former, and thus, a rise in the real wage reduces the number of children.

Substituting (2) into (1), we can express the maximized utility as a function of the real wage as follows:

$$V_i = V(\omega_i) \equiv \left[\frac{\omega_i^{1-\sigma}}{\alpha^\sigma + (1-\alpha)^\sigma (b\omega_i)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}, \quad i \in R.$$

Obviously, V_i increases with respect to ω_i. In Section 4, we assume that individuals move to the regions where they can gain higher utility, which is equivalent to assuming that individuals move to the regions where they can earn higher real wages.

2.2. Production

Next, we turn to the production side of the economy. Each variety γ of differentiated goods is produced by a monopolistic firm indexed by γ. All firms in all regions have the same increasing-returns technology, which uses labor only, with a fixed input of f and marginal input of a. Then, the labor input requirement for the production of a quantity $q_j(\gamma)$ of any variety $\gamma \in \varGamma_j$ at any given region j is given by $l_j(\gamma) = f + aq_j(\gamma)$.

We represent transport costs incurred for the shipment of goods between regions by *iceberg* form: that is, even if τ_{ji} ($\geqq 1$) units of any variety of differentiated goods are shipped from region j to region i, only one unit actually arrives.[9] We assume that intra-regional

[9]The iceberg form of transport costs is first introduced by Von Thünen (1826) and then formalized by Samuelson (1952).

transport costs are zero and inter-regional transport costs are symmetric, that is, $\tau_{jj} = 1$ and $\tau_{ji} = \tau_{ij}$ for any $i, j \in R$.

Given the transportation technology, the output of firm $\gamma \in \mathit{\Gamma}_j$ in region $j \in R$ is equal to $q_j(\gamma) = \sum_{i \in R} \tau_{ji} x_{ji}(\gamma) L_i$, where L_i is the number of individuals (workers) in region i. Then, with given prices in each region, firm γ's profit in region j is given by

$$\pi_j(\gamma) = \sum_{i \in R} p_{ji}(\gamma) x_{ji}(\gamma) L_i - w_j \left(f + a \sum_{i \in R} \tau_{ji} x_{ji}(\gamma) L_i \right), \quad \gamma \in \mathit{\Gamma}_j,\ j \in R.$$

Given the demand for each variety (4), each firm γ chooses its prices, $p_{ji}(\gamma)$, to maximize profit, which leads to a well-known pricing rule as follows:

$$p_{ji}(\gamma) = p_{ji} \equiv \frac{a}{\rho_X} \tau_{ji} w_j, \quad \gamma \in \mathit{\Gamma}_j,\ i, j \in R. \tag{5}$$

This yields the following maximized profit of firm γ in region j:

$$\pi_j(\gamma) = w_j \left[\frac{a q_j(\gamma)}{\sigma_X - 1} - f \right], \quad \gamma \in \mathit{\Gamma}_j,\ j \in R.$$

Allowing free entry and exit must drive the profits of any surviving firm to zero. Thus, we have the equilibrium output, $q^* = (\sigma_X - 1) f / a$, which yields the associated equilibrium labor input, $l^* = \sigma_X f$. Obviously both q^* and l^* are constant common to all surviving firms in all regions. Because the labor supply per individual in region j is $\mu(w_j, P_j)$, the equilibrium number of firms located in region j is given by

$$N_j^* \equiv \frac{\mu(w_j, P_j) \lambda_j L}{f \sigma_X}, \quad j \in R, \tag{6}$$

where L is the total number of workers and λ_j ($\equiv L_j / L$) is the share of workers in region j.

2.3. Market Equilibrium

We now describe the market equilibrium. Substituting pricing rule (5), we rewrite price index (3) in the following form:

$$P_i = \frac{a}{\rho_X} \left[\sum_{j \in R} \phi_{ji} N_j w_j^{1 - \sigma_X} \right]^{\frac{1}{1 - \sigma_X}}, \quad i \in R, \tag{7}$$

where ϕ_{ji} ($\equiv \tau_{ji}^{1 - \sigma_X} \in (0, 1]$) is a measure of the freeness of trade from region j to region i, which rises as τ_{ji} falls and is equal to one under free trade (i.e., $\tau_{ji} = 1$). Thus, from (6), we can express region i's price index as a function of $\boldsymbol{\phi}_i = (\phi_{1i}, \cdots, \phi_{ri})$, L, $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_r)$, $\boldsymbol{w} = (w_1, \cdots, w_r)$ and $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_r)$ as follows:

$$P_i = P(\boldsymbol{\phi}_i, L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu}) \equiv \frac{a}{\rho_X}\left[\frac{L}{f\sigma_X}\sum_{j\in R}\phi_{ji}\mu_j\lambda_j w_j^{1-\sigma_X}\right]^{\frac{1}{1-\sigma_X}}, \quad i \in R, \tag{8}$$

where $\mu_j = \mu(w_j, P_j)$. This equation exhibits increasing returns to scale at the economy level. Suppose that $\boldsymbol{\mu}$, $\boldsymbol{\lambda}$, and $\boldsymbol{w}$ are constant. Then, it is apparent that increasing the total number of workers reduces the price indexes in all regions, simply because the total number of varieties increases. This brings about higher real wages in all regions.

Note that the condition that firms make positive or negative profits is equivalent to the condition that their output becomes larger or smaller than q^*. In addition, any surviving firm in region i must allocate its output to consumers in each region as $\sum_{j\in R}\tau_{ij}x_{ij}(\gamma)L_j = q$. Thus, using (4) and (5), we have the following relationship:

$$\pi_i \gtreqless 0 \iff w_i \lesseqgtr w_i^P \equiv \left(\frac{a}{\rho_X}\right)^{\frac{1-\sigma_X}{\sigma_X}}\left[\frac{1}{\sigma_X f}\sum_{j\in R}\phi_{ij}Y_j P_j^{\sigma_X-1}\right]^{\frac{1}{\sigma_X}}, \quad i \in R, \tag{9}$$

where Y_j ($\equiv w_j\mu_j L_j$) is the income of region j. Given the income levels, price indexes, and trade freeness, w_i^P gives the wage rate that firms can afford to pay in region i. In the short run, w_i^P need not be equal to region i's actual wage rate, w_i. When w_i^P is higher than w_i, firms in region i can earn rent as they are protected from competition with other firms. In this case, however, through the entry of firms, w_i must be adjusted to w_i^P, which cancels out the rent in the long run. In this sense, we call w_i^P the *Wage Potential* in region i; this is the wage rate that workers potentially can earn in region i. Of course, $w_i = w_i^P$ is satisfied in equilibrium, which is what Fujita et al. (1999) call the *wage equation*.

Substituting (8) and $Y_j = w_j\mu_j\lambda_j L$, region i's wage potential can be written as a function of $\boldsymbol{\phi}_1$, $\cdots$, $\boldsymbol{\phi}_r$, $\boldsymbol{\lambda}$, $\boldsymbol{w}$ and $\boldsymbol{\mu}$ as follows:

$$w_i^P(\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_r, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu}) \equiv \left[\sum_{j\in R}\frac{\phi_{ij}\mu_j\lambda_j w_j}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]^{\frac{1}{\sigma_X}}, \quad i \in R.$$

Then the market equilibrium is given by

$$\mu(w_i^*, P(\boldsymbol{\phi}_i, L, \boldsymbol{\lambda}, \boldsymbol{w}^*, \boldsymbol{\mu}^*)) = \mu_i^*, \quad i \in R,$$
$$w_i^P(\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_r, \boldsymbol{\lambda}, \boldsymbol{w}^*, \boldsymbol{\mu}^*) = w_i^*, \quad i \in R.$$

We can then express $\boldsymbol{w}^*$ and $\boldsymbol{\mu}^*$ as a function of $\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_r$, L and $\boldsymbol{\lambda}$. Clearly, if $\boldsymbol{w}^*$ is an equilibrium wage rate vector, for any scalar $c > 0$, $c\boldsymbol{w}^*$ is also an equilibrium wage rate vector. Hence, without loss of generality, we can normalize wage rates as $\sum_{i\in R} w_i = 1$. We can prove the following proposition.

Proposition 1. *Suppose that* $1 < \sigma < \sigma_X$ *holds. Then, for any* $\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_r$ $(0 < \phi_{ij} < 1,\ i, j \in R, i \neq j)$, $L > 0$ *and* $\boldsymbol{\lambda}$ $(0 < \lambda_i < 1,\ \sum_{i\in R} \lambda_i = 1,\ i \in R)$, *the equilibrium wage rate vector,* $\boldsymbol{w}^*$, *exists and it is always in* $\mathbb{R}^r_{++}$. *In particular, for two-region case, the equilibrium wage ratio,* w_1^*/w_2^*, *is determined uniquely.*

Proof See Appendix A.

3. Agglomeration Force and Spatial Equilibrium

3.1. Agglomeration Force: Price Index Effect and Home Market Effect

We now explore the *agglomeration force* in the economy, which causes a concentration of the population in particular regions. First, suppose that a fraction of firms in region j changes their location to region i, holding other things constant. Then, the relationship between the change in the number of firms in region j and that in region i is represented by $\mathrm{d}N_j = -\mathrm{d}N_i$. Using this relationship and differentiating price index (7) while keeping all other things constant, we obtain

$$\frac{N_i}{P_i}\frac{\mathrm{d}P_i}{\mathrm{d}N_i} = \frac{N_i}{\sigma_X - 1}\frac{\phi_{ji} w_j^{1-\sigma_X} - w_i^{1-\sigma_X}}{\sum_{k\in R}\phi_{ki} N_k w_k^{1-\sigma_X}} \lesseqqgtr 0 \Leftrightarrow \left(\frac{w_i}{w_j}\right)^{1-\sigma_X} \gtreqqless \phi_{ji}, \quad i, j \in R.$$

Thus, when the trade freeness between region j and region i is sufficiently low, the price index declines in region i because of the relocation of firms. In particular, if the wage rates are the same between regions j and i, the price index always falls. This is called the *Price Index Effect*; it implies that, all other things being equal, the price index becomes lower in a region with the larger number of firms because this region's consumers can access a larger proportion of differentiated goods without transport costs.

Next, suppose that a fraction of the income in region j is transferred to region i while other things are constant. Then, the relationship between the change in the income level

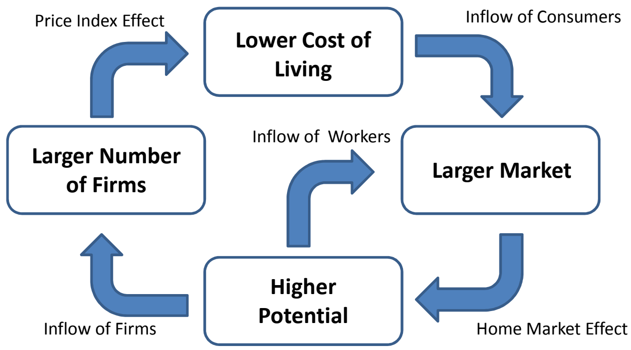


Figure 4: Circular Causality in Agglomeration

of region j and that of region i is represented by $\mathrm{d}Y_j = -\mathrm{d}Y_i$. Using this relationship and keeping all other things constant, differentiating wage potential (9) yields the following relationship:

$$\frac{Y_i}{w_i^P}\frac{\mathrm{d}w_i^P}{\mathrm{d}Y_i} = \frac{Y_i}{\sigma_X}\frac{P_i^{\sigma_X-1} - \phi_{ij}P_j^{\sigma_X-1}}{\sum_{k\in R}\phi_{ik}Y_kP_k^{\sigma_X-1}} \gtreqless 0 \;\Leftrightarrow\; \left(\frac{P_i}{P_j}\right)^{\sigma_X-1} \gtreqless \phi_{ij}, \quad i,j\in R.$$

Thus, when the trade freeness between region i and region j is sufficiently low, the wage potential rises in region i because of the transfer of income. Specifically, if the price indexes are the same between regions i and j, the wage potential always rises. This is called the *Home Market Effect*; it implies that, all other things being equal, the wage potential becomes higher in the region with the larger market because this region's firms can access higher demand without transport costs. The wage rate in the large region tends to be high because of the home market effect.

Figure 4 depicts the circular causality in the spatial agglomeration. Because the region with the larger number of firms has the lower cost of living, consumers migrate to that region. This, in turn, induces an increase in the market size of the region with the larger number of firms. On the other hand, because the region with the larger market has the higher wage potential, workers and firms flow into it. This enlarges the number of firms and the size of the market in that region. Obviously, this circular causality leads to a concentration of population in particular regions.

In the above discussion, we arbitrarily hold some endogenous variables constant. This special treatment goes a long way toward understanding the agglomeration force. Even so, unfortunately, we cannot say the price index effect and the home market effect hold in the general equilibrium framework since these variables do not have to be constant.[10] However, by reducing the number of regions to two, we obtain the following proposition:

Proposition 2. *Suppose $r = 2$ and $1 < \sigma < \sigma_X$. Then, the equilibrium wage ratio, w_1^*/w_2^*, and the equilibrium real wage ratio, ω_1^*/ω_2^*, monotonically increase with respect to the share of workers of region 1, while the equilibrium price index ratio, P_1^*/P_2^*, decreases. When both regions have the same share of workers, w_1^*/w_2^*, ω_1^*/ω_2^* and P_1^*/P_2^* are equal to one. In addition, when $\lambda_1 > \lambda_2$ ($\lambda_1 < \lambda_2$) holds, an increase in the total number of workers, L, reduces (raises) w_1^*/w_2^* and ω_1^*/ω_2^*, but raises (reduces) P_1^*/P_2^*.*

Proof See Appendix B.

Proposition 2 states that population concentration is always the cause of the wider real wage gap between regions, while population growth works as a device that reduces the regional gap. Therefore, when the economy is in a phase in which population grows rapidly, a regional gap is not apparent, which leads to a lower population concentration. However, if population growth stops, the regional gap caused by the difference in population share is actualized and population concentration advances rapidly. Especially, in a phase where population decreases, regional gap rapidly expands.

3.2. Spatial Equilibrium

According to the usual NEG model, spatial equilibrium is defined as the state in which no agents have an incentive to change location. In this regard, the spatial equilibrium is defined by

$$\left(\frac{\omega_i^s}{\bar{\omega}^s} - 1\right) \leqq 0 \text{ and } \left(\frac{\omega_i^s}{\bar{\omega}^s} - 1\right) \lambda_i^s = 0, \quad i \in R, \tag{10}$$

where $\bar{\omega}$ ($\equiv \sum_{i \in R} \lambda_i \omega_i$) is the average real wage among regions and the superscript "s" represents the value in the spatial equilibrium. This condition requires that the real wage is equalized in all regions that have a positive share of workers.

[10] Conditions under which the home market effect holds are studied by Davis (1998), Yu (2005) and Behrens et al. (2009).

