Predetermined Exchange Rate, Monetary Targeting, and Inflation Targeting Regimes

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Abstract

Many works analyzing the Mundell-Fleming dictum compare the predetermined exchange rate regime and the monetary targeting regime under flexible exchange rates. Reflecting on the fact that many emerging market countries have shifted to the regime of inflation targeting, this paper aims to extend the literature to include the inflation targeting regime. The results of our analysis show that the interest rule with an inflation target is superior (or at least equal) to the two abovementioned regimes in absorbing both real and monetary shocks.

Keywords: optimal exchange rate regimes, predetermined exchange rate, flexible exchange rate, the Mundell-Fleming dictum, small open economy

1 Introduction

The choice of exchange rate regime has been one of the most important issues in open macroeconomic policy analysis ever since Friedman (1953). According to the conventional wisdom, both the flexible exchange rate regime and the predetermined exchange rate regime have their advantages and disadvantages. If the internal prices are sticky, the predetermined exchange rate regime makes better adjustments to monetary shocks, while the flexible exchange rate regime makes better adjustments to real shocks. Végh (2013) provides an excellent intuitive explanation of the conventional wisdom (chapter 11, page 512). Real money balances play the key role of absorbing monetary shocks. Since under flexible exchange rates, policy makers set the path of money supply, real money balances are predetermined (if internal prices are sticky). The economy therefore cannot absorb monetary shocks through the instantaneous adjustment of real money balances under flexible exchange rates. In contrast, under predetermined exchange rates, policy makers set the path of nominal exchange rate. Since real money balances are not predetermined in this case, the economy can absorb monetary shocks (at least partially) through the adjustment of real money balances. On the other hand, the opposite is true for real shocks. The real exchange rate plays the key role of absorbing real shocks. Since the real exchange rate is not predetermined under flexible exchange rates, the economy can absorb real shocks (at least partially) through the adjustment of real exchange rate. In contrast, under predetermined exchange rates, the economy cannot absorb real shocks through the adjustment of real exchange rate, since the real exchange rate is predetermined (if the internal prices are sticky). Hence, we can say that the predetermined exchange rate regime has an advantage in its flexibility to absorb monetary shocks, while the flexible exchange rate regime has an advantage in its flexibility to absorb real shocks.

There have been many works on the optimal exchange rate regimes, and the advantages and disadvantages of different exchange regimes have been still discussed. For example, Lahiri et al. (2007) revisited the issue of the optimal exchange rate regime in a model with flexible prices and asset market frictions, arguing that the asset market frictions may be as prevalent as the goods market frictions. They showed that in this case, contrary to the conventional wisdom, the flexible exchange rates are optimal in the presence of monetary shocks, whereas the fixed exchange rates are optimal in the presence of real shocks. They concluded that the choice of an optimal exchange rate regime should also depend on the type of frictions as well as the type of shocks.

In examining the conventional wisdom on the predetermined and flexible ex-

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1Friedman (1953) argued that if internal prices are sticky, the exchange rate regime matters and the flexible exchange rate regime makes better adjustments in response to the changes in external conditions.

2Lahiri et al. (2006) refer to the conventional wisdom as the Mundell-Fleming dictum.

3These results will be replicated in Sections 3.1, 3.2, 4.1, and 4.2.


5Lahiri et al. (2006) also obtained similar results using a perfect-foresight version of the stochastic model in Lahiri et al. (2007).
change rates, the rigorous mathematical models assume that under the flexible exchange rates, policy makers set a path of money supply. In other words, the literature regarding the conventional wisdom has been concerned mainly with the choice between the predetermined exchange rate regime and the monetary targeting regime under flexible exchange rates. However, in reality, as argued by Frankel (2010), many countries have shifted to the regime of inflation targeting. Not only developed countries such as New Zealand, Canada, the United Kingdom, and Sweden but also many emerging market countries such as Brazil, Chile, Colombia, Mexico, the Czech Republic, Hungary, Poland, Israel, Korea, South Africa, Thailand, Indonesia, Romania, and Turkey shifted to the regime of inflation targeting after (or in the middle of) the series of currency crises in the emerging markets in East Asia and Latin America from the mid 1990s to the early 2000s. This recent shift of many emerging market countries to the regime of inflation targeting motivates our paper.

Our paper is not the first to examine these three cases. Indeed, for example, Mishkin and Savastano (2001) already discuss the advantages and disadvantages of a hard exchange rate peg, monetary targeting, and inflation targeting. However, they specifically focus on monetary policy strategies in Latin America and are concerned with the advantages and disadvantages of the three regimes from a more pragmatic perspective, taking into consideration the institutional environment or fiscal situation in each Latin American country rather than developing a rigorous mathematical model. On the other hand, as previously discussed, the literature comprised of rigorous mathematical models, such as Helpman and Razin (1979), Helpman (1981), Helpman and Razin (1982), Calvo (1999), Devereux and Engel (2003), Céspedes et al. (2004), and Lahiri et al. (2007), only consider the predetermined exchange rate regime and the monetary targeting regime when comparing exchange rate regimes. Our paper aims to extend the literature of rigorous mathematical models regarding the Mundell-Fleming dictum to include the inflation targeting regime. As a paper considering the inflation targeting regime, we specifically consider nominal interest rate rules with an inflation target. For example, Reinhart (1992), Végh (2001), and Calvo (2007) develop models comprising the nominal interest rate rule with an inflation target. Reinhart (1992) shows that the introduction of a nominal interest rate rule with an inflation target into a macro model can avoid the indeterminacy problems identified by Sargent and Wallace (1975). Végh (2001) shows that under certain conditions, some equivalences exist among the three regimes: (a) a “k-percent” money growth rule, (b) a nominal interest rate rule with an inflation target, and (c) a real interest rate rule with an inflation target. Calvo (2007) examines the role of a nominal interest rate rule with an inflation target in the context of an imperfectly credible stabilization program and shows that the resulting transitional dynamics in the case of this interest rate rule are opposite to those in the case of an exchange rate-based stabilization program. This paper introduces a nominal interest rate rule with an inflation target developed by the abovementioned authors into the literature of rigorous mathematical models regarding the Mundell-Fleming dictum.

