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Effects on the Cross-Country Difference in the Minimum Wage on International Trade, Growth and Unemployment *

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Effects on the cross-country difference in the minimum wage on international trade, growth and unemployment*

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Abstract

We construct a dynamic general-equilibrium North-South growth model with international trade with both homogenous and heterogeneous firms, endogenous northern economic growth, and unemployment. Unemployment is emerged from the imbalance between the endogenous labor supply and the firms' labor demand under binding the minimum wage policy. The north produces two goods, high-tech good and low-tech good, while the South produces only low-tech good by the scarcity of technology. Both goods are traded between the countries. The production of the high-tech good needs R&D activity for variety creation, which is a source of economic growth.

In this setting, we analyze the southern policy change that increases the southern minimum wage, and show that the increase in the southern minimum wage affects the structure of international trade and the northern growth rate and unemployment.

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1. Introduction

Recently, some developing countries have rapidly grown, and affect the economy of the advanced countries through international transactions. Especially, China has grown quickly since the 1980s and the volume of international trade has also been increasing. Per capita real GDP in terms of U.S. dollar was 307.35 in 1980 and is 6077.65 in 2012[1]. According to Autor, et al. (2013), the amount of U.S. manufacturing imports from China increased from 2.9% in 1991 to 11.7% in 2007. While the expenditure share of the U.S. on Chinese goods was 0.6% in 1991, the rate raises up to 4.6% in 2007. The competition between Chinese goods and the U.S. goods is fierce and affects the labor market in the U.S. Autor et al. (2013) have examined the effects of import competition on the U.S. labor market, and showed that its competition has a negative effects on employment from 1990 to 2007. A situation like this occurs even in EU. Bloom et al. (2011) show that the import from China to EU has grown dramatically from 1980 to 2007. Moreover, they showed that it leads low productive firms exit and there exists a positive effect on innovation and a negative effect on employment in manufacturing. The import competition with China also contributes to 15% of increases in innovation in EU between 2000 and 2007. An increases in imports by 10% decreases employment by 3.5%. They showed that import from China affects economic growth in developed countries.

However, with globalization in China, the government of China has increased the minimum wage. The labor law regarding minimum wage in China was first settled in 1994 to protect vulnerable workers from the most egregious aspects of intensively competitive markets. According to Fang and Lin (2013) and Wang and Gunderson (2011), the minimum wages in China have increased recently. Wang and Gunderson

[1] The data is from World Economic Outlook Databases in 2013.

(2011) showed that the increase in the minimum wage has the negative effects on Chinese economy. In other developing countries such as Thailand, Vietnam and so on, the local governments increase the legal minimum wages in the 2010s. These countries also contribute to the expansion of the world trade, and largely affect the economy of the advanced countries, i.e., the U.S. and the EU.

How does the recent tide of increasing the minimum wage in developing countries affect the economy of the advanced countries through globalization? To examine the relationship of the minimum wages between the developing country and the advanced country, we construct a two country endogenous growth model. Let us denote the U.S. and the EU by the North and China (or developing countries) by the South, respectively. We examine the effects of increases in the minimum wage in the South on innovation in the North through changes in employment in both countries. In the North, there are three sectors: low-technology and high-technology goods sectors, and R&D sector. In the low-tech sector which exists in both countries, there is a continuum of firms with identical technologies. In this sector, a fixed number of firms produces goods with labor. Monopolistically competitive profit is redistributed to consumer in a lump sum distribution.

The high-tech producer is well-differentiated and supplied by monopolistic competitors. We consider two cases of the high-tech sector: homogenous firms and heterogeneous firms. In the case of homogenous firms, once a firm succeeds in R&D activity of creating a high-tech brand (variety), it can sell the variety monopolistically and permanently. In the case of firm heterogeneity, we use the framework of *à la* Melitz (2003). Before entering the market, each firm incurs the sunk cost to find out productivity. In addition, each firm must pay the sunk cost to enter each market. Based on the realization of productivity, each firm exclusively chooses to serve both the markets, the northern market, and exit.

In the both cases, the R&D sector is a perfectly competitive sector. Following Bloom, et al. (2014), the R&D sector exists only in the North. We assume that there are inter-temporal knowledge spillovers, and then endogenous growth with variety expansion occurs in this model. Production of any goods and R&D activities uses labor. The governments in both countries implement the minimum wage policy. The labor is supplied inelastically. The demand of the workers for each activity also depends on the minimum wage. If the minimum wage is set above the equilibrium wage, unemployment occurs.

Our paper is related to the following two research branches of literature. First is the analysis of international trade between the advanced country and the developing country. Bloom et al. (2014) constructed a North-South model with trade and growth to examine the import competition from China. They assumed that a constant fraction of the patent of goods are produced in the South and allow to exporting to the North. They further assume that increases in its fraction imply trade liberalization and showed that trade liberalization leads to a higher growth rate in their model. They considered the perfectly competitive labor market. Thus, they did not considered the effects on labor markets in the North. Autor et al. (2013) empirically showed that import from China negatively affects employment in U.S.

Secondly, our research is also related to the relationship between international trade and the minimum wage. The minimum wage is one of the most important factors which generates labor market rigidities and then, the involuntary unemployment. Davis (1998), who has first dealt with the relationship between labor market rigidities and international trade, showed that bad foreign labor market institutions exert a positive spillover on domestic workers. On the other hand, Felbermayr et al. (2009) empirically showed that trade partner's real wage rigidity positively affects unemployment in own country for 20 OECD countries between 1982 and 2003.