At the market equilibrium, the wage ratio w_i^*/w_j^* and the real wage ratio ω_i^*/ω_j^* are given by (see Appendix A.2.2.)

$$\frac{w_i^*}{w_j^*} = \left[\frac{\sum_{k\in R}\phi_{ki}\mu_k^*\lambda_k\left(w_k^*\right)^{1-\sigma_X}}{\sum_{k\in R}\phi_{kj}\mu_k^*\lambda_k\left(w_k^*\right)^{1-\sigma_X}}\right]^{\frac{1}{\sigma_X}} \text{ and } \frac{\omega_i^*}{\omega_j^*} = \left(\frac{w_i^*}{w_j^*}\right)^{\frac{2\sigma_X-1}{\sigma_X-1}}, \quad i,j\in R. \tag{11}$$

Thus, if workers are fully agglomerated in region i (i.e., $\lambda_i = 1$ and $\lambda_j = 0$ for $j \neq i$), then we have

$$\frac{\omega_i^*}{\bar{\omega}^*} = 1 \text{ and } \frac{\omega_j^*}{\bar{\omega}^*} = \left(\frac{\phi_{ij}}{\phi_{ii}}\right)^{\frac{1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}} \leqq 1, \quad i,j\in R,\ j\neq i,$$

which means that full agglomeration in any region $i \in R$ is always a spatial equilibrium.

Obviously, it is difficult to say more about the spatial equilibrium in general because we cannot find all of the equilibriums. However, by focusing on a two-region case, we can obtain the following result.

Proposition 3. *Suppose $r = 2$ and $1 < \sigma < \sigma_X$. Then, there exists only three spatial equilibrium as follows:*

$$(\lambda_1^s,\, \lambda_2^s) = \begin{cases} (1,\, 0) \\ (0,\, 1) \\ (1/2,\, 1/2) \end{cases}.$$

Proof This is obvious from (10), (11) and Proposition 2.

Therefore, only two fully agglomerated configurations and one symmetry configuration can be the spatial equilibrium. Because the equilibrium real wage ratio ω_1^*/ω_2^* rises when the share of workers of region 1 becomes large and it is equal to one in the symmetry configuration, any partially agglomerated configurations cannot become the spatial equilibrium. In addition, if population size of the economy is constant, symmetry configuration is unstable.

This result is caused by the absence of *dispersion force* from the existence of the immobile resource (e.g., land), which is assumed in the usual NEG model. The existence of immobile resources provides a clear incentive to migrate from large to small regions because such resources become scarce in large regions compared to small regions. In our model, this type of dispersion force is omitted because the production resource is only the worker who can move between regions. Thus, if the population size were constant, the economy would tend to be fully agglomerated configurations. Later, we show that partially agglomerated configuration arises as a steady state in spite of the absence of immobile resource (see Proposition 4). This is because of the lower fertility rate in the highly populated region.

4. Extension of the Model for Demographic Analysis

We now introduce time and generations to the model. Each individual lives for two periods: childhood and adulthood. At the beginning of the adulthood period, individuals first choose where they live. Then, they choose their number of children and amount of consumption in the region that they chose.

4.1. Population Dynamics

After the location choice, there is $L_{i,t}$ number of adults (workers) in region i at time t. Each adult has $n_{i,t}$ children in region i at time t, which means $n_{i,t}$ also represents the fertility rate in region i at time t. Thus, region i has $n_{i,t}L_{i,t}$ number of children at time t. The total number of children in the economy at time t becomes

$$\sum_{i\in R} n_{i,t}L_{i,t} = \left(\sum_{i\in R} \lambda_{i,t}n_{i,t}\right) L_t = \bar{n}_t L_t, \quad t \in \mathbb{N},$$

where L_t, $\lambda_{i,t}$, and $\bar{n}_t$ ($\equiv \sum_{i\in R} \lambda_{i,t}n_{i,t}$) represent the total number of workers, region i's share of workers, and the average fertility rate in the economy at time t, respectively. Since $\bar{n}_t L_t$ children grow to adulthood in the economy at the beginning of time $t+1$, we have the following law of motion of the total number of adults:

$$L_{t+1} = \bar{n}_t L_t = \left(\prod_{s=0}^{t} \bar{n}_s\right) L_0, \quad t \in \mathbb{N}. \tag{12}$$

4.2. Dynamics of Inter-Regional Population Movement

Next, we consider the process of inter-regional migration. Because $n_{i,t}L_{i,t}$ children grow to adulthood in region i at the beginning of time $t+1$, the number of pre-movement workers in region i at time $t+1$ is equal to $n_{i,t}L_{i,t}$. Thus, region i's share of pre-movement workers at time $t+1$, $\lambda^{pre}_{i,t+1}$, becomes

$$\lambda^{pre}_{i,t+1} \equiv \frac{n_{i,t}L_{i,t}}{\sum_{j\in R} n_{j,t}L_{j,t}} = \frac{\lambda_{i,t}n_{i,t}}{\bar{n}_t}, \quad i \in R, \ \ t \in \mathbb{N}. \tag{13}$$

Similar to Fujita et al. (1999), we assume workers gradually move to regions where they can earn a higher real wage. We capture this adjustment process by the following replicator dynamics:[11]

$$\lambda_{i,t+1} - \lambda_{i,t+1}^{pre} = \nu \left(\frac{\omega_{i,t}}{\bar{\omega}_t} - 1 \right) \lambda_{i,t}, \quad i \in R, \ t \in \mathbb{N},$$

where $\bar{\omega}_t$ ($\equiv \sum_{i \in R} \lambda_{i,t}\omega_{i,t}$) is the average real wage among regions at time t and ν (> 0) is the adjustment parameter which we call freeness of migration. Using (13), we can rewrite the above system as follows:

$$\lambda_{i,t+1} - \lambda_{i,t} = \left(\frac{n_{i,t}}{\bar{n}_t} - 1 + \nu \left(\frac{\omega_{i,t}}{\bar{\omega}_t} - 1 \right) \right) \lambda_{i,t}, \quad i \in R, \ t \in \mathbb{N}, \tag{14}$$

which are equivalent essentially to the replicator dynamics, but the natural change of the population is introduced into the equation. Then, we have the following law of motion of the number of workers in region i:

$$L_{i,t+1} = \left(\frac{n_{i,t}}{\bar{n}_t} + \nu \left(\frac{\omega_{i,t}}{\bar{\omega}_t} - 1 \right) \right) \bar{n}_t L_{i,t}, \quad i \in R \quad t \in \mathbb{N},$$

which implies that the total change of workers in region i at time t, $TC_{i,t}$ ($\equiv L_{i,t+1} - L_{i,t}$), can be divided into natural change, $NC_{i,t}$, and social change, $SC_{i,t}$, as $TC_{i,t} = NC_{i,t} + SC_{i,t}$, where $NC_{i,t}$ and $SC_{i,t}$ are defined by

$$\begin{aligned} NC_{i,t} &= (n_{i,t} - 1)L_{i,t}, \quad i \in R, \ t \in \mathbb{N}, \\ SC_{i,t} &= \nu \left(\frac{\omega_{i,t}}{\bar{\omega}_t} - 1 \right) \bar{n}_t L_{i,t} \quad i \in R, \ t \in \mathbb{N}. \end{aligned}$$

4.3. Definition of Steady State

In a steady state, both the total number of workers in the economy and the share of workers in each region are stationary such that $L_{t+1} = L_t = L^{**}$ and $\lambda_{i,t+1} = \lambda_{i,t} = \lambda_i^{**}$. From (12) and (14), this steady state is given by

$$\bar{n}^{**} = 1 \text{ and } \left(\frac{n_i^{**}}{\bar{n}^{**}} - 1 + \nu \left(\frac{\omega_i^{**}}{\bar{\omega}^{**}} - 1 \right) \right) \lambda_i^{**} = 0, \quad i \in R, \tag{15}$$

where the superscript "$**$" represents the value in the steady state.

[11]The replicator dynamics are used often in evolutionary game theory. See Weibull (1995).

As mentioned at (8), a larger size of the total number of workers implies a lower value of price indexes in all regions. This may lead to a lower fertility rate in all regions by raising the real wages. Thus, population size would not grow forever. We make this point clear in the special two-region case. In this case, the equilibrium average fertility rate in the economy, $\bar{n}^*$, becomes a function of the total number of workers, L, and region 1's share of workers, λ, as $\bar{n}^*(L, \lambda)$. It is seen easily that $\lim_{L\to 0} \bar{n}^*(L, \lambda) = 1/b$ and $\lim_{L\to\infty} \bar{n}^*(L, \lambda) = 0$. In addition, we can see that if λ is fixed, $\bar{n}^*(L, \lambda)$ decrease with respect to L (see Appendix C). Hence, if $\lambda_{1,t}$ is fixed at λ and $0 < b < 1$, then L_t is always adjusted until $\bar{n}^*(L_t, \lambda)$ reaches to one and it does not grow forever. This results means that we have function $\mathcal{L}(\cdot)$ such that $\bar{n}^*(\mathcal{L}(\lambda), \lambda) = 1$ for any λ.[12]

Let us turn to the relationship between the spatial equilibrium and the steady state. We have already seen that, in the spatial equilibrium, the real wage must be equalized in all regions endowed a positive number of workers. This leads to identical fertility rates in these regions. Therefore, the second condition of (15) is always satisfied in the spatial equilibrium. In particular, for the two-region case, we have the following three steady states that are the spatial equilibrium (see Appendix D):

$$(L^{**},\, \lambda_1^{**},\, \lambda_2^{**}) = \begin{cases} (D,\, 1,\, 0) \\ (D,\, 0,\, 1) \\ (2D/(1+\phi),\, 1/2,\, 1/2) \end{cases}, \tag{16}$$

where $\phi = \phi_{12} = \phi_{21}$ and $D = \mathcal{L}(1) = \mathcal{L}(0)$ is given by

$$D \equiv \left(\frac{1-\beta}{\beta}\right)^{\frac{\sigma_X - 1}{\sigma - 1}} \left(\frac{a}{\rho_X}\right)^{\sigma_X - 1} \left(\frac{1-b}{b}\right)^{\frac{\sigma_X - \sigma}{\sigma - 1}} \frac{f\sigma_X}{b^{\sigma_X}}.$$

The steady state, however, does not have to be the spatial equilibrium. If some region has a higher real wage, there will be a population inflow, but the fertility rate will be lower in that region. Therefore, it is possible for the second condition of (15) to be satisfied in a steady state that is not the spatial equilibrium. In the two-region case, if $\lambda_{1,t} \geqq 1/2$, the

[12] Note that for any λ, there exists L uniquely such that $\bar{n}^*(L, \lambda) = 1$.

dynamics of population distribution given by (14) can be rewritten as

$$\lambda_{1,t+1} \gtreqless \lambda_{1,t} \iff \mathit{\Lambda}(L_t, \lambda_{1,t}) \gtreqless 0,$$

where $\mathit{\Lambda}$ is given by

$$\mathit{\Lambda}(L_t, \lambda_{1,t}) \equiv \frac{(1-\lambda_{1,t})(\mu^*_{2,t} - \mu^*_{1,t})}{\lambda_{1,t}(1-\mu^*_{1,t}) + (1-\lambda_{1,t})(1-\mu^*_{2,t})} + \nu \left(\frac{\omega^*_{1,t}/\omega^*_{2,t}}{\lambda_{1,t}\left(\omega^*_{1,t}/\omega^*_{2,t}\right) + 1 - \lambda_{1,t}} - 1 \right).$$

Substituting $L_t = \mathcal{L}(\lambda_{1,t})$ in $\mathit{\Lambda}(L_t, \lambda_{1,t})$ yields

$$\lim_{\lambda_{1,t}\to 1/2} \mathit{\Lambda}(L_t, \lambda_{1,t}) = \lim_{\lambda_{1,t}\to 1} \mathit{\Lambda}(L_t, \lambda_{1,t}) = 0.$$

Thus, if

$$\operatorname{sign}\left(\lim_{\lambda_{1,t}\to\frac{1}{2}} \frac{\mathrm{d}\mathit{\Lambda}(L_t, \lambda_{1,t})}{\mathrm{d}\lambda_{1,t}} \right) = \operatorname{sign}\left(\lim_{\lambda_{1,t}\to 1} \frac{\mathrm{d}\mathit{\Lambda}(L_t, \lambda_{1,t})}{\mathrm{d}\lambda_{1,t}} \right)$$

there is at least one steady state such that $1/2 < \lambda_1^{**} < 1$, which is a partially agglomerated steady state. The following proposition gives a more formal condition.

Proposition 4. *Suppose $r = 2$, $1 < \sigma < \sigma_X$, and $0 < b < 1$. Then, a steady state exists such that $\lambda_1^{**} \in (0, 1)$ and $\lambda_1^{**} \neq 1/2$, if either of the following* (a) *or* (b) *is satisfied:*

$$\begin{aligned} &(\mathrm{a}):\ 0 < \phi < 1,\ \ \nu_b < \nu < \nu_s \\ &(\mathrm{b}):\ \nu_b < \nu < \frac{1-b}{b},\ \ 0 < \phi < \phi_s \end{aligned},$$

where ν_s and ν_b are given by

$$\begin{aligned} \nu_s &\equiv \frac{1-b}{b + (1-b)\phi^{\frac{\sigma-1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}}} \frac{1 - \phi^{\frac{\sigma-1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}}}{1 - \phi^{\frac{1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}}}, \\ \nu_b &\equiv (\sigma-1)(1-b) \end{aligned}$$

and ϕ_s is defined implicitly by

$$(1-b) - b\nu - (1-b)(1+\nu)\phi_s^{\frac{\sigma-1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}} + \nu\left[b\phi_s^{\frac{1}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}} + (1-b)\phi_s^{\frac{\sigma}{\sigma_X}\frac{2\sigma_X-1}{\sigma_X-1}} \right] = 0.$$

Proof See Appendix E.