We develop a small open economy model with a cash-in-advance constraint and examine the economy’s responses to real shocks and monetary shocks, respectively.

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6 See Frankel (2010) for a comprehensive survey.
First, we replicate the well-known results regarding the difference between the economy’s responses to each shock in the predetermined exchange rate regime and in the monetary targeting regime under flexible exchange rates. It is shown that in response to a real shock, consumption fluctuates less in the monetary targeting regime under flexible exchange rates than in the predetermined exchange rate regime. In this case, the real exchange rate, which is not a predetermined variable in the monetary targeting regime, plays the key role of absorbing the real shock. In contrast, in response to a monetary shock, consumption fluctuates less in the predetermined exchange rate regime than in the monetary targeting regime under flexible exchange rates. In this case, real money balances, which is not a predetermined variable in the predetermined exchange rate regime, plays the key role of absorbing the monetary shock.

Next, we consider how the economy’s responses to each shock in an inflation targeting regime differ from those in either the predetermined exchange rate regime or monetary targeting regime. We consider a specific form of the interest rate rule with an inflation target, such as the one employed by Calvo (2007), as an inflation targeting regime. According to this rule, policy makers increase (decrease) the level of the nominal interest rate if the actual inflation level is above (below) the target level set in advance by policy makers. This construction allows the nominal interest rate to jump on impact. We focus on the case in which policy makers adjust the nominal interest rate such that the Taylor principle is fulfilled. That is, the nominal interest rate will be adjusted such that the real interest rate increases (decreases) in the face of an increase (decrease) in inflation. We refer to this type of interest rate rule as the interest rate rule incorporating the Taylor principle.

Then, we examine the economy’s response to real shocks and monetary shocks, respectively, under the inflation targeting regime. It is shown that in response to real shocks, consumption fluctuates less in the inflation targeting regime than in the predetermined exchange rate and monetary targeting regimes. In addition, the inflation targeting regime is shown to be superior (or at least equal) to the predetermined exchange rates and monetary targeting regimes in absorbing monetary shocks. Under the inflation targeting regime, both the real exchange rate and real money balances play key roles in absorbing real and monetary shocks, respectively, since neither the real exchange rate nor real money balances are predetermined. Therefore, the economy can make better (or at least equivalent) adjustments to both real and monetary shocks under the inflation targeting regime compared with the predetermined exchange rates and monetary targeting regimes.

To demonstrate that our results are robust, we include an appendix where we examine the economy’s responses to real and monetary shocks under another type of interest rate rule with an inflation target, such as that employed by Reinhart (1992) and Végh (2001). According to this policy rule, policy makers increase (decrease) the rate of change, and not the level, of the nominal interest rate if the actual inflation level is above (below) the target level set by policy makers in advance. This type of interest rate rule implies that the nominal interest rate is a predetermined variable and moves in a sticky way. We refer to this type of interest rate rule as the sticky interest rate rule. The results obtained under the sticky interest rate rule in response to both real and monetary shocks are exactly the same as those obtained in the case of the interest rate rule incorporating the Taylor principle.
described earlier in the main text.

The remainder of the paper is organized in four sections. In section 2, we present a basic model to analyze the impact of real and monetary shocks on the economy under alternative regimes. Section 3 compares how the economy’s response to real shocks varies under predetermined exchange rates, monetary targeting, and inflation targeting. Section 4 compares the economy’s response to monetary shocks under the different regimes. Section 5 concludes.

2 Model

Consider a small open economy that is perfectly integrated with the rest of the world in goods and capital markets. The free movement of goods implies that the law of one price holds for the tradable good: \( P_T(t) = E(t)P^*(t) \), where \( E(t) \), \( P_T(t) \), and \( P^*(t) \), respectively, denote the nominal exchange rate, the domestic price of the tradable good, and the foreign price of the good at time \( t \). For simplicity, we assume that \( P^*(t) = 1 \), so that there is no foreign inflation. Perfect capital mobility implies that the interest parity condition holds:

\[
i(t) = r + \varepsilon(t),
\]

where \( i(t) \) is the domestic nominal interest rate, \( r \) is the (constant) world real interest rate, and \( \varepsilon(t) = \frac{E(t)}{E(0)} \) is the rate of devaluation (or depreciation).