Egger, Egger and Marksen (2012) constructed a two country static model of trade with firm heterogeneity, where there exists unemployment in both countries. Moreover, they examined an increase in the minimum wages in one country affects negatively on international trade and employment in both countries. The one-side increase in the minimum wage in the foreign country decreases the number of varieties the country produces, which decreases the available varieties in the world. The decrease in the available varieties reduces the domestic country's demand for goods, and enforces firm to exit in from the domestic market. As a result, the domestic country is also harmed by the increase in the foreign country's minimum wage. However, they did not consider the effects of minimum wage on economic growth through employment. Moreover, whether their arguments are consistent with the context of North-South model is not clear.

Sener (2006) examined the effects of changes in the minimum wage on unemployment in the EU and economic growth in the EU and the U.S. However, his model considers only unemployment in one country not in both. Grieben (2009) constructed a North-South R&D based growth model of trade. He examined the effects of globalization through increases in population in the South on innovation in both countries and the northern employment. But, there exists no southern unemployment in his model.

The remainder of the paper is as follow. Section 2 explains the base of the model and states the characteristic of the steady state equilibrium. Section 3 analyzes the effects of the minimum wage on international trade, growth and unemployment. Section 4 concludes with summary and future research.

2. The model

There are two countries in the world, North and South. The households in both countries obtain the utility from leisure and consumption of two goods: the high-tech good and the low-tech good. In each countries, the worker's wage is protected by the minimum wage set by the government, w^N, w^S. Under the minimum wage, each worker decides the allocation of working. Firms also decide how much workers they employ for profit maximization. If the minimum wage is higher than the equilibrium wage where the labor supply and the labor demand is equalized, the unemployment occurs.

2.1 The household

We consider the representative household. The household lives infinitely and obtain the utility from consumption of two types of differentiated goods: the high-tech goods D^i and the low-tech goods $Y^i, i = N, S$. He inelastically supplies L^i for working and earns the wage $w^i, i = N, S$. w^i is the minimum wage set by the government.

The North The northern consumer uses a part of the wage income for the consumption and saves the residual of the income. The northern household maximizes the following lifetime utility:

$$U(t) = \int_t^{\infty} e^{-\rho(\tau-t)}[\alpha \log D^N(\tau) + \beta \log Y^N(\tau)]d\tau, \alpha, \beta \in (0,1), \alpha + \beta = 1 \tag{1}$$

where ρ is the subjective discount rate. $D^N(t)$ and $Y^N(t)$ are the sub-utility of the consumption for the high-tech good and the low-tech good at time t. The sub-utility of the high-tech good has the following quantity index given by:

$$D^N(t) = \left[\int_0^{m_D(t)} x_D^N(\kappa)^{\frac{\sigma-1}{\sigma}} d\kappa\right]^{\frac{\sigma}{\sigma-1}}, \sigma > 1 \tag{2}$$

where $x_D^N(\kappa)$ is the consumer's quantity consumed of a product κ at time t, and $m_D(t)$ is the number of available variety of the high-tech good in an economy at time t. The parameter σ measures the degree of product differentiation. The sub-utility of the low-tech good also has the following quantity index given by

$$Y^N(t) = \left[\int_0^{m_Y} x_Y^N(\kappa)^{\frac{\sigma-1}{\sigma}} d\kappa\right]^{\frac{\sigma}{\sigma-1}}, \sigma > 1 \tag{3}$$

where $x_Y^N(\kappa)$ is the consumer's quantity consumed of a product κ at time t, and m_Y is the number of available variety of the low-tech good in an economy.

The northern household maximizes the utility subjected to the following budget constraint:

$$\dot{A}^N(t) = r^N A^N(t) + w^N L^N(t) - E^N(t)$$

where $A^N(t)$ is the stock or saving of the household and E^N is defined as the expenditure for the consumption goods, D^i and $Y^i (i = N, S)$:

$$E^i \equiv P_D^N D^N + P_Y^N Y^N$$

where P_D^N is the price index of the high-tech good and P_Y^N is the price index of the low-tech good. Since the consumptions of the high-tech good and the low-tech good have the Cobb-Douglas form, each expenditure for both goods is

$$P_D^N D^N = \alpha E^N, P_Y^N Y^N = \beta E^N \tag{4}$$

The budget constraint for the high-tech goods is

$$\int_0^{m_D} p_D^N(\kappa) x_D^N(\kappa)\, d\kappa = \alpha E^N \tag{5}$$

With the sub-utility in Eq. (2), solving the static optimization problem yields the demand function of the high-tech good:

$$x_D^N(\kappa) = \frac{p_D^N(\kappa)^{-\sigma} \alpha E^N}{\{P_D^N(t)\}^{1-\sigma}} \tag{6}$$

where

$$P_D^N(t) = \left[\int_0^{m_D} p_D^N(\kappa)^{1-\sigma}\, d\kappa\right]^{\frac{1}{1-\sigma}}$$

Similarly, the budget constraint for the low-tech goods is

$$\int_0^{m_Y} p_Y^N(\kappa) x_Y^N(\kappa)\, d\kappa = \beta E^N \tag{7}$$

With the sub-utility in Eq. (3), solving the static optimization problem yields the demand function of the low-tech good:

$$x_Y^N(\kappa) = \frac{p_Y^N(\kappa)^{-\sigma} \beta E^N}{\{P_Y^N(t)\}^{1-\sigma}} \tag{8}$$

where

$$P_Y^N = \left[\int_0^{m_Y} p_Y^N(\kappa)^{1-\sigma}\, d\kappa\right]^{\frac{1}{1-\sigma}}$$

To solve the dynamic optimization problem, we define the following Hamiltonian:

$$\mathcal{H} \equiv e^{-\rho}[\alpha \log D^N(t) + \beta \log Y^N(t)] + \lambda[r^N(t)A^N(t) + w^N(t)L^N - E^N(t)]$$

Substituting Eq. (4) into $D^N(t)$ and $Y^N(t)$ in the Hamiltonian and solving the Hamiltonian, we obtain a general Euler equation:

$$\frac{\dot{E}^N}{E^N} = r^N - \rho$$

The South Different from the northern household, the southern consumer cannot save the income for the next time. In other words, the southern household sorts the wage income into the consumption of the high-tech good and the low-tech good at every time.