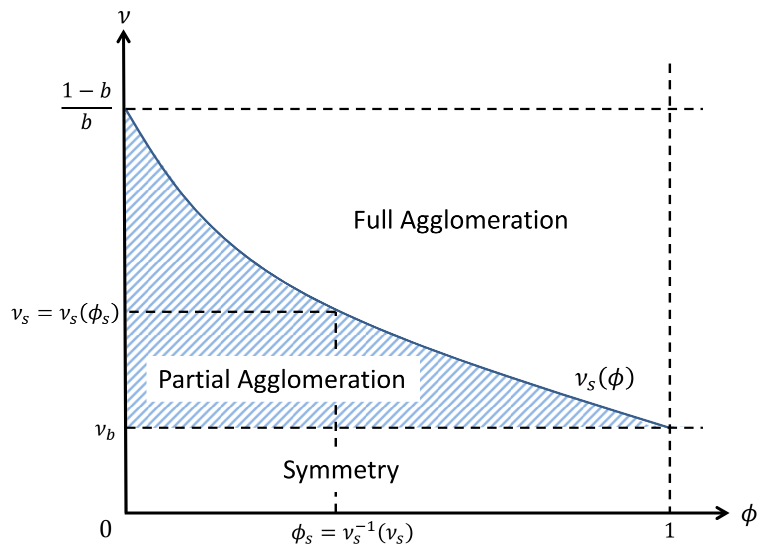


Figure 5: Condition for the Existence of Partial Agglomeration

Both condition (a) and (b) imply

$$\text{sign}\left(\lim_{\lambda_{1,t}\to\frac{1}{2}} \frac{\mathrm{d}\Lambda(L_t, \lambda_{1,t})}{\mathrm{d}\lambda_{1,t}}\right) = \text{sign}\left(\lim_{\lambda_{1,t}\to 1} \frac{\mathrm{d}\Lambda(L_t, \lambda_{1,t})}{\mathrm{d}\lambda_{1,t}}\right) > 0.$$

ϕ_s is what Fujita et al. (1999) call the *sustain point*; that is, if trade freeness ϕ is higher than the sustain point ϕ_s, fully agglomerated configurations become sustainable. We can interpret ν_s analogically as the *sustain point* of migration freeness. In addition, ν_b is analogous to what Fujita et al. (1999) call the *break point*; that is, if migration freeness is higher than the break point, symmetry configuration is broken.

The relationships among ν_b, ν_s, and ϕ_s are described in Figure 5. The downward-sloped curve $\nu_s(\phi)$ gives the sustain point ν_s for trade freeness ϕ. When ν becomes large over the $\nu_s(\phi)$ curve, the steady state of the economy tends to be full agglomeration, because workers can easily move to a region where they can earn a higher real wage. On the other hand, if ν is smaller than ν_b, then the spatial configuration tends to be symmetry, regardless of trade freeness, because it is too difficult for workers to move between regions. If pair (ϕ, ν) is in the shaded area of Figure 5, a partially agglomerated steady state exists. Conventional NEG models, which mainly focus on the relationship between trade freeness and spatial structure, can determine spatial structure regardless of whether workers can move easily between regions. However, our model suggests that migration freeness is also a key factor for determining spatial structure. The change in the share of workers of each region depends on not only social change but also natural change. The partially agglomerated steady state is determined as social change offsets natural change. Therefore, migration freeness is important. This result is obtained by introducing endogenous population growth.

5. Numerical Examples

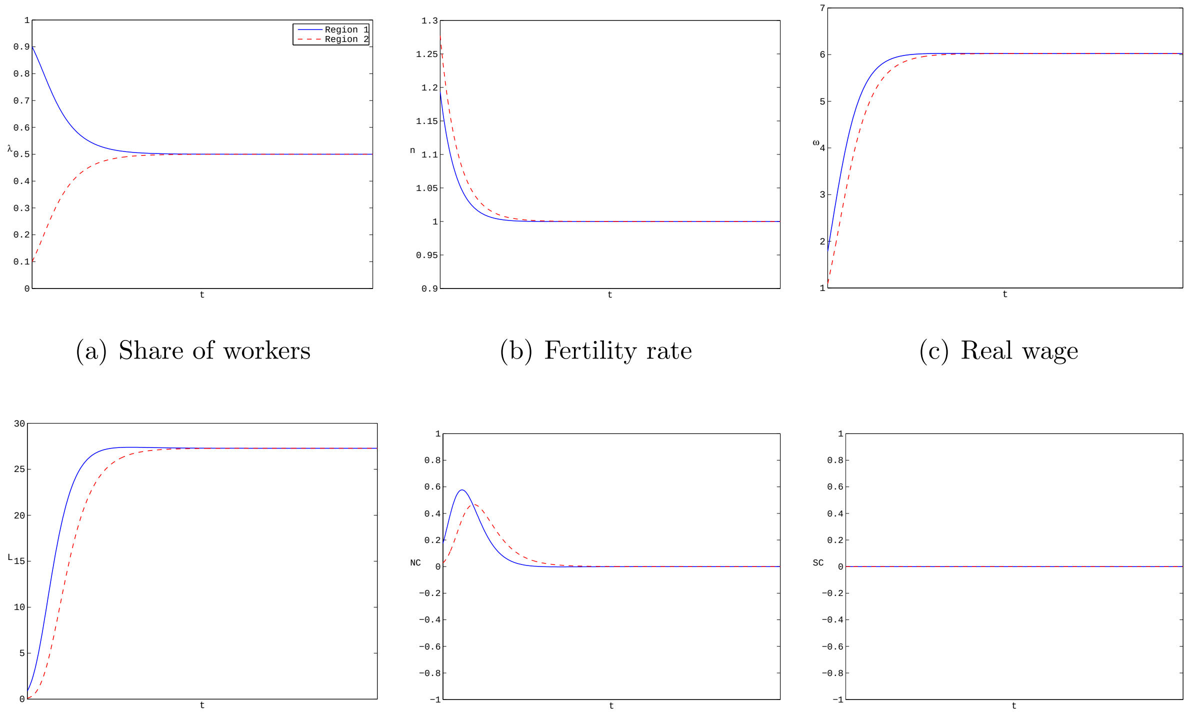
In this section, we analyze how economic integration affects population demographics. Since our model is highly non-linear, it is difficult to obtain analytical results. Therefore, we employ a numerical simulation method and show some examples.

5.1. The Case of Two Regions

Here, we show a two-region case, first with no migration and then with migration. Figure 6 describes the demographics for no migration, in which workers cannot move between regions (i.e., $\nu = 0$). The economy initially has one unit of labor (workers) and region 1's initial share of workers is set as 0.9. Then, we illustrate the dynamic paths of the share of workers ($\lambda_{i,t}$), fertility rate ($n_{i,t}$), real wage ($\omega_{i,t}$), number of workers ($L_{i,t}$), and natural change ($NC_{i,t}$) and social change ($SC_{i,t}$) in each region.

In the two-region case, the home market effect and the price index effect always make the real wage higher in the highly populated region (see Proposition 2). Thus, the fertility rate in the highly populated region will be lower than in the less populated region since a higher real wage rate means a lower fertility rate. Therefore, the real wage of region 1 is higher than that of region 2, while it leads to a lower fertility rate in region 1 compared to region 2. Consequently, if workers cannot move between regions, the share of workers would equalize gradually over time (see Figure 6(a)). In this sense, we call this *dispersion force*; this is the power that makes the population distribution over the regions tend toward uniformity.

On the other hand, the number of workers monotonically increases in regions 1 and 2, which raises real wages in both regions (see Figures 6(c) and 6(d)). This rise of the real wage results in fertility rates decreasing (see Figure 6(b)). Natural changes in regions 1 and 2 become large in the early phase, but reach their peak at a certain point in time,



(a) Share of workers (b) Fertility rate (c) Real wage

(d) Number of workers (e) Natural change (f) Social change

Figure 6: Demographics with No Migration

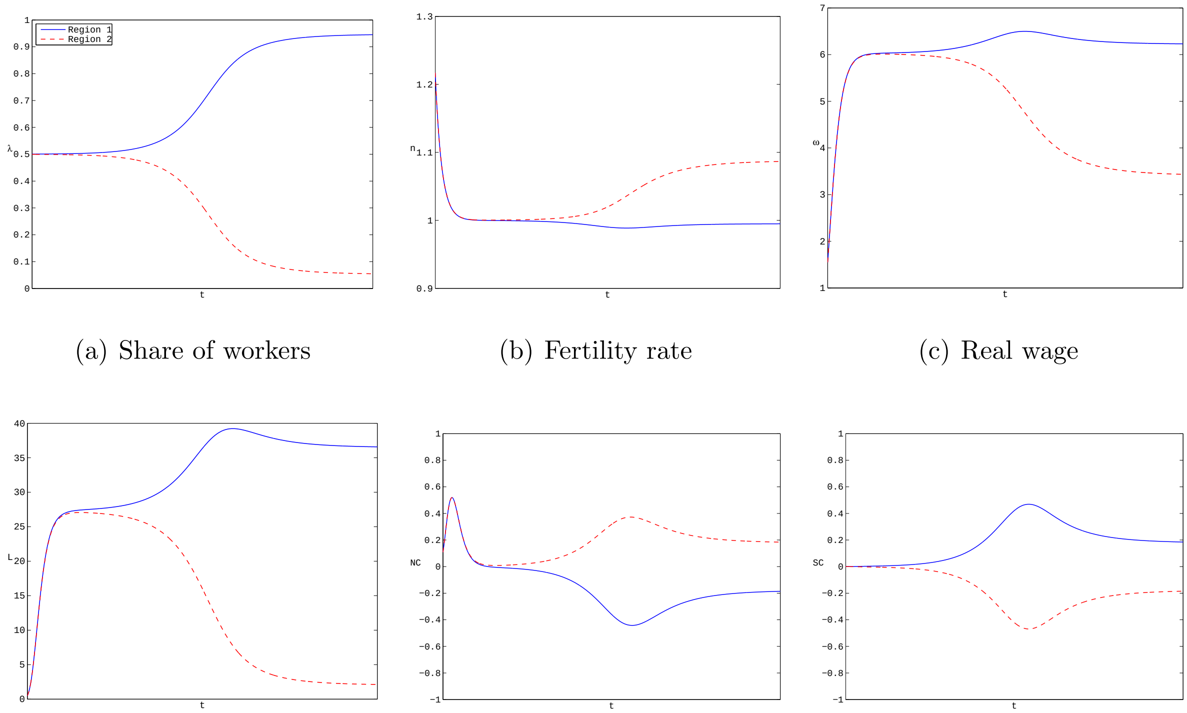
$\sigma = 1.2,\ \sigma_X = 4,\ \alpha = 0.7,\ b = 0.25,\ \phi = 0.25,\ f = 1/3,\ a = 1/3,\ \nu = 0$

and thereafter, decrease until zero (see Figure 6(e)). Because we consider the case of no migration, the social change is always zero (see Figure 6(f)).

Next, we consider the case with migration, in which workers can move between regions. Figure 7 describes the dynamic paths of the variables under the same parameters as the case of no migration except for the freeness of migration, ν.

Permitting the migration of workers greatly changes the dynamic path from Figure 6. In Figure 7, we set the initial share of workers to be almost the same in both regions, but the share in region 1 is slightly larger.[13] As the home market effect and the price index effect make the real wage of region 1 higher than that of region 2, the social change, $SC_{i,t}$, is positive in region 1, but negative in region 2 (see Figure 7(f)). This agglomeration force encourages the share of workers in region 1 to increase over time (see Figure 7(a)). On

[13] We set $\lambda_{1,0} = 1001/2000$ and $\lambda_{2,0} = 999/2000$.



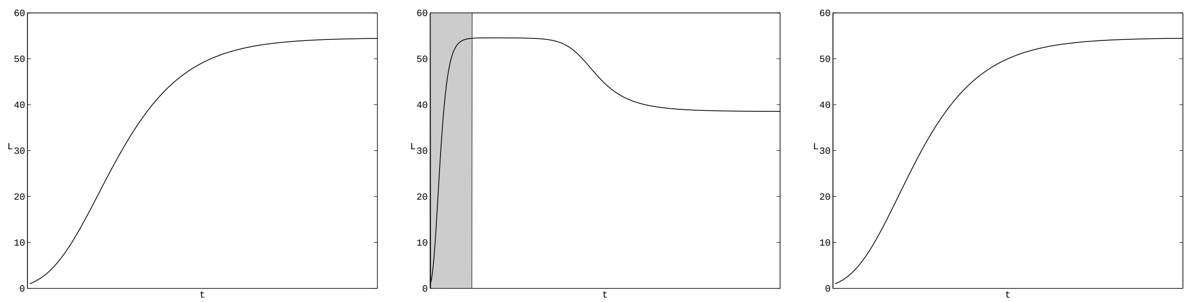
(a) Share of workers
(b) Fertility rate
(c) Real wage
(d) Number of workers
(e) Natural change
(f) Social change

Figure 7: Demographics with Migration

$\sigma = 1.2$, $\sigma_X = 4$, $\alpha = 0.7$, $b = 0.25$, $\phi = 0.25$, $f = 1/3$, $a = 1/3$, $\nu = 0.2$

the other hand, the natural change, $NC_{i,t}$, is small in region 1 and large in region 2 (see Figure 7(e)) since a higher real wage reduces the fertility rate (dispersion force). When the difference in the share of workers between regions becomes large, the agglomeration force and the dispersion force are balanced and the economy is in a steady state. Interestingly, in this steady state, the social change of region 1 is positive, which means that this economy is not in spatial equilibrium. In particular, in the steady state, the two regions differ in characteristics: one has a positive natural change but a negative social change, while the other has a positive social change but a negative natural change.

The paths of the number of workers in the whole economy in the cases of no migration and migration are described in Figure 8. In the case of no migration (i.e., $\nu = 0$), the path has the form of a logistic curve (see Figure 8(a)). When there are few workers in the economy, since the real wage is low, the fertility rate is high and the population grows rapidly. As the population grows, the real wage becomes higher and this suppresses the fertility rate.