2.1 Consumers

The economy is inhabited by a large number of identical, indefinitely living individuals. The representative household’s instantaneous utility depends on the consumption of tradables \( c^T(t) \) and non-tradables (or home goods), \( c^N(t) \). The lifetime utility as of time 0 can therefore be written as follows:

\[
\int_0^\infty \left\{ \gamma \log(c^T(t)) + (1 - \gamma) \log(c^N(t)) \right\} e^{-\beta t} \, dt,
\]

where \( \beta(>0) \) is the rate of time preference. An individual holds assets in the form of foreign bonds, \( B(t) \), and domestic money, \( M(t) \). The individual’s financial wealth (in nominal terms of the domestic price of tradables) is denoted by \( A(t) \) and is given by

\[
A(t) = M(t) + E(t)B(t).
\]

Dividing equation (3) by \( P_T(t) \), we obtain

\[
\frac{A(t)}{P_T(t)} = \frac{M(t)}{P_T(t)} + \frac{E(t)B(t)}{P_T(t)}.
\]

Since the law of one price holds (i.e., \( P_T(t) = E(t)P^*(t) \)), we also obtain

\[
\frac{E(t)B(t)}{P_T(t)} = \frac{B(t)}{P^*(t)}.
\]
Hence, from equations (4) and (5), we obtain

\[ a(t) = m(t) + b(t), \tag{6} \]

where \( a(t) = A(t)/P_T(t) \), \( m(t) = M(t)/P_T(t) \), and \( b(t) = B(t)/P_T(t) \) denote the individual’s real financial wealth, real money balances, and real foreign bond holdings, respectively.

The individual has a constant endowment flow of tradables, \( y^T \), while the output of non-tradables, \( y^N(t) \), is demand-determined. The individual’s lifetime constraint is given by

\[ a_0 + \int_0^\infty \left( y^T + \frac{y^N(t)}{e(t)} + \tau(t) \right) e^{-rt} dt = \int_0^\infty \left( c^T(t) + \frac{c^N(t)}{e(t)} + i(t)m(t) \right) e^{-rt} dt, \tag{7} \]

where \( e(t) \equiv \frac{P_T(t)}{P_N(t)} \) is the real exchange rate (i.e., the relative price of tradables in terms of non-tradables) and \( \tau(t) \) denotes the government lump-sum transfers.

We assume that transactions require holding cash in advance:

\[ m(t) = \alpha \left( c^T(t) + \frac{c^N(t)}{e(t)} \right), \quad \alpha > 0. \tag{8} \]

By maximizing the lifetime utility (2) subject to the individual’s budget constraint (7) and the cash-in-advance constraint (8), the following first-order conditions are obtained:

\[ \frac{\gamma}{c^T(t)} = \lambda (1 + \alpha i(t)), \tag{9} \]

and

\[ \frac{(1 - \gamma)}{c^N(t)} = \lambda (1 + \alpha i(t)) \frac{e(t)}{e(t)}, \tag{10} \]

where \( \lambda \) is the Lagrange multiplier. From (9) and (10), we can express the real exchange rate as

\[ e(t) = \left( \frac{\gamma}{1 - \gamma} \right) \frac{c^N(t)}{c^T(t)}. \tag{11} \]

Following Calvo and Végh (1999) and Calvo (2007), we assume that the non-tradables sector operates under the sticky price setting and the output of non-tradables is demand determined. Formally, we adopt the sticky price model in Calvo (1983) that implies that the rate of change in the inflation rate is a negative function of excess demand:

\[ \dot{\pi}(t) = -\theta \left( y^N(t) - \bar{y}^N \right), \quad \theta > 0, \tag{12} \]
where $\pi(t) \equiv \frac{\dot{p}_N(t)}{p_N(t)}$ is the inflation rate of non-tradable goods and $\bar{y}^N$ can be interpreted as the “full employment” level of output.

By substituting (11) into (8), we obtain

$$m(t) = \left(\frac{\alpha}{\gamma}\right) c^T(t).$$

Let us denote the real money balances in terms of non-tradable goods as $n(t) \equiv \frac{M(t)}{p_N(t)}$.

It follows from the definition of $e$ (i.e., $e \equiv \frac{p_T(t)}{p_N(t)}$) that the real exchange rate can be rewritten as

$$e(t) = \frac{n(t)}{m(t)},$$

From (11), (13), and (14), we have

$$n(t) = \left(\frac{\alpha}{1 - \gamma}\right) c^N(t).$$

### 2.2 Government

The government’s intertemporal constraint is given by

$$\int_0^\infty \tau(t)e^{-rt}dt = h_0 + \int_0^\infty (\dot{m}(t) + \varepsilon(t)m(t)) e^{-rt}dt,$$

where $h_0$ is the initial stock of international reserves. The quantity $\dot{m}(t) + \varepsilon(t)m(t)$ is equal to $\frac{M(t)}{p_T(t)}$ and indicates revenues from money creation (i.e., seigniorage flow).

Equation (16) indicates that the government must finance the present discounted value of transfers using the initial assets and seigniorage revenues.