The demands of the consumptions goods are the same ones in the North, Eq. (6) and (8).

2.2 Firms

2.2.1 Low-tech good

Both countries produce the low-tech goods and export them to each other. The low-tech goods is differentiated and sold under monopolistic competition market. Each firm in the differentiated sector is able to produce a unique variety of low-tech good. Country $i, i = N, S$ is endowed with a fixed mass m_Y^i of these varieties. For simplicity, we assume that $\bar{m}_Y = m_Y^N = \frac{m_Y^S}{\gamma}$, where $\gamma \geq 0$ is the exogenous parameter. The productivity of the low-tech goods' producer is homogenous, and all varieties are produced and exported. The production of the low-tech good uses one unit of labor. The profit of low-tech good becomes

$$\pi_Y^i(\kappa) = p_Y^i(\kappa)x_Y^i(\kappa) - w^i x_Y^i(\kappa) + p_Y^i(\kappa)x_Y^j - w^i x_Y^j(\kappa), i, j = N, S, i \neq j$$

All firms face the demand of the low-tech good in Eq. (8) and set the price as

$$p_Y^i(\kappa) = p_Y^i = \frac{\sigma w^i}{\sigma - 1}, i = N, S$$

The operating profit is

$$\pi_Y^i = \frac{(w^N)^{1-\sigma}\beta(E^N + E^S)}{\sigma\bar{m}_Y[(w^N)^{1-\sigma} + \gamma(w^S)^{1-\sigma}]}, i = N, S \tag{9}$$

Since free entry is not allowed, the positive profit, $m_Y^i \pi_Y^i$, occurs in equilibrium. The profit is distributed into all the households. The labor demand in the low-tech good's production equals that the number of variety times the amount of production, $m_Y^i x_Y^i, i = N, S$. Therefore, the wage income from the low-tech good production is expressed as

$$w^N L_Y = m_Y^N \pi_Y^N + m_Y^N \pi_Y^S = \frac{\beta(E^N + E^S)}{1 + \gamma\omega^{\sigma-1}} \tag{10}$$

where $\omega \equiv \frac{w^N}{w^S}$. Similarly, the southern wage income from the low-tech good production is

$$w^S L_Y^S = \frac{\beta(E^N + E^S)}{\frac{1}{\gamma\omega^{\sigma-1}} + 1} \tag{11}$$

2.2.2 High-tech goods}

The high-tech industry is only located in the North and supplies the goods for the North and the south. The high-tech good is also well-differentiated but the variety of the high-tech good is created by R&D activity. Differentiated with low-tech goods, the measure of the high-tech good increases over time. Each firm in the high-tech industry owns one variety. The high-tech goods are produced only by using labor and sold under monopolistic competition. In this section, all the firms are homogenous.

Production Each high-tech firm earns the following profit:

$$\pi_D(\kappa) = p_D^N(\kappa)x_D^N(\kappa)x_D^N(\kappa) - w^N x_D^N(\kappa) + p_D^S(\kappa)x_D^S(\kappa) - w^N x_D^S(\kappa)$$

From profit maximization, the monopolistic price of the variety is set as

$$p_D^N(\kappa) = \frac{\sigma w^N}{\sigma - 1} \tag{12}$$

Using Eq. (13), we calculate the operation profit:

$$\pi_D^N(\kappa) = \frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(w^N)^{1-\sigma}\alpha E^N(t)}{\{P_D^N(t)\}^{1-\sigma}} \tag{13}$$

Innovation We assume that the patent term of a variety is permanent. The value of firm is defined as the sum of the present discount value of profit:

$$V \equiv \int_t^\infty \pi_D(\tau) e^{-\int_t^\tau r(s)ds}\, d\tau$$

Differentiating the above equation, we obtain no-arbitrage condition. The no-arbitrage condition requires that the return from risk-free asset market equals to the one of equity security which consists of capital gain (or loss) and dividend.

$$\frac{\dot{V}(t)}{V(t)} + \frac{\pi_D}{V(t)} = r(t)$$

Rearranging the no-arbitrage condition, the value of firm is calculated as

$$V^i(t) = \frac{\pi_D}{r(t) - \frac{\dot{V}(t)}{V(t)}} \tag{14}$$

In the innovation sector, firms engage in R&D activities. The unit labor requirement associated with creating a variety is $b_I(t)$. In other words, it takes $b_I(t)$ units of labor at time t to create one variety. Each individual firm treats $b_I(t)$ as a parameter, but it can change over time due to knowledge spillovers. We assume perfect knowledge spillover among firms, therefore,

$$b_I(t) = \frac{1}{m_D(t)} \tag{15}$$

Free entry is allowed in the innovation sector. Under an equilibrium that a new variety is continuously created, excess profit must equal to the cost of R&D activity. Therefore, the following free entry condition is established:

$$V(t) = b_I(t)w^N \tag{16}$$

Steady-state equilibrium We examine the steady state equilibrium of the model. The steady state equilibrium is defined as an equilibrium where all endogenous variables grow at constant (not necessarily identical) rates over time. Following Grossman and Helpman (1991), the world expenditure is normalized. Therefore, the market interest rate $r^N(t)$ must be constant over time and equal the discount rate in any steady-state equilibrium, $r^N(t) = \rho$.