(a) No-migration case (b) Migration case (c) Shadow area of (b)

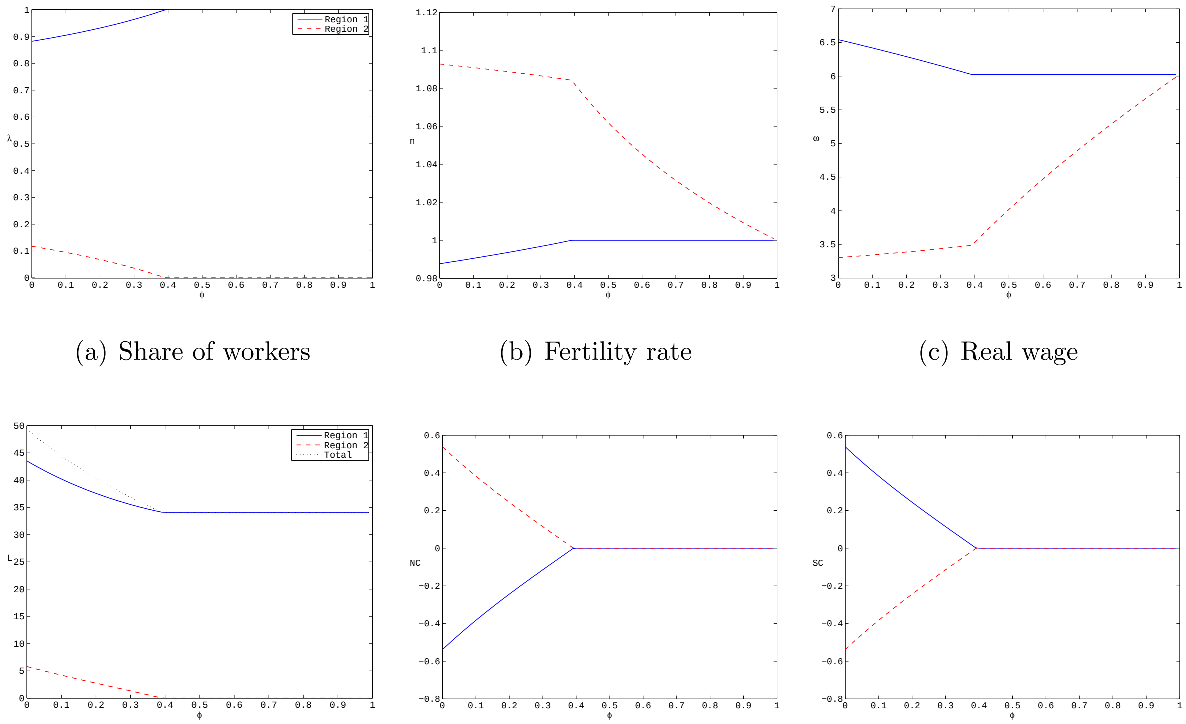
Figure 8: Number of Workers in the Whole Economy

$\sigma = 1.2$, $\sigma_X = 4$, $\alpha = 0.7$, $b = 0.25$, $\phi = 0.25$, $f = 1/3$, $a = 1/3$ $\nu = 0.2$

Therefore, population growth gradually decreases and the economy reaches a steady state.

In the case of migration (i.e., $\nu > 0$), the shape of the path stands in sharp contrast to the case of no migration, as in Figure 8(b). The population reaches its peak before the steady state and then decreases. In the early phase, because the difference in the shares of workers between regions is not sizable, the real wages in both regions are nearly the same. Hence, the migration rate of workers is small and the population growth path in the economy is similar to that of the case of no migration. Figure 8(c) illustrates this phase, which corresponds to the shaded area of Figure 8(b). In this phase, the path of population growth is also in the form of a logistic curve.

After the early phase, however, workers migrate to the region with the higher real wage and gradually congregate there, which increases the real wage in the highly populated region (region 1) and decreases the real wage in the less populated region (region 2). This brings about further migration and reduces the fertility rate in region 1. When the difference between the shares of workers becomes large enough, the fertility rate is less than one in region 1. Thus, total population growth becomes negative, even though region 2 has a fertility rate higher than one, and this overall negative growth continues until the negative natural change in region 1 is canceled out by positive natural change in region 2.



(a) Share of workers (b) Fertility rate (c) Real wage

(d) Number of workers (e) Natural change (f) Social change

Figure 9: Effects of the Change of Trade Freeness

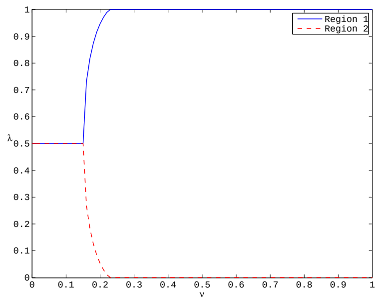
$\sigma = 1.2,\ \sigma_X = 4,\ \alpha = 0.7,\ b = 0.25,\ f = 1/3,\ a = 1/3,\ \nu = 0.2$

5.2. How Integration of the Economy Affects Population Size and Spatial Structure

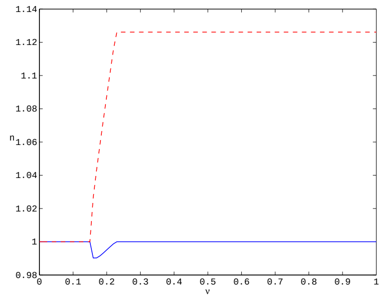
Next, we show how integration of the economy affects population size and spatial structure. We examine how the steady state is affected by changing trade freeness, ϕ, and migration freeness, ν.[14]

First, we show the effects of changing trade freeness. Figure 9 describes the steady state corresponding to each ϕ. A rise in trade freeness increases the share of workers in region 1 (Figure 9(a)). At levels higher than $\phi = \phi_s \approx 0.39$, the economy completely agglomerates. We depict the number of workers in Figure 9(d). Because trade of goods comes to incur few transportation costs, an increase in the trade freeness directly increases the real wage, which reduces the fertility rate. Hence, the steady-state total number of workers decreases

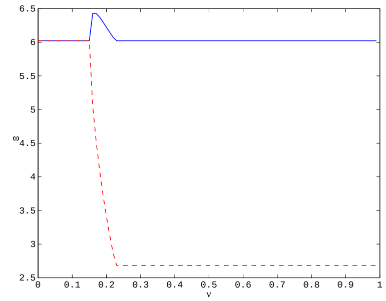
[14] Using the initial distribution of workers and all parameters in the migration case in Subsection 5.1, we calculate the steady state corresponding to each ϕ and ν.



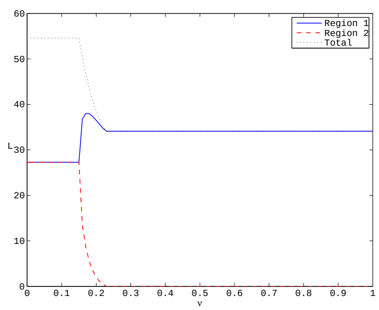
(a) Share of workers



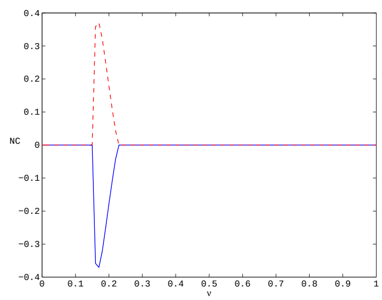
(b) Fertility rate



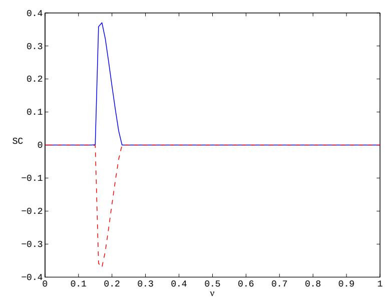
(c) Real wage



(d) Number of workers



(e) Natural change



(f) Social change

Figure 10: Effects of Change of the Freeness of Migration

$\sigma = 1.2$, $\sigma_X = 4$, $\alpha = 0.7$, $b = 0.25$, $\phi = 0.25$, $f = 2/3$, $a = 1/3$

due to a rise of trade freeness. Since this reduction of the total number of workers is large, even in region 1, the number of workers decreases although a rise in trade freeness induces the population concentration to region 1. Hereby, the steady-state fertility rate increases in region 1 because the reduction of the total number of workers decreases the real wage (Figures 9(b) and 9(c)). Moreover, the real wage gap decreases as the trade freeness rises, which brings about reduction in the regional differences of natural change and social change (Figures 9(e) and 9(f)). After complete agglomeration, trade freeness does not affect the number of workers, natural change, and social change in the steady state since, in this case, there is only one region in the economy.

Next, we show the effects of changing the freeness of migration, ν. Figure 10(a) describes the share of workers in the steady state. If freeness of migration is small, it is difficult for workers to move even if there is a difference in the real wages between regions. However, the fertility rate is lower in the highly populated regions; this occurs irrespective of the level of migration freeness. Hence, the distribution of workers between regions tends to be uniform in the steady state, as in the case of no migration. Conversely, if freeness of migration is sufficiently large, a positive social change always overcomes a negative natural change in the large region. Thus, the steady state of the spatial structure of the economy is full agglomeration. Given moderate freeness of migration ($0.15 = \nu_b < \nu < \nu_s \approx 0.23$), the economy converges to be neither symmetric nor in full agglomeration, but it converges to be in partial agglomeration.

Figures 10 (c) and (d) show the real wage and the number of workers. Unlike the trade freeness, the migration freeness does not affect the real wage directly. However, the real wage is affected by the distribution of workers. Higher freeness of migration implies higher population concentration in region 1, which raises the real wage in region 1 given the population. In addition, a higher real wage induces the lower fertility rate. Therefore, a rise in migration freeness decreases the steady-state total number of workers through lowering the fertility rate in region 1 (Figure 10(b)). In partial agglomeration near the symmetry equilibrium, a rise of freeness of migration increases the regional difference of social change, not only directly, but also indirectly through population concentration which raises the real wage gap. This large gap of the real wage also leads to a large difference of natural change between regions. On the other hand, in partial agglomeration near the full agglomeration, the real wage gap decreases as the freeness of migration rises, which brings about reduced regional differences of natural and social change. Therefore, as freeness of migration rises, the regional differences of natural change and social change increase at first, and then, they decrease when the spatial structure is sufficiently close to full agglomeration (Figures 10(e) and 10(f)). When the economy reaches the full agglomeration, natural change and social change become zero.

From the above discussion, we see that if the economy is more integrated, the number of workers decreases. This is why economic integration not only raises the real wage directly (consider the effects of a rise of ϕ) but also induces spatial agglomeration which in turn raises the real wage. Hence, the fertility rate decreases given the population, which results in a decline in the total number of workers in the steady state. This result cannot be obtained

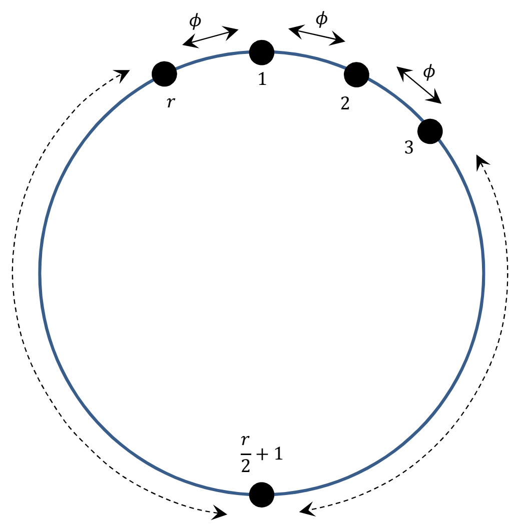


Figure 11: Conceptual Diagram of the Race Track Economy (r is an even number)

in models that consider only natural change or only social change.

5.3. The Case of Multi-Regions

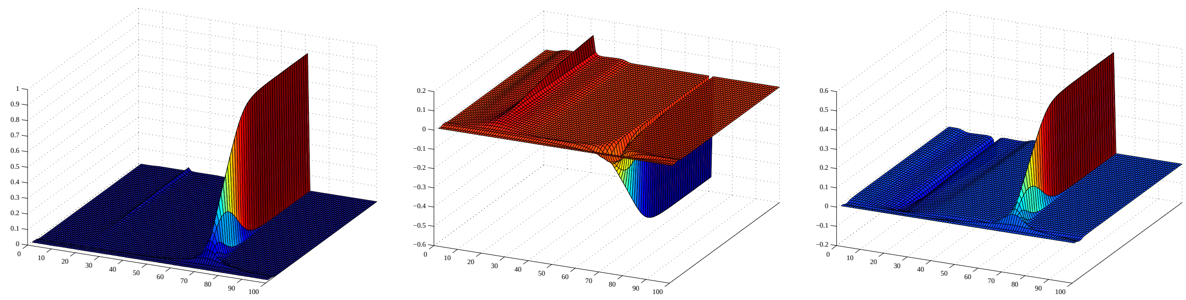
Finally, we examine the multi-region case. If there are many regions, we can consider various geometries of the economy. Here, however, we focus on the special case known as the *racetrack economy* which is described in Figure 11.[15] Each region is arranged at equal intervals on the circumference of a circle and transportation is carried out only along the circumference within the shortest distance. Since we employ iceberg-type transportation costs and regions are located along the circumference at even intervals, the trade freeness between regions i and j becomes

$$\phi_{ij} = \begin{cases} \phi^{|i-j|} & \text{if } |i-j| \leqq r/2 \\ \phi^{r-|i-j|} & \text{if } |i-j| > r/2 \end{cases},$$

where $\phi \in (0, 1]$ is constant.

In this economy, we can easily confirm that the uniform distribution of workers, called the *flat earth* in Fujita et al. (1999), is always a spatial equilibrium. However, the flat earth is not always sustainable; that is, the symmetric spatial equilibrium may turn out to be unstable.

[15]The racetrack economy is first introduced in NEG by Krugman (1993).



(a) Share of workers (b) Natural change (c) Social change

Figure 12: Multi-Region Case

$\sigma = 1.2$, $\sigma_X = 4$, $\alpha = 0.7$, $b = 0.25$, $\phi = 0.95$, $f = 1/3$, $a = 1/3$, $\nu = 0.2$, $r = 100$

As Figure 12 shows, we start the simulation from an almost flat but randomly deviated distribution of workers.[16] Even though the deviation is very small, the circular causality of agglomeration shown in Figure 4 can break the flat earth: an almost even distribution of workers eventually develops local concentration of workers. Figure 12(a) shows this process, in which 100 regions are arranged along the front axis in numerical order and the share of workers in each region is indicated by the vertical axis. The almost flat earth evolves over time into a very uneven spatial structure in which workers become concentrated in two regions that are positioned opposite to each other on the circumference.

While this result may appear to be the same as that obtained by Fujita et al. (1999), there are some major differences. Figure 12(c) illustrates the social change of each region over time. It shows that workers migrate from small regions to large regions and that the social change of the largest region is extremely positive even after a sufficiently long time. As a result, the region that initially has the largest number of workers tends to be extremely large in the steady state. It has been noted that workers flow from the smaller of two regions in which workers are concentrated. The smaller region has a large share of workers in the steady state because its natural change is highly positive. Figure 12(b) shows the natural change of each region over time. When workers concentrate in one region, the real wage becomes

[16] First, we draw L_i randomly from the interval $[0.95,\ 1.05]$. Then we normalize the total number of workers to one.

lower in the opposite region since a larger fraction of consumption must incur transportation costs. This leads to a higher fertility rate and increases the natural change in the region that is positioned opposite to the region with the largest number of workers. Therefore, the two regions in which workers are concentrated have markedly different characteristics and sizes. These results can be obtained only by considering the endogenous fertility rate, which is ignored in Fujita et al. (1999).

6. Summary and Discussion

In this article, we constructed a model to describe regional population changes in a market economy. Using this model, the effects of economic integration on population change were analyzed.

If workers can migrate among regions, then regional differences in the real wage, the natural population change, and the social population change become larger with a snowball effect, even though there are only subtle differences initially. The population concentrates in the region that initially has a larger population share. The region in which the population concentrates has a higher real wage, which results in a lower fertility rate and higher net migration compared to other less concentrated regions. Thus, regions have very different population changes. In particular, the regional difference in the real wage does not disappear even in the long run, which means that the steady state is not spatial equilibrium; that is, workers have an incentive to change their location. This result differs widely from the usual NEG model and is consistent with the facts we presented in the introduction, which enhances the legitimacy of our model. In addition, we derived a prediction for the population growth path of the whole economy; it resembles a logistic curve in the early phase, but the population decreases in the last phase.