### 2.3 Equilibrium conditions

Equilibrium in the non-tradable goods sector requires

$$y^N(t) = c^N(t).$$

By combining the consumer’s constraint (7) and the government’s constraint (16), and using (1) and (17), we obtain this economy’s resource constraint:

$$k_0 + \frac{y^T}{r} = \int_0^\infty c^T(t)e^{-rt}dt,$$

where $k_0(\equiv b_0 + h_0)$ denotes the economy’s net stock of foreign assets (at time 0).
3 Real shocks

In this section, we analyze how the economy responds to real shocks under different exchange rate regimes. Following Végh (2013), we consider a shock that shifts demand from non-tradables to tradables as a real shock. Formally, we assume that there is an unanticipated and permanent increase in $\gamma$ as shown in Panel A of Figure 2.

We first replicate the well-known results about the difference between the economy’s response to real shocks in the predetermined exchange rate regime and that in the monetary targeting regime under flexible exchange rates, and will then compare them with that in the inflation targeting regime.9

3.1 Predetermined exchange rates

Under predetermined exchange rates, policy makers set the path of nominal exchange rate $E(t)$. We assume that the devaluation rate is set at a constant level: $\bar{\varepsilon}$. From the interest parity (1), the nominal interest rate is therefore constant over time (Panel B in Figure 2):

$$i(t) = r + \bar{\varepsilon}.$$ (19)

Considering that $i(t)$ is constant and the consumer’s first-order condition (9), we can get that the consumption of tradables will be constant along a perfect foresight equilibrium path. From the economy’s resource constraint, we obtain the constant level of tradables consumption:

$$c^T(t) = rk_0 + y^T.$$ (20)

Using (11) and (17), we can rewrite (12) as

$$\dot{\pi}(t) = \theta \left\{ \bar{y}^N - \frac{1 - \gamma}{\gamma} \left( rk_0 + y^T \right) e(t) \right\}.$$ (21)

From the definition of the real exchange rate (i.e., $e(t) = \frac{P^T(t)}{P^N(t)}$) and $P^T(t) = E(t)$, we have

$$\dot{e}(t) = e(t) \left( \bar{\varepsilon} - \pi(t) \right),$$ (22)

under predetermined exchange rates.

Equations (21) and (22) constitute a system of differential equations in $\pi$ and $e$. In the steady state, we have

$$e_s = \left( \frac{\gamma}{1 - \gamma} \right) \left( \bar{y}^N \right),$$ (24)

$$\pi_s = \bar{\varepsilon};$$ (23)

9The model, which is used to replicate the well-known results, heavily draws on Végh (2013) (subchapter 11.4). While Végh (2013) employs a money-in-the-utility-function model, we consider a cash-in-advance economy.
where $\pi_s$ and $e_s$ denote the steady state levels of $\pi(t)$ and $e(t)$, respectively.\footnote{Hereafter, in this paper, a variable with the suffix $s$ denotes its steady state level.} By linearizing the system comprising (21) and (22) around the steady state, we have

$$
\begin{bmatrix}
\dot{\pi}(t) \\
\dot{e}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \theta \left(1 - \frac{1}{\gamma} \right) \left(r k_0 + y^T \right) \\
-e_s & 0
\end{bmatrix}
\begin{bmatrix}
\pi(t) - \pi_s \\
e(t) - e_s
\end{bmatrix}.
$$

The determinant of the matrix on the right-hand side is denoted as $\Delta$ and has the value

$$
\Delta = -\theta \bar{y}^N < 0.
$$

Around the steady state, the system is saddle-path stable. Note that $e(t) \equiv \frac{p^T(t)}{p^N(t)} = \frac{E(t)}{P^N(t)}$ is a predetermined variable, since we have predetermined exchange rates $E(t)$ and the sticky price of non-tradables $P^N(t)$. Figure 1 shows the corresponding phase diagram. The initial steady state is at point A. The new steady state associated with a higher value of $\gamma$ becomes point C. On impact, the system jumps from point A to point B and then converges to point C along the saddle path.

![Figure 1: Increase in $\gamma$ under predetermined exchange rates](image)

The paths of the main variables are illustrated in Figure 2. Figure 2 is obtained based on the Mathematica program in Chapter 11 of Végh (2013). Following Végh (2013), the parameter values are set as follows: $r = 0.05$, $k_0 = 0$, $\bar{y}^T = 1$, $\bar{y}^N = 1$, $\theta = 0.5$, $\bar{\varepsilon} = 0.5$, and $\alpha = 1$. The shock consists of an increase in $\gamma$ from 0.5 to 0.6. The path of $\pi(t)$ and that of $e(t)$ are straightforward from Figure 1 (Panels C and D in Figure 2). From the path of $e(t)$, equation (11), and the fact that tradables consumption, $c^T(t)$, is constant, we obtain the path of non-tradables consumption, $c^N(t)$, as shown in Panel E in Figure 2.
Figure 2: Real shock under predetermined exchange rates
3.2 Monetary Targeting

In the monetary targeting regime under flexible exchange rates, policy makers set the path of money supply \( M(t) \). We assume that the rate of money growth \( \mu(t) \) is set at a constant level: \( \bar{\mu} \). From the definition of \( m(t) = \frac{M(t)}{P(t)} = \frac{M(t)}{E(t)} \), it follows that

\[
\frac{\dot{m}(t)}{m(t)} = \bar{\mu} - \varepsilon(t). \tag{25}
\]

From the interest parity (1), equation (25) can be rewritten as

\[
\frac{\dot{m}(t)}{m(t)} = r + \bar{\mu} - i(t). \tag{26}
\]