We define the (constant) growth rate of creating variety as $g \equiv \dot{m}_D/m_D$. From Eq. (14) and (16), the steady-state-equilibrium growth rate of variety g is obtained as:

$$g = \frac{\alpha}{\sigma w^N} - \rho \tag{17}$$

Since the labor supply is exogenous as L^N and L^S, the unemployment, U^N and U^S, are calculated by subtracting the labor demand from the labor supply. In the South, the labor is used only by the low-tech sector:

$$LD^S = \frac{\beta\gamma\omega^{\sigma-1}}{(1+\gamma\omega^{\sigma-1})w^S}$$

Therefore, the southern unemployment is

$$U^S = L^S - \frac{\beta\gamma\omega^{\sigma-1}}{(1+\gamma\omega^{\sigma-1})w^s} \tag{18}$$

In the North, labor force is used by R&D activity, production in the high-tech sector, and production in the low-tech sector:

$$\begin{aligned} LD^N &= g + \frac{\alpha(\sigma-1)}{\sigma w^N} + \frac{\beta(\sigma-1)}{\sigma(1+\gamma\omega^{\sigma-1})} \\ &= \frac{\alpha}{w^N} + \frac{\beta(\sigma-1)}{\sigma(1+\gamma\omega^{\sigma-1})w^N} - \rho \end{aligned}$$

Therefore, the Northern unemployment is

$$U^N = L^N - \left[\frac{\alpha}{w^N} + \frac{\beta(\sigma-1)}{\sigma(1+\gamma\omega^{\sigma-1})w^N} - \rho\right] \tag{19}$$

2.2.3 Effects of the minimum wage

This section analyzes how a change in the minimum wages affects international trade, growth, and unemployment. As explained in introduction, we have the interest on the effects of the policy change in the developing country on the advanced countries.

Therefore, we investigate the role of the developing country's minimum wage,w^S. When w^S increases, the northern relative wage rate w^N/w^S decreases, and then ω decreases. The change in ω affects the low-tech good in both countries through the price index. With reducing ω, the low-tech goods produced in the North relatively become cheaper than that of the South, and the North has an advantage on the low-tech good. This advantage increases the labor demand of the low-tech good in the North $(\frac{\partial LD^N}{\partial w^S} > 0)$, and raises the northern wage income in low-tech sector, which increases the northern expenditure. Therefore, the effect of the increase in the southern minimum wage on the northern unemployment is negative:

$$\frac{\partial U^N}{\partial w^S} = \frac{\gamma\beta(\sigma-1)^2\omega^{\sigma-2}}{\sigma(w^S)^2(1+\gamma\omega^{\sigma-1})^2} < 0$$

In other hand, since the South is suffered from disadvantage in low-tech sector, the southern labor demand decreases: $\frac{\partial LD^S}{\partial w^S} < 0$. Similarly, the effect of the increase in the southern minimum wage on the southern unemployment is positive:

$$\frac{\partial U^S}{\partial w^S} = \frac{\beta\gamma\omega^{\sigma-1}}{(1+\gamma\omega^{\sigma-1})^2(w^S)^3}[(\gamma\omega^{\sigma-1} + \sigma)w^S] > 0$$

As seen in Eq. (17), the northern growth rate g does not depend on w^S. Therefore, the increase in the southern minimum wage does not affect the northern growth. The main reason is the assumption of homogenous firms. In the high-tech sector, all firms produce goods and export them to the South. Although the increase in w^S changes the sizes of the market, each firms' decision is not changed. Moreover, since the decrement of the southern expenditure is offset by the increment of the northern expenditure in the model, R&D activities of firms are not affected by increasing w^S. Therefore, the growth rate is constant even if the southern minimum wage increases.

2.2.4 Firms heterogeneity and the minimum wage

In this section, we consider the case of heterogeneous firms in the high-tech sector. Since the household optimization and the profit maximization of the low-tech firms are the same as the case of homogenous firms, we omit the explanation of these problems. In the high-tech sector, a firm first has to develop a new variety as the case of homogenous firms. After having incurred the fixed cost, the firm receives a patent to exclusively produce the new variety and learns the labor productivity associated with its production. The productivity is drawn from a probability density function, so different firms have different marginal costs of production. As Melitz (2003), the productivities of firms are heterogeneous among firms.

After having developed a new variety and learned its labor productivity, the firm decides whether or not to incur the one-time fixed costs of selling the variety in the local and foreign markets. We think of these market-entry costs as reflecting the costs of adapting the variety to market-specific standards, regulations, and norms. The firm needs to draw a sufficiently high productivity to justify the entrance to the domestic market and an even more favorable productivity to justify entering the foreign market. Therefore, the decisions of R&D activities vary with the productivity of firms.

Innovation To develop a new variety, a firm needs to create F_I units of knowledge in the innovation sector. Thus, the cost of developing a new variety is $w^N b_I(t) F_I$ at time. Knowledge development is also involved in adapting a variety to market-specific standards, regulations, and norms. To sell a new variety in the local market, a firm needs to create F_N units of knowledge at cost $w^N b_I(t) F_N$ and to sell a new variety in the southern market, a firm needs to create F_S units of knowledge at cost $w^N b_I(t) F_S$.

Once a firm has developed a new variety, it learns the labor productivity φ associated with its production. The labor productivity φ is drawn from a probability density function $g(\varphi)$ with support $(1,\infty)$ and the corresponding cumulative distribution function $G(\varphi)$. Once drawn, the labor productivity of a firm associated with producing a particular variety does not change over time. We assume that the labor productivity is subject to the Pareto distribution, that is,

$$g(\varphi) = k\varphi^{-k-1}, G(\varphi) = 1 - \varphi^{-k}, \varphi \in (1,\infty), k > 2 \tag{20}$$

where k is the shape parameter.