In addition, we showed that high freeness to migrate and trade of goods lead to population concentration and decrease of the total population. If inter-regional migration were permitted, workers would move to regions where they could earn higher real wages. This increases the populations of regions with higher real wages. Moreover, the existence of transportation costs leads to higher real wages in highly populated regions compared to less-populated regions since a larger fraction of goods must incur transportation costs in the latter case. This circular causality induces a population concentration in a particular region. Specifically, we showed that the greater is the freeness of migration and trade of goods, the more the population is concentrated. Furthermore, higher trade freeness means a higher real wage because trade of goods incurs few transportation costs, which implies a lower fertility rate given the population. Therefore, an increase of trade freeness decreases the total population in the steady state. On the other hand, higher freeness of migration itself does not mean a higher real wage directly. However, by concentrating the population, the fertility rate in the higher populated region decreases given the population. Hence, in the steady state, the total population decreases.

Proposition 2 suggests that population growth reduces regional inequality even if there is a population gap between regions. When the population grows rapidly, regional inequality does not widen significantly, that is, population growth keeps regional inequality hidden. On the other hand, when population growth stops, regional inequality becomes evident. In particular, in an economy with a decreasing population, such as Japan, regional inequality may be a severe problem.

In addition, our model suggests that both migration freeness and trade freeness are important. So far, NEG models have focused mainly on the relationship between trade freeness and spatial structure. In these models, population distribution is determined, as social change becomes zero at the steady state; this means that real wages are equalized between regions or that the population is fully agglomerated in one region. Thus, freeness of migration does not affect the steady state spatial structure. However, our model introduces natural change and the steady-state population distribution is given by the state in which social change offsets natural change. Therefore, migration freeness has a key role in our model. We believe that our model is more appropriate because population distribution depends on not only social change but also natural change, which is ignored in conventional NEG models.

Finally, we consider issues for future study. In our model, we did not consider urban costs. Even though urban costs are important factors for people who choose their locations and number of children, we omitted them for simplicity. In addition, in this article, we dealt with only two overlapping generations, that is, childhood and adulthood. We paid little attention to the composition of the population. In demographic studies, the composition of the population is typically a matter of primary importance, as its structure changes over time, just as its size and distribution do. However, for simplicity, we omitted population structure from our analysis. In spite of this limitation, we believe that our analysis identifies new aspects of the relationship between economic integration and population change. We leave the consideration of urban costs and population structure for future research.

Appendix A. Proof of Proposition 1

We divide the problem into two steps. In the first step, under condition $1 < \sigma < \sigma_X$, we prove that; for all $L > 0$, $\boldsymbol{\lambda}$ ($0 < \lambda_i < 1$, $\sum_{i \in R} \lambda_i = 1$, $i \in R$) and $\boldsymbol{w} \in \mathbb{R}^r_{++}$, there exist $\boldsymbol{\mu}$ uniquely such that

$$\mu\left(w_i, P_i(L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu})\right) = \mu_i > 0, \quad i \in R.$$

This $\boldsymbol{\mu}$ can be expressed as a function of L, $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ as $\boldsymbol{\mu} = \mathcal{M}(L, \boldsymbol{\lambda}, \boldsymbol{w})$. Then, in the second step, we show the existence of the equilibrium wage vector $\boldsymbol{w}^* \in \mathbb{R}^r_{++}$, which is given by

$$w_i^P(\boldsymbol{\lambda}, \boldsymbol{w}^*, \mathcal{M}(L, \boldsymbol{\lambda}, \boldsymbol{w}^*)) = w_i^*, \quad i \in R.$$

Appendix A.1. The first step

For given $L > 0$, $\boldsymbol{\lambda}$ ($0 < \lambda_i < 1$, $\sum_{i \in R} \lambda_i = 1$, $i \in R$) and $\boldsymbol{w} \in \mathbb{R}^r_{++}$, we define a function $F_i : [0,1]^r \to [0,1]$ as follows:

$$\forall \boldsymbol{\mu} \in I \equiv [0,1]^r : \ F_i(\boldsymbol{\mu}) = \begin{cases} \mu\left(w_i, P_i(L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu})\right) & \text{if } \boldsymbol{\mu} \neq \mathbf{0} \\ 0 & \text{if } \boldsymbol{\mu} = \mathbf{0} \end{cases}, \quad i \in R,$$

where $\mu(w_i, P_i)$ and $P_i(L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu})$ are defined by

$$\mu(w_i, P_i) \equiv \frac{\beta P_i^{1-\sigma}}{\beta P_i^{1-\sigma} + (1-\beta)(b w_i)^{1-\sigma}}, \quad i \in R,$$

$$P_i(L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu}) \equiv \frac{a}{\rho_X} \left[\frac{L}{f \sigma_X} \sum_{j \in R} \phi_{ji} \mu_j \lambda_j w_j^{1-\sigma_X} \right]^{\frac{1}{1-\sigma_X}}, \quad i \in R.$$

If $1 < \sigma < \sigma_X$ holds, we have

$$\lim_{\boldsymbol{\mu} \to 0} P_i(L, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\mu}) = \infty \quad \text{and} \quad \lim_{P_i \to \infty} \mu(w_i, P_i) = 0, \quad i \in R,$$

which means that $F(\cdot) = (F_1(\cdot), \cdots, F_r(\cdot))$ is a continuous function from I to I.

Let us define a function $G(\cdot) = (G_1(\cdot), \cdots, G_r(\cdot))$ as follows:

$$\forall \boldsymbol{\mu} \in I: \; G_i(\boldsymbol{\mu}) \equiv F_i(\boldsymbol{\mu}) - \mu_i, \quad i \in R.$$

We then show that there exists $\boldsymbol{\mu} \in \mathrm{int}I$ uniquely such that $G(\boldsymbol{\mu}) = 0$. Note that because when we write $\boldsymbol{\mu}_{-i} \equiv (\mu_1, \cdots, \mu_{i-1}, \mu_{i+1}, \cdots, \mu_r)$ and $\boldsymbol{\mu} = (\mu_i; \boldsymbol{\mu}_{-i})$ for all $i \in R$,

$$\begin{aligned} &\forall \boldsymbol{\mu}_{-i} \in [0,1]^{r-1}: \; \boldsymbol{\mu}_{-i} \neq \mathbf{0} \implies G_i(0; \boldsymbol{\mu}_{-i}) > 0, \quad i \in R, \\ &\forall \boldsymbol{\mu}_{-i} \in [0,1]^{r-1}: \; G_i(1; \boldsymbol{\mu}_{-i}) < 0, \quad i \in R, \end{aligned} \tag{A.1}$$

are satisfied, no boundary points of I ever become roots of G except $\boldsymbol{\mu} = \mathbf{0}$.

To show the existence and uniqueness of the root of G, we construct closed and bounded intervals on $\mathbb{R}^r$, $J^1, J^2, \cdots$ that satisfy $I \supset J^1 \supset J^2 \supset \cdots$ and $J \equiv \lim_{n\to\infty} J^n = \{\boldsymbol{c}\}$ where $G(\boldsymbol{c}) = 0$, and show that $I - J$ never contains the root of G except $\boldsymbol{\mu} = \mathbf{0}$.

For all $\boldsymbol{\mu} \neq \mathbf{0}$, the partial differentiation of F_i with respect to $\mu_j > 0$ becomes

$$F_{ij}(\boldsymbol{\mu}) \equiv \frac{\partial F_i(\boldsymbol{\mu})}{\partial \mu_j} = \frac{\sigma - 1}{\sigma_X - 1} \frac{F_i(\boldsymbol{\mu})(1 - F_i(\boldsymbol{\mu}))}{\mu_j} \frac{\phi_{ji}\mu_j\lambda_j w_j^{1-\sigma_X}}{\sum_{k\in R} \phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}, \quad i, j \in R. \tag{A.2}$$

Hence, for all $i \in R$ we have

$$\begin{aligned} \lim_{\mu_i \to 0} \frac{\partial G_i(\mu_i; \mathbf{0})}{\partial \mu_i} &= \lim_{\mu_i \to 0} \left(\frac{\sigma - 1}{\sigma_X - 1} \frac{F_i(\mu_i; \mathbf{0})\,(1 - F_i(\mu_i; \mathbf{0}))}{\mu_i} - 1 \right) \\ &= \frac{\sigma - 1}{\sigma_X - 1} \left(\lim_{\mu_i \to 0} \frac{F_i(\mu_i; \mathbf{0})}{\mu_i} \right) - 1 = \infty, \end{aligned} \tag{A.3}$$

where the last equality holds under $1 < \sigma < \sigma_X$. Together with $G_i(0; \mathbf{0}) = 0$, (A.3) implies that real numbers $\varepsilon_1, \cdots, \varepsilon_r$ exist, which are sufficiently close to zero and satisfy the following

$$G_i(\mu_i; \mathbf{0}) > 0, \quad \mu_i \in (0, \, \varepsilon_i], \; i \in R.$$

On the other hand, from (A.1), $G_i(1; \mathbf{0}) < 0$ holds for all $i \in R$. Thus, we have $\underline{\mu}_i^1 \in (\varepsilon_i, \, 1)$ which satisfies $G_i(\underline{\mu}_i^1; \mathbf{0}) = 0$. Because of (A.2), $\partial G_i(\boldsymbol{\mu})/\partial \mu_i < 0$ at any point $\boldsymbol{\mu}$ that satisfies $G_i(\boldsymbol{\mu}) = 0$. Hence, such $\underline{\mu}_i^1$ is unique. This implies:

$$G_i(\mu_i^{''}; \mathbf{0}) < 0 < G_i(\mu_i^{'}; \mathbf{0}), \quad \mu_i^{'} \in (0, \, \underline{\mu}_i^1), \; \mu_i^{''} \in (\underline{\mu}_i^1, \, 1], \; i \in R.$$

Therefore, because under $1 < \sigma < \sigma_X$ it holds that

$$\forall \boldsymbol{\mu}^{'}, \boldsymbol{\mu}^{''} \in I: \; \boldsymbol{\mu}^{'} \leqq \boldsymbol{\mu}^{''} \text{ and } \boldsymbol{\mu}^{'} \neq \boldsymbol{\mu}^{''} \implies F(\boldsymbol{\mu}^{'}) << F(\boldsymbol{\mu}^{''}), \tag{A.4}$$

we have the following for all $i \in R$:

$$(\mu_i; \boldsymbol{\mu}_{-i}) \neq (\underline{\mu}_i^1; \mathbf{0}), (0; \mathbf{0}) \implies G_i(\mu_i; \boldsymbol{\mu}_{-i}) > 0, \quad \mu_i \in [0,\, \underline{\mu}_i^1],\ \boldsymbol{\mu}_{-i} \in [0,\, 1]^{r-1}. \tag{A.5}$$

Similarly, we can show that $\bar{\mu}_i^1 \in (0,\, 1)$ exists uniquely, which satisfies $G_i(\bar{\mu}_i^1; \mathbf{1}) = 0$ and

$$G_i(\mu_i^{''}; \mathbf{1}) < 0 < G_i(\mu_i^{'}; \mathbf{1}), \quad \mu_i^{'} \in [0,\, \bar{\mu}_i^1),\ \mu_i^{''} \in (\bar{\mu}_i^1,\, 1],\ i \in R.$$

Then, using (A.4), we have the following relationship for all $i \in R$:

$$(\mu_i; \boldsymbol{\mu}_{-i}) \neq (\bar{\mu}_i^1; \mathbf{1}) \implies G_i(\mu_i; \boldsymbol{\mu}_{-i}) < 0, \quad \mu_i \in [\bar{\mu}_i^1, 1],\ \boldsymbol{\mu}_{-i} \in [0,\, 1]^{r-1}. \tag{A.6}$$

Note that $\underline{\mu}_i^1 < \bar{\mu}_i^1$ holds for all $i \in R$. In fact, if $\underline{\mu}_i^1 \geqq \bar{\mu}_i^1$ holds, then we have

$$0 < G_i(\bar{\mu}_i^1; \boldsymbol{\mu}_{-i}) < 0, \quad \boldsymbol{\mu}_{-i} \in (0,\, 1)^{r-1},\ i \in R,$$

where the first inequality is due to (A.5) while the second inequality is derived from (A.6). However, this relationship is a contradiction. Thus, we can define J^1 as $J^1 \equiv \prod_{i=1}^{r} [\underline{\mu}_i^1,\, \bar{\mu}_i^1]$, and (A.5) and (A.6) imply that the root of G does not exist in $I - J^1$ except $\boldsymbol{\mu} = \mathbf{0}$. Therefore, we only need to search for the root of G in J^1.