It follows from (13) that

\[
\frac{\dot{m}(t)}{m(t)} = \frac{\dot{c}(t)}{c(t)}. \tag{27}
\]

By differentiating both sides of equation (9) with respect to time, we obtain

\[
-\frac{\dot{c}(t)}{c(t)} = \frac{\alpha}{1 + \alpha i(t)} i(t). \tag{28}
\]

By combining (26), (27), and (28), we obtain

\[
i(t) = \frac{1 + \alpha i(t)}{\alpha} (i(t) - r - \bar{\mu}). \tag{29}
\]

Using (15) and (17), we can rewrite (12) as

\[
\dot{\pi}(t) = \theta \left\{ \bar{y}^N - \frac{1 - \gamma}{\alpha} n(t) \right\}. \tag{30}
\]

From the definition of \( n(t) = \frac{M(t)}{P(t)} \), we have

\[
\dot{n}(t) = n(t) (\bar{\mu} - \pi(t)). \tag{31}
\]

Equations (29), (30), and (31) constitute a system of differential equations in \( i, \pi, \) and \( n \). In the steady state, we have

\[
i_s = r + \bar{\mu}, \tag{32}
\]

\[
\pi_s = \bar{\mu}, \tag{33}
\]

\[
n_s = \frac{\alpha}{1 - \gamma} \bar{y}^N. \tag{34}
\]

By linearizing the system comprising (29), (30), and (31) around the steady state, we have

\[
\begin{bmatrix}
\dot{i}(t) \\
\dot{\pi}(t) \\
\dot{n}(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{1 + \alpha i_s}{\alpha} & 0 & 0 \\
0 & -\theta \frac{1 - \gamma}{\alpha} & 0 \\
0 & -n_s & 0
\end{bmatrix}
\begin{bmatrix}
\dot{i}(t) - i_s \\
\dot{\pi}(t) - \pi_s \\
\dot{n}(t) - n_s
\end{bmatrix}.
\]
The determinant of the matrix on the right-hand side is denoted as $\Delta$ and has the value

$$\Delta = -\frac{\theta (1 - \gamma)}{\alpha^2} (1 + \alpha i_{ss}) n_{ss} < 0.$$ 

The trace of the matrix on the right-hand side is denoted as $Tr$ and has the value

$$Tr = \frac{1 + \alpha i_{ss}}{\alpha} > 0.$$ 

Therefore, we get that there exist one negative and two positive roots. Note that

$n(t) \left( \equiv \frac{M(t)}{P^N(t)} \right)$ is a predetermined variable, since $M(t)$ is predetermined under monetary targeting and the price of non-tradables $P^N(t)$ is sticky. Since the number of negative roots is equal to the number of predetermined variables (i.e., $n(t)$), we can pin down the saddle path converging to the steady state for a given value of $i_0$, $\pi_0$, and $n_0$.\(^{11}\)

Let $\delta_1$ denote the negative root. Let $h_{1j}, j = 1, 2, 3$ denote the elements of the eigenvector associated with the root $\delta_1$. By setting the constants corresponding to the two unstable roots as zero, we can express the solution of this dynamic system as

$$i(t) - i_s = \omega_1 h_{11} \exp(\delta_1 t), \quad \pi(t) - \pi_s = \omega_1 h_{12} \exp(\delta_1 t), \quad n(t) - n_s = \omega_1 h_{13} \exp(\delta_1 t), \quad (35$$

where $\omega_1$ is the constant associated with the root $\delta_1$. To obtain $h_{1j}, j = 1, 2, 3$, we solve

$$\begin{bmatrix} 1 + \alpha i_{ss} - \delta_1 & 0 & 0 \\ 0 & -\delta_1 & -\theta \frac{1 - \gamma}{\alpha} \\ 0 & -n_s & -\delta_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (36)$$

From $\left(\frac{1 + \alpha i_{ss}}{\alpha} - \delta_1\right) > 0$, we have that

$$h_{11} = 0 \quad (38)$$

and

$$\frac{h_{12}}{h_{13}} = -\frac{\theta (1 - \gamma)}{\alpha \delta_1} > 0. \quad (39)$$

From (35) and (38), we can get that $i(t)$ remains at the initial steady state. Equations (36), (37), and (39) imply that $\pi(t)$ and $n(t)$ will converge to their steady state values from the same direction.\(^{12}\)

Figure 3 illustrates the paths of the main variables. As argued above, $i(t)$ remains at the steady state (Panel B in Figure 3). $\pi(t)$ and $n(t)$ converge to their

\(^{11}\)For details on the analysis of the system with three roots, see, for example, Végh (2001) or Végh (2013) (subchapter 8.5.2).

\(^{12}\)We can pin down $\omega_1$ by setting $h_{13} = 1$, since $n_0$ is its initial steady state value and $n_s$ is its new steady state value. (Note that $n(t)$ is a predetermined variable.)
steady state values from the same direction (Panels C and E in Figure 3). From (15), we can obtain the path of $c^N(t)$ (Panel F in Figure 3). Considering that $i(t)$ is constant and the consumer’s first-order condition (9), the consumption of tradables is constant along a perfect foresight path equilibrium (i.e., $c^T(t) = rk_0 + y^T$). Thus, from the path of $c^N(t)$ and (11), the path of $e(t)$ must look as in Panel D of Figure 3. Note that under monetary targeting, the real exchange rate $e(t)$ is not predetermined and can jump on impact. From (11) and the fact that the consumption of tradables is constant along a perfect foresight path equilibrium, we can know that the real exchange rate $e(t)$ plays the role of absorbing the real shock. That is, the consumption of non-tradables fluctuates less under monetary targeting than under predetermined exchange rates.