Due to the heterogeneity in unit labor requirements, there are three types of firms: nonproducing firms, domestic firms, and exporting firms. Firms that get sufficiently unfavorable draws (too low φ) choose not to produce and exit, firms that get intermediate draws choose to just produce only for the domestic market, and firms that get the most favorable draws choose to produce for both the domestic and the foreign markets.

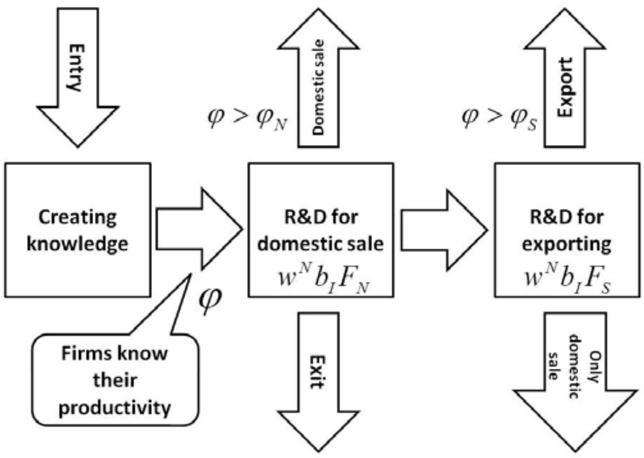

Figure 1: High-tech firm's decision and productivity

Figure 1 describes the relationship between R&D decisions and firm's productivity. For a firm that develops a new variety, let φ_N denote the cutoff of labor productivity at which the firm is indifferent between selling in the local market with incurring the fixed cost $w^N b_I(t)F_N$ and immediately shutting down production. Similarly, let φ_S denote the cutoff of labor productivity at which the firm is indifferent between selling in the local market only and incurring the additional fixed cost $w^N b_I(t)F_S$ to export its high-tech good.

Production Given a draw of $\varphi > \varphi^N$ from the common density function $g(\varphi)$ firm's profits from local sales of variety are given by

$$\pi^N(\varphi) = p_D^N(\varphi)x_D^N(\varphi) - \frac{w^N}{\varphi}x_D^N(\varphi)$$

where $x_D^N(\varphi)$ is the demand for a sold variety φ in the North and $p_D^N(\varphi)$ is the corresponding price. From the pricing rule, the price of the high-tech good sold in the northern market becomes

$$p_D^N(\varphi) = \frac{\sigma w^N}{(\sigma - 1)\varphi} \tag{21}$$

Substituting the price into profit, we obtain operating profits of firms selling the domestic market:

$$\pi_D^N(\varphi) = \frac{\sigma^\sigma(\sigma - 1)^{\sigma-1}(w^N)^{1-\sigma}\alpha E^N(t)}{\varphi^{1-\sigma}\{P_D^N(t)\}^{1-\sigma}} \tag{22}$$

Similarly, given a draw $\varphi < \varphi^S$, additional profits from exports are given by

$$\pi_D^S(\varphi) = p_D^S(\varphi)x_D^S(\varphi) - \frac{w^N}{\varphi}x_D^S(\varphi)$$

where $x_D^S(\varphi)$ is the foreign demand for the exported variety φ and $p_D^S(\varphi)$ is the price of the exported variety φ. The corresponding price of the high-tech good sold in the southern market is

$$p_D^S(\varphi) = \frac{\sigma w^N}{(\sigma - 1)\varphi} \tag{23}$$

Substituting for the price, the operating profits earned by an exporting firm are

$$\pi_D^S(\varphi) = \frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(w^N)^{1-\sigma}\alpha E^S(t)}{\varphi^{1-\sigma}\{P_D^S(t)\}^{1-\sigma}} \tag{24}$$

Market entry From the profits of firms in selling in domestic market and exporting to the foreign market, we consider firm's decision for market entry. Similar to the case of homogenous firms, the no-arbitrage condition of supplying to i country becomes

$$\frac{\dot{V}^i}{V^i} + \frac{\pi_D^i(\varphi)}{V^i(\varphi)} = r^N(t), i = N, S$$

where $V^i(\varphi), i = N, S,$ is the value of firm selling the high-tech good in the market i. Solving for $V^i(\varphi)$ yields

$$V^i(\varphi) = \frac{\pi_D^i(\varphi)}{r^N(t) + \frac{\dot{V}^i(\varphi)}{V^i(\varphi)}}, i = N, S$$

Given that the firms with the cutoff levels, φ^Nandφ^S, are indifferent among exit, entering the local market, and exporting, respectively, the costs of entry have to equal the benefits of entry:

$$V^i(\varphi^i) = w^N b_I(t) F_S, i = N, S \tag{25}$$

Substituting Eq. (22) and (24) into Eq. (25), we obtain the northern market entry condition

$$\frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(w^N)^{1-\sigma}(\varphi^N)\alpha E^N}{\{P_D^N(t)\}^{1-\sigma}\left[r^N(t) - \frac{\dot{b}_I(t)}{b_I(t)}\right]} = w^N b_I(t) F_N \tag{26}$$

Similarly, the southern market entry condition is obtained as

$$\frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(w^N)^{1-\sigma}(\varphi^S)\alpha E^S}{\{P_D^S(t)\}^{1-\sigma}\left[r^N(t) - \frac{\dot{b}_I(t)}{b_I(t)}\right]} = w^N b_I(t) F_S \tag{27}$$

With Eq. (26) and (27), the ratio of the cutoff for domestic market and exporting is

$$\left(\frac{\varphi^{N}}{\varphi^{S}}\right)^{\sigma-1}=\frac{F_N E^S(t)}{F_S E^N(t)}\left[\frac{P_D^N(t)}{P_D^S(t)}\right]^{1-\sigma} \tag{28}$$