Next, we define J^2 in a similar way. Let us denote the vectors $(\underline{\mu}_1^1, \cdots, \underline{\mu}_{i-1}^1, \underline{\mu}_{i+1}^1, \cdots, \underline{\mu}_r^1)$ and $(\bar{\mu}_1^1, \cdots, \bar{\mu}_{i-1}^1, \bar{\mu}_{i+1}^1, \cdots, \bar{\mu}_r^1)$ by $\underline{\boldsymbol{\mu}}_{-i}^1$ and $\bar{\boldsymbol{\mu}}_{-i}^1$, respectively. Then we have

$$\begin{aligned} G_i(\bar{\mu}_i^1; \underline{\boldsymbol{\mu}}_{-i}^1) < 0 < G_i(\underline{\mu}_i^1; \underline{\boldsymbol{\mu}}_{-i}^1), \quad i \in R, \\ G_i(\bar{\mu}_i^1; \bar{\boldsymbol{\mu}}_{-i}^1) < 0 < G_i(\underline{\mu}_i^1; \bar{\boldsymbol{\mu}}_{-i}^1), \quad i \in R. \end{aligned}$$

Thus, the intermediate value theorem ensures the existence of $\underline{\mu}_i^2, \bar{\mu}_i^2 \in (\underline{\mu}_i^1, \bar{\mu}_i^1)$, which satisfy

$$G_i(\underline{\mu}_i^2; \underline{\boldsymbol{\mu}}_{-i}^1) = G_i(\bar{\mu}_i^2; \bar{\boldsymbol{\mu}}_{-i}^1) = 0, \quad i \in R.$$

Because $\partial G_i(\boldsymbol{\mu})/\partial \mu_i < 0$ holds for any point $\boldsymbol{\mu}$ that satisfies $G_i(\boldsymbol{\mu}) = 0$, such $\underline{\mu}_i^2$ and $\bar{\mu}_i^2$ are determined uniquely. Therefore, the following relationships can be obtained:

$$\begin{aligned} G_i(\mu_i^{''}; \underline{\boldsymbol{\mu}}_{-i}^1) < 0 < G_i(\mu_i^{'}; \underline{\boldsymbol{\mu}}_{-i}^1), \quad \mu_i^{'} \in [\underline{\mu}_i^1,\, \underline{\mu}_i^2),\ \mu_i^{''} \in (\underline{\mu}_i^2,\, \bar{\mu}_i^1],\ i \in R, \\ G_i(\mu_i^{''}; \bar{\boldsymbol{\mu}}_{-i}^1) < 0 < G_i(\mu_i^{'}; \bar{\boldsymbol{\mu}}_{-i}^1), \quad \mu_i^{'} \in [\underline{\mu}_i^1,\, \bar{\mu}_i^2),\ \mu_i^{''} \in (\bar{\mu}_i^2,\, \bar{\mu}_i^1],\ i \in R. \end{aligned}$$

These imply that

$$\begin{aligned} (\mu_i; \boldsymbol{\mu}_{-i}) \neq (\underline{\mu}_i^2; \underline{\boldsymbol{\mu}}_{-i}^1) \implies G_i(\mu_i; \boldsymbol{\mu}_{-i}) > 0, \quad \mu_i \in [\underline{\mu}_i^1,\, \underline{\mu}_i^2],\ \boldsymbol{\mu}_{-i} \in J_{-i}^1,\ i \in R, \\ (\mu_i; \boldsymbol{\mu}_{-i}) \neq (\bar{\mu}_i^2; \bar{\boldsymbol{\mu}}_{-i}^1) \implies G_i(\mu_i; \boldsymbol{\mu}_{-i}) < 0, \quad \mu_i \in [\bar{\mu}_i^2,\, \bar{\mu}_i^1],\ \boldsymbol{\mu}_{-i} \in J_{-i}^1,\ i \in R, \end{aligned}$$

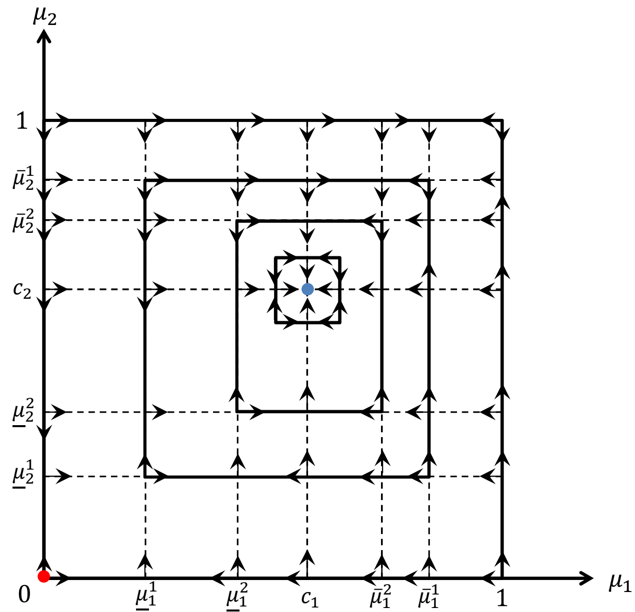


Figure A1: Vector Field G and Construction of the Steady State

where $J_{-i}^1 \equiv \prod_{j\neq i}[\underline{\mu}_j^1, \bar{\mu}_j^1]$. $\underline{\mu}_i^2 \geqq \bar{\mu}_i^2$ leads to $0 < G_i(\bar{\mu}_i^2; \boldsymbol{\mu}_{-i}) < 0$ for all $\boldsymbol{\mu}_i \in \text{int}J_{-i}^1$, which is a contradiction. Thus, $\underline{\mu}_i^2 < \bar{\mu}_i^2$ must hold. Then, we can define $J^2 \equiv \prod_{i=1}^r[\underline{\mu}_i^2, \bar{\mu}_i^2]$, and $J^1 - J^2$ never contains the root of G.

We continue this process as depicted by Figure A1; then, we obtain the sequence of closed and bounded interval, $(J^m)_{m=1}^\infty$, such that $J^m = \prod_{i=1}^r[\underline{\mu}_i^m, \bar{\mu}_i^m]$ and $J^1 \supset J^2 \supset \cdots$. Therefore, $(J^m)_{m=1}^\infty$ converges to a non-empty, closed and bounded interval $J = \prod_{i=1}^r[\underline{c}_i, \bar{c}_i]$ and the root of G never exists outside of J.

Because $G_i(\underline{\mu}_i^{m+1}; \underline{\boldsymbol{\mu}}_{-i}^m) = G_i(\bar{\mu}_i^{m+1}; \bar{\boldsymbol{\mu}}_{-i}^m) = 0$ for $m = 1, 2, \cdots$, we have

$$G_i(\underline{c}_i; \underline{\boldsymbol{c}}_{-i}) = \lim_{m\to\infty} G_i(\underline{\mu}_i^{m+1}; \underline{\boldsymbol{\mu}}_{-i}^m) = 0 = \lim_{m\to\infty} G_i(\bar{\mu}_i^{m+1}; \bar{\boldsymbol{\mu}}_{-i}^m) = G_i(\bar{c}_i; \bar{\boldsymbol{c}}_{-i}), \quad i \in R.$$

Thus, $\underline{\boldsymbol{c}} = (\underline{c}_1, \cdots, \underline{c}_r)$ and $\bar{\boldsymbol{c}} = (\bar{c}_1, \cdots, \bar{c}_r)$ are the roots of G.

Now, we show that $\underline{\boldsymbol{c}} = \bar{\boldsymbol{c}}$. Suppose that $\underline{\boldsymbol{c}} \neq \bar{\boldsymbol{c}}$. Let us define that $\boldsymbol{\mu}(\theta) = \theta\bar{\boldsymbol{c}} + (1-\theta)\underline{\boldsymbol{c}}$ and $H_i(\theta) = G_i(\boldsymbol{\mu}(\theta))$ for all $i \in R$. From (A.2), we have for all $\boldsymbol{\mu} \in \text{int}I$:

$$\frac{\partial^2 G_i(\boldsymbol{\mu})}{\partial\mu_l\partial\mu_k} = -\left[1 - \frac{\sigma-1}{\sigma_X-1}(1-2F_i(\boldsymbol{\mu}))\right]\frac{F_{il}(\boldsymbol{\mu})}{\mu_k}\frac{\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}{\sum_{j\in R}\phi_{ji}\mu_j\lambda_j w_j^{1-\sigma_X}} < 0, \quad i, k, l \in R,$$

where $F_{il}(\boldsymbol{\mu}) = \partial F_i(\boldsymbol{\mu})/\partial \mu_l$. Thus, it holds that

$$\frac{\mathrm{d}^2 H_i(\theta)}{\mathrm{d}\theta^2} = \sum_{k\in R}\sum_{l\in R} \frac{\partial^2 G_i(\boldsymbol{\mu}(\theta))}{\partial \mu_l \partial \mu_k}(\bar{c}_k - \underline{c}_k)(\bar{c}_l - \underline{c}_l) < 0, \quad i \in R,$$

which implies that $H_i(\theta)$ is a concave function. Because $H_i(1) = H_i(0) = 0$, we have

$$\frac{\mathrm{d}H_i(0)}{\mathrm{d}\theta} = \sum_{j\in R} \frac{\partial G_i(\underline{\boldsymbol{c}})}{\partial \mu_j}(\bar{c}_j - \underline{c}_j) > 0, \quad i \in R,$$
$$\frac{\mathrm{d}H_i(1)}{\mathrm{d}\theta} = \sum_{j\in R} \frac{\partial G_i(\bar{\boldsymbol{c}})}{\partial \mu_j}(\bar{c}_j - \underline{c}_j) < 0, \quad i \in R.$$

Hence, for $\theta < 0$ and $i \in R$ we have

$$\boldsymbol{\mu}(\theta) \in I \implies G_i(\boldsymbol{\mu}(\theta)) < 0.$$

However, this contradicts (A.5).

Appendix A.2. The second step

For any $L > 0$, $\boldsymbol{\lambda}$ $(0 < \lambda_i < 1, \sum_{i\in R} \lambda_i = 1, i \in R)$ and $\boldsymbol{w} \in \mathbb{R}^r_{++}$, the wage potential in region i, w_i^P, becomes

$$w_i^P(\boldsymbol{\lambda}, \boldsymbol{w}, \mathcal{M}(L, \boldsymbol{\lambda}, \boldsymbol{w})) = \left[\sum_{j\in R} \frac{\phi_{ij}\mu_j\lambda_j w_j}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]^{\frac{1}{\sigma_X}}, \quad i \in R,$$

where $\boldsymbol{\mu} = \mathcal{M}(L, \boldsymbol{\lambda}, \boldsymbol{w}) = (\mathcal{M}_1(L, \boldsymbol{\lambda}, \boldsymbol{w}), \cdots, \mathcal{M}_r(L, \boldsymbol{\lambda}, \boldsymbol{w}))$. The equilibrium wage vector $\boldsymbol{w}^* = (w_1^*, \cdots, w_r^*)$ is given by

$$w_i^P(\boldsymbol{\lambda}, \boldsymbol{w}^*, \mathcal{M}(L, \boldsymbol{\lambda}, \boldsymbol{w}^*)) = w_i^*, \quad i \in R.$$

Let us define function $Z(\cdot) = (Z_1(\cdot), \cdots, Z_r(\cdot))$ as

$$Z_i(\boldsymbol{w}) = \mu_i\lambda_i \left[w_i^{-\sigma_X} \sum_{j\in R} \frac{\phi_{ij}\mu_j\lambda_j w_j}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}} - 1\right], \quad i \in R.$$

Then, $\boldsymbol{w}^*$ is given by $Z(\boldsymbol{w}^*) = 0$. We show that under the condition $1 < \sigma < \sigma_X$, $\boldsymbol{w}^*$ exists in int$S$, where S is the unit simplex on $\mathbb{R}^r$. In particular, when $r = 2$, w_1^*/w_2^* is determined uniquely.

Appendix A.2.1. Existence

It is straightforward to see that for any $\boldsymbol{w} \in \mathbb{R}^r_{++}$ the function Z satisfies the following properties:[17]

$$
\begin{aligned}
&(\mathrm{B}): \ Z(\boldsymbol{w}) \geqq -\boldsymbol{\lambda}, \\
&(\mathrm{C}): \ Z \text{ is continuous at } \boldsymbol{w}, \\
&(\mathrm{H}): \ Z(c\boldsymbol{w}) = Z(\boldsymbol{w}), \quad c \in \mathbb{R}, \ i \in R, \\
&(\mathrm{W}): \ \sum_{i \in R} w_i Z_i(\boldsymbol{w}) = 0.
\end{aligned}
$$

We shall show that for any $\boldsymbol{w}^o = (w_1^o, \cdots, w_r^o) \in \partial\mathbb{R}^r_+$ the function Z satisfies

$$
(\mathrm{C}'): \ \boldsymbol{w}^o \neq \boldsymbol{0} \implies \lim_{\boldsymbol{w} \to \boldsymbol{w}^o} \sum_i Z_i(\boldsymbol{w}) = \infty.
$$

Then, under (B), (C), (H), (W), and (C$'$), the existence of an equilibrium $\boldsymbol{w}^*$ in int$S$ is proved by Arrow and Hahn (1971).[18]

Let us take any sequence $(\boldsymbol{w}^m)_{m=0}^\infty$ on int$S$ that converges to $\boldsymbol{w}^o \in \partial\mathbb{R}^r_+$ $(\neq \boldsymbol{0})$ and define the sequence $(\mu_i^m)_{m=0}^\infty$ as $\mu_i^m = \mathcal{M}_i(L, \boldsymbol{\lambda}, \boldsymbol{w}^m)$. We then have

$$
(\exists \epsilon_i > 0)(\forall m \in \mathbb{N}): \ \mu_i^m \geqq \epsilon_i, \quad i \in R. \tag{A.7}
$$

In fact, if the above relationship is not true, we have a subsequence $(\mu_i^{m(l)})_{l=0}^\infty$ of $(\mu_i^m)_{m=0}^\infty$ such that $\lim_{l\to\infty} \mu_i^{m(l)} = 0$. Because of $\mu_i^{m(l)} = \mathcal{M}_i(L, \boldsymbol{\lambda}, \boldsymbol{w}^{m(l)})$, the following relationship must hold:

$$
\forall l \in \mathbb{N}: \ \lim_{l\to\infty} \left[\beta\mu_i^{m(l)} + (1-\beta)\left(b\left(\mu_i^{m(l)}\right)^{\frac{1}{1-\sigma}} \omega_i^{m(l)} \right)^{1-\sigma} \right] = \beta, \quad i \in R,
$$

where $\omega_i^{m(l)}$ is given by

$$
\forall l \in \mathbb{N}: \ \omega_i^{m(l)} = \frac{\rho_X}{a} \left[\frac{L}{f\sigma_X} \left(\mu_i^{m(l)}\lambda_i + \sum_{j\neq i} \phi_{ji}\mu_j^{m(l)}\lambda_j \left(w_j^{m(l)}/w_i^{m(l)} \right)^{1-\sigma_X} \right) \right]^{\frac{1}{\sigma_X - 1}}, \quad i \in R.
$$

[17]Note that $\mathcal{M}$ is continuous and homogeneous of degree zero for $\boldsymbol{w}$ on $\mathbb{R}^r_{++}$.

[18]See Arrow and Hahn (1971, Section 8 of Chapter 2).

However, if $(\mu_i^{m(l)})_{l=0}^{\infty}$ converges to zero, $1 < \sigma < \sigma_X$ implies

$$\lim_{l\to\infty}(\mu_i^{m(l)})^{\frac{1}{1-\sigma}}\omega_i^{m(l)} = \lim_{l\to\infty}\frac{\rho_X}{a}\left[\frac{L}{f\sigma_X}(\mu_i^{m(l)})^{\frac{\sigma_X-\sigma}{1-\sigma}}\left(\lambda_i + \frac{\sum_{j\neq i}\phi_{ji}\mu_j^{m(l)}\lambda_j\left(w_j^{m(l)}\right)^{1-\sigma_X}}{\mu_i^{m(l)}\left(w_i^{m(l)}\right)^{1-\sigma_X}}\right)\right]^{\frac{1}{\sigma_X-1}}$$
$$\geqq \lim_{l\to\infty}\frac{\rho_X}{a}\left[\frac{L}{f\sigma_X}(\mu_i^{m(l)})^{\frac{\sigma_X-\sigma}{1-\sigma}}\lambda_i\right]^{\frac{1}{\sigma_X-1}} = \infty.$$

Therefore, if $\lim_{l\to\infty}\mu_i^{m(l)} = 0$, then we have

$$0 < \beta = \lim_{l\to\infty}\left[\beta\mu_i^{m(l)} + (1-\beta)\left(b(\mu_i^{m(l)})^{\frac{1}{1-\sigma}}\omega_i^{m(l)}\right)^{1-\sigma}\right] = 0,$$

which is a contradiction.