### 3.3 Inflation targeting

We now consider nominal interest rate rules with an inflation target. Policy makers announce a target level of inflation $\bar{\pi}$. We consider the interest rate rule incorporating the Taylor principle as follows:

$$i(t) = r + \bar{\pi} + \phi (\pi(t) - \bar{\pi}), \quad \phi > 1.$$  \hfill (40)

This rule is employed in, for example, Calvo (2007). We will focus on the case where policy makers adjust the nominal interest rate in such a way that the Taylor principle is satisfied.\(^\text{13}\) The condition $\phi > 1$ is the so-called Taylor principle. According to this policy rule, policy makers increase (reduce) the level of the nominal interest rate if the actual inflation level is above (below) the target level set by the policy makers in advance. In this set-up, by construction, the nominal interest rate can jump on impact. That is, the nominal interest rate will be adjusted such that the real interest rate is increased (reduced) in the face of an increase (decrease) in inflation.

In order to analyze the dynamic adjustment of this economy under the nominal interest rate rule, we follow Calvo (2007) and introduce another variable $x(t) \left( \equiv \frac{\lambda}{e(t)} \right)$ into the system. Using the variable $x(t)$ and (10), we have

$$c^N(t) = \frac{1 - \gamma}{(1 + \alpha i(t)) x(t)}. \hfill (41)$$

By combining (17), (40), and (41) with (12), we obtain

$$\dot{\pi}(t) = \theta \left\{ \tilde{y}^N - \frac{1 - \gamma}{[1 + \alpha \{r + \bar{\pi} + \phi (\pi(t) - \bar{\pi})\}] x(t)} \right\}. \hfill (42)$$

From the definition of $x(t) \left( \equiv \frac{\lambda}{e(t)} \right)$ and $e(t) = \frac{E(t)}{P^N(t)}$, we have

$$\frac{\dot{x}(t)}{x(t)} = -\frac{\dot{e}(t)}{e(t)} = -\varepsilon(t) + \pi(t).$$

From the interest rate parity (1), we obtain

$$\dot{x}(t) = (r + \pi(t) - i(t)) x(t). \hfill (43)$$

\(^{13}\)For the Taylor principle, see, for example, Gali (2008).
Figure 3: Real shock under monetary targeting
By substituting (40) into (43), we obtain

\[ \dot{x}(t) = x(t) \left\{ (1 - \phi) (\pi(t) - \bar{\pi}) \right\}. \]  \hspace{1cm} (44)

Equations (42) and (44) constitute a system of differential equations in \( \pi \) and \( x \). In the steady state, we have

\[ \pi_s = \bar{\pi}, \]  \hspace{1cm} (45)

\[ x_s = \frac{1 - \gamma}{\{1 + \alpha(r + \bar{\pi})\} \bar{y}^N}. \]  \hspace{1cm} (46)

By linearizing the system comprising (42) and (44) around the steady state, we have

\[
\begin{bmatrix}
\dot{\pi}(t) \\
\dot{x}(t)
\end{bmatrix}
= \begin{bmatrix}
\frac{\theta(1-\gamma)}{x_s} \frac{\alpha \phi}{(1+\alpha(r+\bar{\pi}))^2} & \frac{\theta(1-\gamma)}{1+\alpha(r+\bar{\pi})} x_s^2 \\
\frac{1-\phi}{x_s (1-\phi)} & 0
\end{bmatrix}
\begin{bmatrix}
\pi(t) - \pi_s \\
x(t) - x_s
\end{bmatrix}.
\]

The trace of the matrix on the right-hand side is denoted as \( Tr \) and has the value

\[ Tr = \frac{\theta(1-\gamma)}{x_s} \frac{\alpha \phi}{\{1 + \alpha (r + \bar{\pi})\}^2} > 0. \]

The determinant of the matrix on the right-hand side is denoted as \( \Delta \) and has the value

\[ \Delta = -\frac{\theta(1-\phi)(1-\gamma)}{\{1 + \alpha (r + \bar{\pi})\} x_s} = \begin{cases} > 0 & \text{if } \phi > 1, \\ \leq 0 & \text{if } \phi \leq 1. \end{cases} \]

The Taylor principle implies that there are two positive roots in this system. Note that neither \( \pi(t) \) nor \( x(t) \) are predetermined. Therefore, as argued in Calvo (2007), we need two positive roots so that the system has a unique equilibrium around the steady state. Figure 4 shows the corresponding phase diagram. The initial steady state is at point A. The new steady state associated with a higher value of \( \gamma \) becomes point B. On impact, the system jumps from point A to point B.