Next, we determine the incentive to develop new variety. We assume that there is free entry into variety innovation. Since any firm can develop a new variety, the *ex ante* expected benefit of developing a new variety must equal the cost of variety innovation. Therefore, the free entry condition of firms is

$$\int_{\varphi^N}^{\infty}[V^N(\varphi)-w^N b_I(t)F_N]g(\varphi)\,d\varphi+\int_{\varphi^S}^{\infty}[V^S(\varphi)-w^N b_I F_S]g(\varphi)\,d\varphi=w^N b_I F_I$$

Rewriting the free entry condition, we obtain the expected value of firms:

$$E[V]\equiv\frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(w^N)^{1-\sigma}\alpha\Delta}{r^N(t)-\dot{b}_I/b_I}=w^N b_I(t)\bar{F} \tag{29}$$

where

$$\Delta\equiv\frac{E^N}{\{P_D^N(t)\}^{1-\sigma}}\int_{\varphi^N}^{\infty}\varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi^N)}d\varphi+\frac{E^S}{\{P_D^S(t)\}^{1-\sigma}}\int_{\varphi^S}^{\infty}\varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi^N)}d\varphi \tag{30}$$

and

$$\begin{aligned}\bar{F}&\equiv\frac{F_I}{1-G(\varphi^N)}+\int_{\varphi^N}^{\infty}F^N\frac{g(\varphi)}{1-G(\varphi^N)}d\varphi+\int_{\varphi^S}^{\infty}F_S\frac{g(\varphi)}{1-G(\varphi^N)}d\varphi\\&=F_I(\varphi^N)^k+F_N+F_S\left(\frac{\varphi^N}{\varphi^S}\right)^k\end{aligned} \tag{31}$$

Δ is the average profit of producing firms, and $\bar{F}$ is the ex ante expected fixed cost of developing a profitable variety measured in units of knowledge created.

The flow of new varieties is determined by the labor, L_R, devoted to R&D divided by the labor units required for successful innovation $b_I(t)\bar{F}$. Therefore, the flow of new variety is obtained as

$$\dot{m}_D(t)=\frac{L_R}{b_I\bar{F}} \tag{32}$$

Steady state equilibrium As the case of homogenous firms, we define the steady-state equilibrium as an equilibrium where all endogenous variables grow at constant (not necessarily identical) rates over time. The world expenditure is normalized. Therefore, the market interest rate $r^N(t)$ must be constant over time and equal the discount rate in any steady-state equilibrium, $r^N(t) = \rho$. It then follows from studying Eq. (26) and (27) that, as the growth rates of the endogenous variables $P_D^i, i = N, S$ and $b_I(t)$ are all constant over time, $\bar{F}$ must grow at a constant rate. But, as F_I is a constant, Eq. (31) implies that $\bar{F}$ cannot grow at a constant rate unless the cutoff φ^N and φ^S is constant. Hence $\bar{F}$, φ^N and φ^S are all constants in any steady-state equilibrium.

The price index for local and export market is

$$P_D^N(t)^{1-\sigma} = \left(\frac{\sigma w^N}{\sigma-1}\right)^{1-\sigma} \frac{k m_D(t)(\varphi^N)^{\sigma-1}}{1+k-\sigma} \tag{33}$$

$$P_D^S(t)^{1-\sigma} = \left(\frac{\sigma w^N}{\sigma-1}\right)^{1-\sigma} \frac{k m_D(t)(\varphi^S)^{\sigma-1}}{1+k-\sigma} \left(\frac{\varphi^N}{\varphi^S}\right)^k \tag{34}$$

Substituting Eq. (33) and (34) into Eq. (28) yields the ratio of cutoff of local and export markets:

$$\left(\frac{\varphi^N}{\varphi^S}\right)^k = \frac{F_N E^S}{F_S E^N} \tag{35}$$

Furthermore, substituting the price index in Eq. (33) and (34) into the northern cutoff condition in Eq. (26) and the free entry condition in Eq. (29) yields

$$\frac{(1+k-\sigma)\alpha E^N}{\sigma k(\rho+g)} = w^N F_N \tag{36}$$

$$\frac{\alpha E^N}{\sigma(\rho+g)}\left(1+\frac{E^S}{E^N}\right) = w^N\left[F_I(\varphi^N)^k + F_N + F_S\left(\frac{\varphi^N}{\varphi^S}\right)^k\right] \tag{37}$$

From Eq. (37), the cutoff level depends on the expenditures in the northern country and the southern country, E^N and E^S. We solve these expenditures from the household's budget constraint. In the steady state equilibrium, the expenditure and the saving is constant, $\dot{E}^i = \dot{A}^N = 0, i = N, S$. Therefore, the expenditure in both countries is expressed as

$$E^N = \rho A^N + w^N L^N, \qquad E^S = w^S L^S$$

where A^Nequals the northern investment to firms and L^i is labor demand. Since the southern households do not save, the southern expenditure equals to the total income earned from the low-tech sector. Therefore, the southern expenditure is calculated as

$$E^S = \frac{\beta\gamma\omega^{\sigma-1}}{1+\gamma\omega^{\sigma-1}} \tag{38}$$

Since the world expenditure is one, the northern expenditure is calculated by

$$E^N = 1 - \frac{\beta\gamma\omega^{\sigma-1}}{1+\gamma\omega^{\sigma-1}} = \frac{1+(1-\beta)\gamma\omega^{\sigma-1}}{1+\gamma\omega^{\sigma-1}} \tag{39}$$