Let us define R_o as a subset of R such that $w_i^o = 0$ for any $i \in R_o$. Because of (A.7), $\lim_{m\to\infty} Z_i(\boldsymbol{w}^m) < -\epsilon_i\lambda_i$ is satisfied for any $i \in R - R_o$. Thus, (W) leads to

$$\lim_{m\to\infty}\left(\sum_{i\in R_o} w_i^m Z_i(\boldsymbol{w}^m)\right) = -\lim_{m\to\infty}\left(\sum_{i\in R-R_o} w_i^m Z_i(\boldsymbol{w}^m)\right) > \sum_{i\in R-R_o}\epsilon_i\lambda_i w_i^o > 0,$$

which implies that there exist $i \in R_o$ such that $\lim_{m\to\infty} Z_i(\boldsymbol{w}^m) = \infty$. Thus we have (C$'$).

Appendix A.2.2. Uniqueness

Let us define a_{ij} as

$$a_{ij} \equiv \phi_{ij}\mu_i\lambda_i w_i^{1-\sigma_X}\mu_j\lambda_j w_j^{1-\sigma_X}\left[1 - \left(\frac{w_j}{w_i}\right)^{\sigma_X}\frac{\sum_{k\in R}\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right], \quad i, j \in R.$$

Obviously $a_{ii} = 0$ for all $i \in R$. In addition, since $w_i = w_i^P$ holds at the equilibrium for all $i \in R$, it holds that for all $i \in R$:

$$\sum_{j\in R} a_{ij} = \mu_i\lambda_i w_i^{1-\sigma_X}\sum_{j\in R}\phi_{ij}\mu_j\lambda_j w_j^{1-\sigma_X}\left[1 - \left(\frac{w_j}{w_i}\right)^{\sigma_X}\frac{\sum_{k\in R}\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]$$
$$= \mu_i\lambda_i w_i^{1-\sigma_X}\left[\sum_{j\in R}\phi_{ij}\mu_j\lambda_j w_j^{1-\sigma_X} - \frac{\sum_{k\in R}\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}{w_i^{\sigma_X}}\sum_{j\in R}\frac{\phi_{ij}\mu_j\lambda_j w_j}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]$$
$$= \mu_i\lambda_i w_i^{1-\sigma_X}\left[\sum_{j\in R}\phi_{ij}\mu_j\lambda_j w_j^{1-\sigma_X} - \frac{\sum_{k\in R}\phi_{ik}\mu_k\lambda_k w_k^{1-\sigma_X}}{w_i^{\sigma_X}}w_i^{\sigma_X}\right] = 0.$$

Thus we have

$$0 = \sum_{i\in R}\sum_{j\in R} a_{ij} = \sum_{i\in R}\left[\sum_{j>i}(a_{ij} + a_{ji})\right],$$

where

$$\operatorname{sgn}(a_{ij}+a_{ji}) = \operatorname{sgn}\left(\left[\left(\frac{w_i}{w_j}\right)^{\sigma_X} - \frac{\sum_{k\in R}\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]\left[\left(\frac{w_j}{w_i}\right)^{\sigma_X} - \frac{\sum_{k\in R}\phi_{kj}\mu_k\lambda_k w_k^{1-\sigma_X}}{\sum_{k\in R}\phi_{ki}\mu_k\lambda_k w_k^{1-\sigma_X}}\right]\right)$$
$$\leqq 0.$$

This means that the following equation is satisfied at the equilibrium:

$$\frac{w_i^*}{w_j^*} = \left[\frac{\sum_{k\in R}\phi_{ki}\mu_k^*\lambda_k\left(w_k^*\right)^{1-\sigma_X}}{\sum_{k\in R}\phi_{kj}\mu_k^*\lambda_k\left(w_k^*\right)^{1-\sigma_X}}\right]^{\frac{1}{\sigma_X}}, \quad i,j\in R. \tag{A.8}$$

Suppose that $r=2$. We define a function $\mathcal{G}$ as

$$\mathcal{G}\left(\lambda_1,\lambda_2,\mu_1,\mu_2,W\right) = \left[\frac{\mu_1\lambda_1 W^{1-\sigma_X}+\phi\mu_2\lambda_2}{\phi\mu_1\lambda_1 W^{1-\sigma_X}+\mu_2\lambda_2}\right]^{\frac{1}{\sigma_X}},$$

where $W = w_1/w_2$ and $\phi=\phi_{12}=\phi_{21}$. Substituting $\mu_i = \mathcal{M}_i(L,\lambda_1,\lambda_2,W,1)$ into $\mathcal{G}$, let us define $\mathcal{H}$ as $\mathcal{H}\left(L,\lambda_1,\lambda_2,W\right) = \mathcal{G}\left(\lambda_1,\lambda_2,\mathcal{M}_1(L,\lambda_1,\lambda_2,W,1),\mathcal{M}_2(L,\lambda_1,\lambda_2,W,1),W\right)$. Then, taking the differential of $\mathcal{H}$ with respect to W, we have

$$\begin{aligned}\frac{W}{\mathcal{H}}\frac{\partial\mathcal{H}}{\partial W} &= \frac{W}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial W} + \left(\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1}\right)\left(\frac{W}{\mu_1}\frac{\partial\mathcal{M}_1}{\partial W}\right) + \left(\frac{\mu_2}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_2}\right)\left(\frac{W}{\mu_2}\frac{\partial\mathcal{M}_2}{\partial W}\right)\\ &= \left[1-\sigma_X+\frac{W}{\mu_1}\frac{\partial\mathcal{M}_1}{\partial W}-\frac{W}{\mu_2}\frac{\partial\mathcal{M}_2}{\partial W}\right]\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1}\\ &= \frac{(\mu_1-\mu_1F_{11}-\mu_2F_{12})(\mu_2-\mu_1F_{21}-\mu_2F_{22})}{\mu_1\mu_2\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]}\frac{W}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial W},\end{aligned} \tag{A.9}$$

where we use

$$\begin{aligned}\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1} &= -\frac{\mu_2}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_2} = \frac{1}{1-\sigma_X}\frac{W}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial W}\\ &= \frac{(1-\phi)(1+\phi)}{\sigma_X}\frac{\mu_1\lambda_1W^{1-\sigma_X}}{\mu_1\lambda_1W^{1-\sigma_X}+\phi\mu_2\lambda_2}\frac{\mu_2\lambda_2}{\phi\mu_1\lambda_1W^{1-\sigma_X}+\mu_2\lambda_2},\end{aligned}$$

and

$$\begin{aligned}\frac{W}{\mu_1}\frac{\partial\mathcal{M}_1}{\partial W} &= (\sigma_X-1)\frac{F_{12}}{\mu_1}\frac{\mu_2-\mu_1F_{21}-\mu_2F_{22}}{(1-F_{11})(1-F_{22})-F_{12}F_{21}},\\ \frac{W}{\mu_2}\frac{\partial\mathcal{M}_2}{\partial W} &= (1-\sigma_X)\frac{F_{21}}{\mu_2}\frac{\mu_1-\mu_1F_{11}-\mu_2F_{12}}{(1-F_{11})(1-F_{22})-F_{12}F_{21}},\end{aligned} \tag{A.10}$$

where $F_{ij} = \partial F_i/\partial\mu_j$. Because (A.2) and $1<\sigma<\sigma_X$, the following relationship holds:

$$\mu_i-\mu_1F_{i1}-\mu_2F_{i2} = \mu_i\left(1-\frac{\sigma-1}{\sigma_X-1}(1-\mu_i)\right) > 0, \quad i=1,2.$$

Because we have

$$\begin{aligned}\mu_1\mu_2\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right] &= (\mu_1-\mu_1F_{11}-\mu_2F_{12})(\mu_2-\mu_1F_{21}-\mu_2F_{22})\\ &\quad+\mu_1F_{21}(\mu_1-\mu_1F_{11}-\mu_2F_{12})\\ &\quad+\mu_2F_{12}(\mu_2-\mu_1F_{21}-\mu_2F_{22}),\end{aligned}$$

(A.9) implies $\partial\mathcal{H}/\partial W<0$, which means W^* must be unique.

Appendix B. Proof of Proposition 2

Appendix B.1. Effect of change of λ

When we write $\lambda_1=\lambda$ and $\lambda_2=1-\lambda$, then W^* is defined implicitly as a function of L and λ by

$$W^*(L,\lambda)\equiv\mathcal{H}\left(L,\lambda,1-\lambda,W^*(L,\lambda)\right). \tag{B.1}$$

Thus we have

$$\begin{aligned}\frac{\partial W^*}{\partial\lambda} &= \frac{1}{1-(\partial\mathcal{H}/\partial W)}\left[\left(\frac{\partial\mathcal{G}}{\partial\lambda_1}-\frac{\partial\mathcal{G}}{\partial\lambda_2}\right)+\left(\frac{\partial\mathcal{M}_1}{\partial\lambda_1}-\frac{\partial\mathcal{M}_1}{\partial\lambda_2}\right)\frac{\partial\mathcal{G}}{\partial\mu_1}+\left(\frac{\partial\mathcal{M}_2}{\partial\lambda_1}-\frac{\partial\mathcal{M}_2}{\partial\lambda_2}\right)\frac{\partial\mathcal{G}}{\partial\mu_2}\right]\\ &= \frac{W^*}{1-(\partial\mathcal{H}/\partial W)}\frac{\mu_2\lambda_1(\mu_1-\mu_1F_{11}-\mu_2F_{12})+\mu_1\lambda_2(\mu_2-\mu_1F_{21}-\mu_2F_{22})}{\mu_1\mu_2\lambda_1\lambda_2\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]}\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1}>0,\end{aligned} \tag{B.2}$$

where we use

$$\begin{aligned}\frac{\partial\mathcal{M}_i}{\partial\lambda_i} &= \frac{\mu_i}{\lambda_i}\frac{F_{ii}(1-F_{jj})+F_{12}F_{21}}{(1-F_{11})(1-F_{22})-F_{12}F_{21}}, \quad i,j=1,2,\ i\neq j,\\ \frac{\partial\mathcal{M}_i}{\partial\lambda_j} &= \frac{\mu_j}{\lambda_j}\frac{F_{ij}}{(1-F_{11})(1-F_{22})-F_{12}F_{21}}, \quad i,j=1,2,\ i\neq j.\end{aligned} \tag{B.3}$$

(B.2) means that W^* $(=w_1^*/w_2^*)$ monotonically increases with respect to population share of region 1. Because (A.8) leads to

$$\frac{\omega_1^*}{\omega_2^*}(L,\lambda)=\left(\frac{w_1^*}{w_2^*}(L,\lambda)\right)^{\frac{2\sigma_X-1}{\sigma_X-1}}=\left(\frac{P_1^*}{P_2^*}(L,\lambda)\right)^{\frac{1-2\sigma_X}{\sigma_X}}, \tag{B.4}$$

an increase in population share of region 1 also raises the equilibrium real wage ratio, ω_1^*/ω_2^*, but reduces the ratio of the equilibrium price index, P_1^*/P_2^*. In addition, from (A.8) and (B.4), w_1^*/w_2^*, ω_1^*/ω_2^* and P_1^*/P_2^* are equal to one when both regions have the same population, that is $\lambda_1=\lambda_2=1/2$.

Appendix B.2. Effect of change of L

From (B.1), we have

$$\begin{aligned}\frac{L}{W^*}\frac{\partial W^*}{\partial L} &= \frac{1}{1-(\partial\mathcal{H}/\partial W)}\frac{L}{W^*}\left(\frac{\partial\mathcal{G}}{\partial\mu_1}\frac{\partial\mathcal{M}_1}{\partial L}+\frac{\partial\mathcal{G}}{\partial\mu_2}\frac{\partial\mathcal{M}_2}{\partial L}\right)\\ &= \frac{1}{1-(\partial\mathcal{H}/\partial W)}\left(\frac{L}{\mu_1}\frac{\partial\mathcal{M}_1}{\partial L}-\frac{L}{\mu_2}\frac{\partial\mathcal{M}_2}{\partial L}\right)\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1}\\ &= \frac{1}{1-(\partial\mathcal{H}/\partial W)}\frac{\sigma-1}{\sigma_X-1}\frac{\mu_2-\mu_1}{(1-F_{11})(1-F_{22})-F_{12}F_{21}}\frac{\mu_1}{\mathcal{G}}\frac{\partial\mathcal{G}}{\partial\mu_1},\end{aligned} \tag{B.5}$$

where we use

$$\begin{aligned}\frac{L}{\mu_1}\frac{\partial\mathcal{M}_1}{\partial L} &= \frac{F_{12}(\mu_1F_{21}+\mu_2F_{22})+(1-F_{22})(\mu_1F_{11}+\mu_2F_{12})}{\mu_1\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]},\\ \frac{L}{\mu_2}\frac{\partial\mathcal{M}_2}{\partial L} &= \frac{F_{21}(\mu_1F_{11}+\mu_2F_{12})+(1-F_{11})(\mu_1F_{21}+\mu_2F_{22})}{\mu_2\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]}.\end{aligned} \tag{B.6}$$

Because $\mu(\omega_i^*,1) \gtreqless \mu(\omega_j^*,1)$ as $\omega_i^* \gtreqless \omega_j^*$, (B.4) and (B.5) imply that Proposition 2 is true.

Appendix C. Existence of function $\boldsymbol{\mathcal{L}}$

Because $\mu_i^*(L,\lambda) \equiv \mathcal{M}_i(L,\lambda,1-\lambda,W^*(L,\lambda),1)$, the differential of $\bar{n}^*(L,\lambda)$ with respect to L becomes

$$\frac{\partial\bar{n}^*(L,\lambda)}{\partial L} = -\frac{1}{b}\left[\left(\lambda\frac{\partial\mathcal{M}_1}{\partial L}+(1-\lambda)\frac{\partial\mathcal{M}_2}{\partial L}\right)+\left(\lambda\frac{\partial\mathcal{M}_1}{\partial W}+(1-\lambda)\frac{\partial\mathcal{M}_2}{\partial W}\right)\frac{\partial W^*}{\partial L}\right] < 0,$$

where we use (A.2), (A.10), (B.5), (B.6) and

$$\frac{\partial\mu_i^*}{\partial L} = \frac{\partial\mathcal{M}_i}{\partial L}+\frac{\partial\mathcal{M}_i}{\partial W}\frac{\partial W^*}{\partial L}, \quad i=1,2.$$

Thus, if $0<b<1$, there exists L uniquely such that $\bar{n}^*(L,\lambda)=1$, which implies the existence of $\mathcal{L}$ such that $\bar{n}^*(\mathcal{L}(\lambda),\lambda)=1$ for any λ.