Figure 5 shows the paths of the main variables. An increase in \( \gamma \) does not change the steady state level of \( \pi(t) \). Thus, there is no change in \( \pi(t) \) on impact (Panel C in Figure 5). From (40), \( i(t) \) remains at its initial steady state level (Panel B in Figure 5). \( x(t) \) falls to its new steady state level on impact (Panel F in Figure 5). The amounts of change in the steady state levels of \( x(t) \), \( \lambda(t) \), and \( c(t) \) due to an increase in \( \gamma \) are explained in Appendix A2. From the paths of \( \gamma \), \( x(t) \), and \( i(t) \), and equation (41), we obtain the path of \( c^N(t) \) (Panel E in Figure 5) (as explained in Appendix A2). It turns out that the consumption of non-tradables remains unchanged over time.

To show the robustness of our results, Appendix A1 presents another interest rate rule as follows:

\[ \dot{i}(t) = \phi (\pi(t) - \bar{\pi}), \]

which is employed in Reinhart (1992) and Végh (2001). As mentioned in the introduction, we refer to this type of interest rate rule as the sticky interest rate rule, since this type of interest rate rule implies that the nominal interest rate is a predetermined variable that moves in a sticky way. In Appendix A1.1, we show that the real shock considered in this section causes the exact same dynamics as all of the endogenous variables shown in Figure 5.

\[ \text{If } \phi < 1, \text{ we have the saddle-path stability that implies indeterminacy in the system with two non-predetermined variables.} \]
3.4 Comparison of the response to real shocks under alternative regimes

We have analyzed the economy’s response to real shocks under alternative regimes. As a benchmark case, we first replicated the well-known results about the difference in the economy’s response to real shocks under predetermined exchange rates and monetary targeting. By comparing Panel E in Figure 2 and Panel F in Figure 3, we can confirm that in response to the real shock, the consumption of non-tradables fluctuates less under monetary targeting than under predetermined exchange rates. As argued by Végh (2013), Chapter 11, this is because under monetary targeting, the real exchange rate serves as a cushion against real shocks (as is clear from the comparison of Panel D in Figure 2 and Panel D in Figure 3). By comparing Panel E in Figure 5 with Panel E in Figure 2 and Panel F in Figure 3, we can say that the inflation targeting regime is more optimal in making adjustments to real shocks. The reason why the inflation targeting regime can make better adjustments than the predetermined regime is that the real exchange rate under the inflation targeting regime is not predetermined and plays the role of absorbing real shocks as under monetary targeting. In addition, under the inflation targeting regime, real money balances are not predetermined either, and play the role of a shock absorber, in addition to the real exchange rate. It should be noted that even though the real exchange rate is not predetermined, if real money balances are predetermined, (which is the case in monetary targeting), an immediate adjustment to the consumption of non-tradables is impossible (as is clear from equation (15)).

\[\hat{x} = 0\]

Since the real (monetary) shock does not change the steady state level of the consumption of non-tradables in this model, the immediate adjustment implies that there is no convergence process to the steady state after the shock (i.e., non-tradables consumption remains unchanged on impact).
Figure 5: Real shock under the interest rate rule with the Taylor principle
4 Monetary shocks

In this section, we analyze how the economy responds to monetary shocks under the different regimes. Following Végh (2013), we consider a change in money demand as a monetary shock. Formally, we assume that there is an unanticipated and permanent increase in $\alpha$ as shown in Panel A of Figure 6.

We first replicate the well-known results about the difference in the economy’s response to monetary shocks under predetermined exchange rates and monetary targeting, and will then compare them with the economy’s response to monetary shocks under inflation targeting.\footnote{As in the case of real shocks, the model here, which is used to replicate the well-known results in the case of monetary shocks, heavily draws on Végh (2013) (subchapter 11.4). While Végh (2013) employs a money-in-the-utility-function model, we consider a cash-in-advance economy.}

4.1 Predetermined exchange rates

As argued in Section 3.1, equations (21) and (22) constitute the dynamic system under predetermined exchange rates. Note that $e(t) = \frac{E(t)}{P(t)^N(t)}$ is the only predetermined variable under predetermined exchange rates. From (24), we know that a shift in $\alpha$ does not change $e_s$. Therefore, there is no adjustment in response to the monetary shock. Each variable except for real money balances remains at its initial steady state level (Panels B to D and F in Figure 6). From (15) ((13)), $n$ ($m$) jumps to its new steady state level on impact. That is, under predetermined exchange rates, the monetary shock changes only the real money balances.

4.2 Monetary Targeting

As argued in Section 3.2, equations (29), (30), and (31) constitute the dynamic system under monetary targeting. Note that $n(t) = \frac{M(t)}{P(t)^N(t)}$ is the only predetermined variable under monetary targeting. From (34), we know that an increase in $\alpha$ will raise $n_s$. Therefore, we can get that there is adjustment due to the monetary shock. The adjustment process of $n$ to its new steady state level is shown in Panel E in Figure 7. From (15), we can know the path of $c^N(t)$ (Panel F in Figure 7). As shown in Section 3.2, $\pi(t)$ converges to its steady state level from the same direction as $n(t)$ (Panel C in Figure 7), while $i(t)$ remains at its initial steady state (Panel B in Figure 7). Considering that $i(t)$ is constant and the consumer’s first-order condition (9), the consumption of tradables is constant along a perfect foresight path equilibrium (i.e., $c^T(t) = rk_0 + y^T$). Thus, from the path of $c^N(t)$ and (11), we get that the path of $e(t)$ would be as in Panel D of Figure 7.