Substituting Eq. (39) into (36), the northern growth rate is

$$g = \frac{(1+k-\sigma)[1+(1-\beta)\gamma\omega^{\sigma-1}]}{\sigma k w^N F_N (1+\gamma\omega^{\sigma-1})} - \rho \tag{40}$$

Next, let us consider the cutoff of firm productivity. Substituting Eq. (35), (37), (39), and (40) into Eq. (36), we obtain the equation for the domestic cutoff φ^N:

$$\frac{\beta\gamma\omega^{\sigma-1}}{1(1-\beta)\gamma\omega^{\sigma-1}} = \frac{(1+k-\sigma)F_I(\varphi^N)^k - (\sigma-1)F_N}{(\sigma-1)F_N} \tag{41}$$

Figure 2 depicts the determination of φ^N. The LHS of Eq. (41) does not depend on φ^N, and it has the horizontal line in figure 2. On the other hand, the RHS is increasing function of φ^N. To confirm the exist of φ^N, the following condition must be satisfied.

$$\frac{(1+k-\sigma)F_N}{(\sigma-1)F_I} > (\varphi^N)^k$$

We obtain the cutoff value of the domestic sale as

$$\varphi^N = \left[\frac{(1+\gamma\omega^{\sigma-1})(\sigma-1)F_N}{[1+(1-\beta)\gamma\omega^{\sigma-1}](1+k-\sigma)F_I}\right]^{1/k} \tag{42}$$

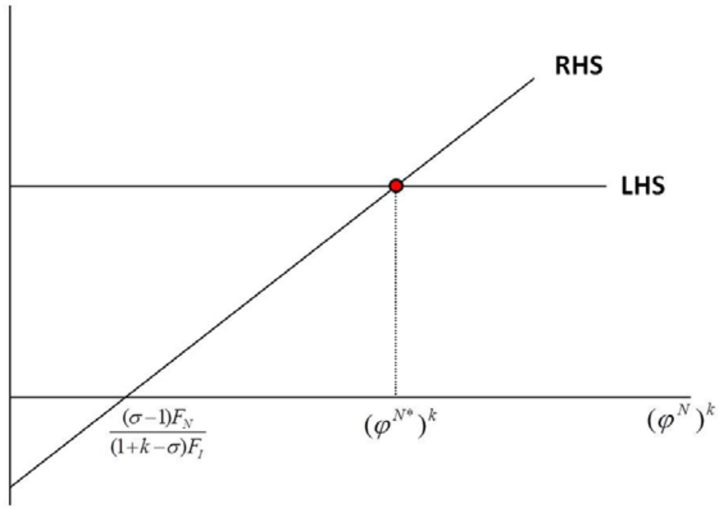

Figure 2: Determination of the northern cutoff φ^N

Finally, we solve the unemployment. The unemployment is solved by subtracting the labor demand from the labor supply. The unemployment in the North and the South is

$$\begin{aligned} U^N = L^N &- \frac{(1+k-\sigma)\{1+(1-\beta)\gamma\omega^{\sigma-1}\} - \rho\sigma k w^N F_N(1+\gamma\omega^{\sigma-1})}{(1+k-\sigma)[1+(1-\beta)\gamma\omega^{\sigma-1}]} \\ &- \frac{(\sigma-1)[\alpha(1+\gamma\omega^{\sigma-1})+\beta\,]}{\sigma w^N(1+\gamma\omega^{\sigma-1})} \end{aligned} \tag{43}$$

$$U^S = L^S - \frac{\gamma\omega^{\sigma-1}(\sigma-1)}{\sigma w^S(a+\gamma\omega^{\sigma-1})} \tag{44}$$

Effects of the minimum wages As section 2.2.3, we analyze effects of an increase in the southern minimum wage w^S. An increase in w^S means a decrease in ω. In the south, the lower ω increases the cost of producing the low-tech good and the price. Since the southern workers earn the wage only from the low-tech sector, the southern income and expenditure clearly decrease. On the other hand, the lower ω has the northern low-tech sector sell the good cheaper than the south and the profits of the northern low-tech firms increase.

Since the northern profits in low-tech sector are given back to the northern consumer, the northern expenditure increases. Figure 2 describe the effects of an increase in w^S on the domestic cutoff φ^N. A decrease in ω drags down the left-hand-side of Eq. (41). A new domestic cutoff is smaller than the previous one. The northern country has a relatively advantage on the low-tech good by lower ω, and the northern workers in the low-tech good earn higher wage income. If other variables (i.e., φ^N) are constant, the increase in the wage income raises the northern expenditure, E^N. Since the firms in the high-tech sector are relatively attracted to the local market, the firm's entry in the local high-tech sector and the cutoff productivity decreases. The lowering φ^{N*} means not only the increase in the number of firms but the decrease in the average productivity because the inefficient firms can entry into the market. Since the effect of increasing he wage income in the low-tech sector is larger than that of decreasing the average productivity, we obtain $\partial E^N / \partial \omega < 0$.

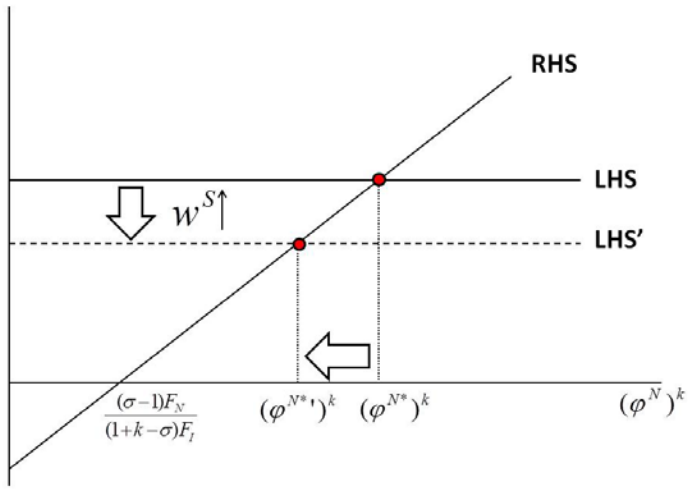

Figure 3: An increase in w^s and the domestic cutoff

The cutoff of local market also affects the cutoff productivity of exporting. With Eq.