Appendix D. Derivation of (16)

At $(\lambda_1,\, \lambda_2) \in \{(1/2,\, 1/2),\, (1,\, 0),\, (0,\, 1)\}$, the second condition of (15) is satisfied. If $(\lambda_1,\, \lambda_2)=(1/2,\, 1/2)$, then $w_1^*=w_2^*$, $\mu_1^*=\mu_2^*$ and $n_1^*=n_2^*$. Thus, $\bar{n}^{**}=1$ is equivalent to

$$\frac{(1-\beta)b^{1-\sigma}}{\beta\left(\frac{a}{\rho_X}\right)^{1-\sigma}\left[\frac{1+\phi}{2}\frac{L^{**}}{f\sigma_X}(1-b)\right]^{\frac{\sigma-1}{\sigma_X-1}}+(1-\beta)b^{1-\sigma}} = b,$$

which implies $L^{**} = 2D/(1+\phi)$. On the other hand, if $(\lambda_1, \lambda_2) \in \{(1, 0), (0, 1)\}$, then $\bar{n}^{**} = 1$ is equivalent to

$$\frac{(1-\beta)b^{1-\sigma}}{\beta\left(\frac{a}{\rho X}\right)^{1-\sigma}\left[\frac{L^{**}}{f\sigma X}(1-b)\right]^{\frac{\sigma-1}{\sigma X-1}}+(1-\beta)b^{1-\sigma}} = b,$$

which implies $L^{**} = D$.

Appendix E. Proof of Proposition 4

Taking the differential of $\mathcal{L}$ with respect to λ, we obtain

$$\frac{\mathrm{d}\mathcal{L}(\lambda)}{\mathrm{d}\lambda} = -\frac{\mu_1^* - \mu_2^* + \lambda\left(\partial\mu_1^*/\partial\lambda\right) + (1-\lambda)\left(\partial\mu_2^*/\partial\lambda\right)}{\lambda\left(\partial\mu_1^*/\partial L\right) + (1-\lambda)\left(\partial\mu_2^*/\partial L\right)}.$$

Using

$$\frac{\partial\mu_i^*}{\partial\lambda} = \left(\frac{\partial\mathcal{M}_i}{\partial\lambda_1} - \frac{\partial\mathcal{M}_i}{\partial\lambda_2}\right) + \frac{\partial\mathcal{M}_i}{\partial W}\frac{\partial W^*}{\partial\lambda}, \quad i = 1, 2,$$

we have

$$\begin{aligned}\Theta &\equiv \mu_1^* - \mu_2^* + \lambda\frac{\partial\mu_1^*}{\partial\lambda} + (1-\lambda)\frac{\partial\mu_2^*}{\partial\lambda} \\ &= \mu_1^* - \mu_2^* + \left(\lambda_1\frac{\partial\mathcal{M}_1}{\partial W} + \lambda_2\frac{\partial\mathcal{M}_2}{\partial W}\right)\frac{\partial W^*}{\partial\lambda} + \left(\lambda_1\frac{\partial\mathcal{M}_1}{\partial\lambda_1} + \lambda_2\frac{\partial\mathcal{M}_2}{\partial\lambda_1}\right) - \left(\lambda_1\frac{\partial\mathcal{M}_1}{\partial\lambda_2} + \lambda_2\frac{\partial\mathcal{M}_2}{\partial\lambda_2}\right).\end{aligned}$$

From (A.2), (A.10), (B.2) and (B.3), we obtain $\lim_{\lambda\to1}\Theta > 0$ and $\lim_{\lambda\to1/2}\Theta = 0$, which means that $\mathcal{L}$ has the following local properties: $\lim_{\lambda\to1}\mathcal{L}'(\lambda) < 0$ and $\lim_{\lambda\to1/2}\mathcal{L}'(\lambda) = 0$.

Taking the differential of $\Lambda\left(\mathcal{L}(\lambda), \lambda\right)$ with respect to λ yields

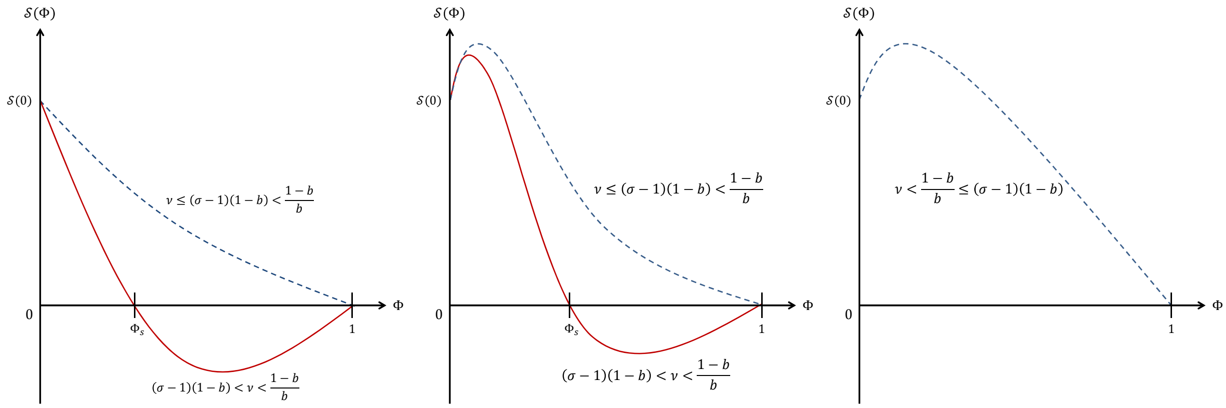
$$\begin{aligned}\frac{\mathrm{d}\Lambda\left(\mathcal{L}(\lambda),\lambda\right)}{\mathrm{d}\lambda} = &-\left[\frac{\mu_2^* - \mu_1^*}{b} + \nu\frac{\omega_1^*/\omega_2^* - 1}{\lambda\left(\omega_1^*/\omega_2^*\right) + 1 - \lambda}\right] \\ &+ \frac{1-\lambda}{b}\left[\left(\frac{\partial\mu_2^*}{\partial\lambda} - \frac{\partial\mu_1^*}{\partial\lambda}\right) + \left(\frac{\partial\mu_2^*}{\partial L} - \frac{\partial\mu_1^*}{\partial L}\right)\frac{\mathrm{d}\mathcal{L}(\lambda)}{\mathrm{d}\lambda}\right] \\ &+ \nu\frac{1-\lambda}{\left[\lambda\left(\omega_1^*/\omega_2^*\right) + 1 - \lambda\right]^2}\left[\frac{\partial\left(\omega_1^*/\omega_2^*\right)}{\partial\lambda} + \frac{\partial\left(\omega_1^*/\omega_2^*\right)}{\partial L}\frac{\mathrm{d}\mathcal{L}(\lambda)}{\mathrm{d}\lambda} - \left(\frac{\omega_1^*}{\omega_2^*} - 1\right)^2\right].\end{aligned}$$

Then, converging λ to $1/2$, the following relationship is obtained:

$$\lim_{\lambda\to1/2}\frac{\mathrm{d}\Lambda\left(\mathcal{L}(\lambda),\lambda\right)}{\mathrm{d}\lambda} \gtreqless 0 \iff \nu \gtreqless \nu_b \equiv (\sigma-1)(1-b). \tag{E.1}$$

Similarly, converging λ to one, we have:

$$\lim_{\lambda\to1}\frac{\mathrm{d}\Lambda\left(\mathcal{L}(\lambda),\lambda\right)}{\mathrm{d}\lambda} \gtreqless 0 \iff \mathcal{S}\left(\Phi(\phi),\nu\right) \gtreqless 0,$$



(d) $1 < \sigma \leqq 2$ (e) $2 < \sigma < (1+b)/b$ (f) $(1+b)/b \leqq \sigma$

Figure A2: The Shape of $\mathcal{S}$

where $\Phi \equiv \phi^{\frac{\sigma-1}{\sigma X}\frac{2\sigma X-1}{\sigma X-1}}$ and $\mathcal{S}(\Phi, \nu)$ is defined by

$$\mathcal{S}(\Phi, \nu) \equiv (1-b) - b\nu - (1-b)(1+\nu)\Phi + \nu\left[b\Phi^{\frac{1}{\sigma-1}} + (1-b)\Phi^{\frac{\sigma}{\sigma-1}}\right].$$

We can readily see that, for any $\phi \in (0,\, 1)$,

$$\mathcal{S}\left(\Phi(\phi), \nu\right) \gtreqless 0 \iff \nu \lesseqgtr \nu_s \equiv \frac{1-b}{b+(1-b)\Phi}\frac{1-\Phi}{1-\Phi^{\frac{1}{\sigma-1}}}$$

is satisfied, which means that the following relationship is true for any $\phi \in (0,\, 1)$ and $\nu \geqq 0$:

$$\lim_{\lambda\to 1}\frac{\mathrm{d}\mathit{\Lambda}\left(\mathcal{L}(\lambda), \lambda\right)}{\mathrm{d}\lambda} \gtreqless 0 \iff \nu \lesseqgtr \nu_s. \tag{E.2}$$

On the other hand, for any $\nu < (1-b)/b$, we can describe the shape of $\mathcal{S}(\Phi, \nu)$ as Figure A2. Thus, for any $\phi \in (0,\, 1)$ and $\nu \geqq 0$, we have

$$(\sigma-1)(1-b) < \nu < (1-b)/b \implies \left(\lim_{\lambda\to 1}\frac{\mathrm{d}\mathit{\Lambda}\left(\mathcal{L}(\lambda), \lambda\right)}{\mathrm{d}\lambda} \gtreqless 0 \ \Leftrightarrow\ \phi \lesseqgtr \phi_s\right), \tag{E.3}$$

where $\phi_s \in (0,\, 1)$ is given by $\mathcal{S}\left(\Phi(\phi_s), \nu\right) = 0$. We conclude that Proposition 4 is true by (E.1), (E.2) and (E.3).

Finally, we show the relationship between ν_s and ϕ_s. Because of $\partial\mathcal{S}/\partial\nu < 0$, we have $\phi_s'(\nu) < 0$ which means that $\nu_s'(\phi) < 0$. Thus, by $\lim_{\phi\to 0}\nu_s = (1-b)/b$ and $\lim_{\phi\to 1}\nu_s = (\sigma-1)(1-b)$, the relationship between ν_s and ϕ_s can be described as Figure 5.

References

Arrow, K.J., Hahn, F.H. (1971) *General Competitive Analysis*. Edinburgh: Oliver and Boyd.

Barro, R., Becker, G.S. (1989) Fertility choice in a model of economic growth. *Econometrica*, 57: 481–501.

Becker, G.S. (1965) A theory of the allocation of time, *Economic Journal*. 75: 493–517.

Becker, G.S., Barro, R. (1988) A reformulation of the theory of fertility. *Quarterly Journal of Economics*, 103: 1–25.

Becker, G.S., Lewis, H.G. (1973) On the interaction between the quantity and quality of children. *Journal of Political Economy*, 81: 279–288.

Becker, G.S., Murphy, K.M., Tamura, R. (1990) Human capital, fertility and economic growth. *Journal of Political Economy*, 98: 12–37.

Behrens, K., Lamorgese, A., Ottaviano, G., Tabuchi, T. (2009) Beyond the home market effect: market size and specialization in a multi-country world. *Journal of International Economics*, 79: 259–265.

Borg, M.O. (1989) The income-fertility relationship: effect of the net price of a child. *Demography*, 26: 301–310.

Davis, D.R. (1998) The home market, trade, and industrial structure. *American Economic Review*, 88: 1264–1276.

Dixit, A.K., Stiglitz, J.E. (1977) Monopolistic competition and optimum product diversity. *American Economic Review*, 67: 297–308.

Docquier, F. (2004) Income distribution, non-convexities and the fertility-income relationship. *Economica*, 71: 261–273.

Fujita, M., Krugman, P., Venables, A.J. (1999) *The Spatial Economy: Cities, Regions and International Trade*. Cambridge, MA: MIT Press.

Galor, O., Weil, D.N. (1996) The gender gap, fertility and growth, *American Economic Review*, 86: 374–387.

Goto, H., Minamimura, K. (2015) Fertility, regional demographics, and economic integration. *RIEB Discussion Paper Series*, 2015-17.

Jones, L.E., Tertilt, M. (2008) An economic history of fertility in the US: 1826–1960. In Rupert, P. (ed.) *Frontiers of Family Economics*, 1: 165–230. Bingley, UK: Emerald Press.

Kondo, K. (2015) Does agglomeration discourage fertility? Evidence from the Japanese general social survey 2000–2010. *RIETI Discussion Paper Series*, 15-E-067.

Krugman, P. (1993) On the number and location of cities. *European Economic Review*, 37: 293–298.

Malthus, R. (1798) *An Essay on the Principle of Population, as it Affects the Future Improvement of Society, with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*. London.

Maruyama, A., Yamamoto, K. (2010) Variety expansion and fertility rates. *Journal of Population Economics*, 23: 57–71.

Ministry of Health, Labour and Welfare. (2014) Vital Statistics.

Moav, O. (2005) Cheap Children and the Persistence of Poverty. *Economic Journal*, 115: 88–110.

Samuelson, P.A. (1952) The transfer problem and transport costs: the terms of trade when impediments are absent. *Economic Journal*, 62: 278–304.

Sato, Y. (2007) Economic geography, fertility and migration. *Journal of Urban Economics*, 61: 372–387.

Sato, Y., Yamamoto, K. (2005) Population concentration, urbanization, and demographic transition. *Journal of Urban Economics*, 58: 45–61.

Schultz, T.P. (1985) Changing world prices, women's wages, and the fertility transition: Sweden, 1860–1910. *Journal of Political Economy*, 93: 1126–1154.

Shoven, J.B. (2008) *Demography and the Economy.* Chicago: The University of Chicago Press.

Statistics Bureau, Ministry of Internal Affairs and Communications, Japan. (2013a) Social Indicators by Prefecture 2013.

Statistics Bureau, Ministry of Internal Affairs and Communications, Japan. (2013b) Japan Statistical Yearbook 2014.

Von Thünen, J.H. (1826) Der Isolierte Staat in Beziehung auf Landtschaft und Nationalökonomie. Hamburg (English translation by Wartenberg, C.M., 1996. Von Thünen's Isolated Stat. Oxford: Pergamon Press.).

Weibull, J.W. (1995) *Evolutionary Game Theory.* Cambridge, MA: MIT Press.

Yu, Z. (2005) Trade, market size, and industrial structure: revisiting the home-market effect. *Canadian Journal of Economics*, 38: 255–272.