(35), (36), and (37), the relationship between $\varphi^{N}$and φ^{S} is

$$\varphi^{S}=\frac{(\sigma-1)F_{S}(\varphi^{N})^{k}}{(1+k-\sigma)F_{I}(\varphi^{N})^{k}-(\sigma-1)F_{N}} \tag{45}$$

From Eq. (45), we obtain $\frac{\partial\varphi^{S}}{\partial\varphi^{N}}<0$. When the southern minimum wage increases, the expenditure in the South decreases. The exporting firms face the decrease in the southern demand of the high-tech goods. Therefore, exporting firms exit and choose to be the local firms.

As seen in Eq. (40), we can affirm that the increase in w^{S} (the decrease in ω) raise the growth rate of varieties: $\frac{\partial g}{\partial\omega}<0$. From Eq. (26), the increase in the northern expenditure and the decrease in the northern price for the high-tech sector index promotes R&D activities. Therefore, the growth rate increases. The result is not observed in the case of the homogenous firms. In the case of the homogenous firms, once a firm succeeds at developing a new variety, the firm can sell for both domestic and foreign markets. Since an increase in w^{S} changes activities of all the firms, it does not affect the growth rate in the north. However, the heterogeneities of firms have each firm choose the number of R&D and the supply destinations. These choices are different with the productivity of the firms. Since an increase in w^{S} reduces the cutoff for domestic firms, more potential firms enter into the domestic market and engage in R&D. As a result, the growth rate increases in the case of firm heterogeneities.

The change in the foreign minimum wage also affects the unemployment rate. Differentiating Eq. (43) with respect to $\omega^{\sigma-1}$, we obtain

$$\frac{\partial U^{N}}{\partial\omega^{\sigma-1}}=\frac{\beta\rho\sigma w^{N}k}{(1+k-\sigma)[1+(1-\beta)\gamma\omega^{\sigma-1}]^{2}}+\frac{\beta\gamma}{\sigma w^{N}[1+(1-\beta)\gamma\omega^{\sigma-1}]^{2}}>0$$

Since both the northern expenditure and the growth rate increase with increasing w^{S}, the northern labor demand in all sectors increase and the unemployment decreases.

The effect on the southern unemployment is solved by Eq. (44) as

$$\frac{\partial U^S}{\partial w^S} = \frac{\sigma - 1}{\sigma}\left[\frac{1}{(w^S)^2} + \frac{\gamma(\sigma - 1)(w^N)^{\sigma-1}}{(w^S)^{\sigma}}\right] > 0$$

The unemployment in the South clearly increases when w^S increases.

An unilateral increase in the minimum wage asymmetrically affects the two countries. We focus on the southern policy of the minimum wage. When the government in the South raises the minimum wage, the southern economy is harmed by the disadvantage on the exporting, which results in decreasing the expenditure and increasing the unemployment. On the other hand, the North has an advantage on the sector which competes with the South, which induces higher expenditure, speeding up the growth rate, and reduction in the unemployment. This result is similar with Davis (1998). He concludes that, through international trade, a country setting the minimum wage is harmed while the other country with a flexible wage obtains positive effects. Unlike Davis (1998), we allow that the minimum wage is bound in both countries and take into account growth effect.

However, the result of our model is not simple. In the low-tech sector, the North produces $\bar{m}_Y$ units of varieties and the South does $\gamma\bar{m}_Y$ units of varieties. Since the household in the North consumes both countries' varieties, they are suffered from the increase in the price of the low-tech good made in the South. The increase in the price may reduce the utility of the northern household. In the high-tech sector, the number of the exporting firms decreases. Since the North has more advantage on the high-tech sector than the South, the contraction of exporting is bad news for the North. Finally, the effects of the foreign minimum wage also increase the northern growth rate. The increase in the growth rate means the household in both countries obtain more varieties of the high-tech sector in the future. This effect may increase the welfare of the southern household.

3. Conclusion

We constructed a North-South-R¥&D-based growth model of trade in two cases: homogeneous firms and heterogeneous firms. We examined effects of southern minimum wage on the northern growth rate and unemployment in both countries. In both cases, higher southern minimum wage leads to the higher relative price of southern low-tech goods, which in turn decreases the southern expenditure. On the other hand, the north takes advance in the low-tech sector, which increases the northern expenditure. In the case of homogenous firm, firm decision are not affected by the change of the southern minimum wage. Therefore, the northern growth rate does not change.

In the case of firm heterogeneity, firms' decisions are different with the productivity. Since an increase in w^S attracts to the domestic sell, more potential firms enter into the domestic market and engage in R&D. The increase in the entry of firms encourages R&D, which increases the northern growth rate. In both cases, the northern unemployment unambiguously decreases and the southern unemployment increases.

We consider the future work. Although we assume that R&D is conducted only in the northern country, the recent developing countries also engage in R¥&D activity. Introducing R¥&D into the southern country, we need analyze the relationship between the minimum wage and economic growth through international trade. Moreover, recent empirical papers show that trade liberalization in the South affects the wage inequality in the north, we need incorporate unskilled worker and skilled worker into our model. Finally, we need analyze the effects of competition between China and Mexico in the

U.S. market on the growth rate and employment in U.S.

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