4.3 Inflation targeting

We now analyze how the economy responds to the monetary shock under the inflation targeting regime. As shown in Section 3.3, equations (42) and (44) constitute the dynamic system under the interest rate rule with the Taylor principle. Since we have no predetermined variables and the system is unstable around the steady state,
Figure 6: Monetary shock under predetermined exchange rates
Figure 7: Monetary shock under monetary targeting
there is no adjustment in response to the monetary shock. From (46), an increase in $\alpha$ will reduce $x_s$. Similarly as in Figure 4, the system jumps to its new steady state like point B.

Figure 8 shows the paths of the main variables. Since an increase in $\alpha$ does not change the steady state level of $\pi(t)$ (from (45)), $\pi(t)$ remains unchanged on impact (Panel C in Figure 8). $x(t)$ falls to its new steady state level on impact (Panel G in Figure 8). From (40), $i(t)$ remains at its initial steady state level (Panel B in Figure 8). The new steady state levels of $x$, $\lambda$, and $e$ due to an increase in $\alpha$ are explained in detail in Appendix A3. From the paths of $\alpha$ and $i(t)$ and by substituting (A21) into (41), we can get that the new steady state level of $c^N(t)$ is equal to its initial level (Panel F in Figure 8). From the path of $c^N(t)$ and (15), therefore, the steady state level of real money balances $n(t)$ increases on impact (Panel E in Figure 8).

To demonstrate the robustness of our results, we include Appendix A1.2, in which we also examine the case with a sticky nominal interest rate. It is shown that the monetary shock considered in this section causes the exact same dynamics as all the endogenous variables shown in Figure 8.

### 4.4 Comparison of the response to monetary shocks under alternative regimes

As a benchmark case, we first replicated the well-known results about the difference in the economy’s response to monetary shocks under predetermined exchange rates and monetary targeting. As is clear from Panel F in Figures 6 and 7, we can confirm the well-known result that in response to monetary shocks, the predetermined exchange rate regime is more optimal than the monetary targeting regime. As argued by Végh (2013), Chapter 11, this is because under predetermined exchange rates, the real money balances serve as a cushion against monetary shocks (as is clear from the comparison of Panel E in Figures 6 and 7). Note that if the real money balances are predetermined, which is the case in monetary targeting, the consumption of non-tradables would have an adjustment (convergence) process to its steady state (from (15)). By comparing Panel F in Figure 8 with Panel F in Figures 6 and 7, we can get that in terms of adjustments to monetary shocks, the inflation targeting regime is more optimal than the monetary targeting regime, and as optimal as the predetermined exchange rate regime. The reason why the inflation targeting regime is more optimal than the monetary targeting regime is that the real money balances under the inflation targeting regime are not predetermined and play the role of absorbing monetary shocks as under predetermined exchange rates.
Figure 8: Monetary shock under the interest rate rule with the Taylor principle
5 Conclusion

Although there have been many works on the conventional wisdom on predetermined and flexible exchange rates, existing studies have been mainly focused on the choice between predetermined exchange rates and monetary targeting under flexible exchange rates. The aim of our study is to extend the literature regarding the Mundell-Fleming dictum to include the inflation targeting regime. After replicating the well-known results about the difference in the economy’s response to the shocks under predetermined exchange rates and monetary targeting, we have compared them with the economy’s response to the shocks under the inflation targeting regime.

The results of our analysis show that the inflation target regime is superior (or at least equal) to predetermined exchange rates and monetary targeting in absorbing real and monetary shocks, respectively. The intuitive explanation for this result is as follows. The real exchange rate plays the key role of absorbing real shocks. Since the real exchange rate is predetermined under predetermined exchange rates, the predetermined exchange rate regime is not as optimal as the monetary targeting regime in the presence of real shocks. On the other hand, the real money balances are the key shock absorber to monetary shocks. Since the real money balances are predetermined under monetary targeting, the economy under monetary targeting cannot make immediate adjustments to monetary shocks. In contrast, under the interest rate rule with an inflation target, neither the real exchange rate nor real money balances are predetermined. Hence, the economy under inflation targeting can make better (or at least equivalent) adjustments to both real and monetary shocks as compared to under predetermined exchange rates and monetary targeting.

Appendix

A1 Sticky interest rate rule

In this appendix, we consider the nominal interest rate rule as follows:

\[ \dot{i}(t) = \phi (\pi(t) - \bar{\pi}) . \]  

(A1)

According to this policy rule, policy makers increase (reduce) the rate of change (not the level) of the nominal interest rate if the actual inflation level is above (below) the target level set by the policy makers in advance.\(^{17}\) This type of nominal interest rate rule implies that the nominal interest rate is a predetermined variable, and sticky.

By substituting (17) and (41) into (12), we obtain

\[ \dot{\pi}(t) = \theta \left\{ \bar{y}^N - \frac{1 - \gamma}{(1 + \alpha i(t)) x(t)} \right\} . \]  

(A2)

Similarly, as in Section 3.3, we have

\[ \dot{x}(t) = (r + \pi(t) - i(t)) x(t) . \]  

(A3)

\(^{17}\)Végh (2013) explains this type of interest rate rule in detail (chapter 9, page 438).
Equations (A1), (A2), and (A3) constitute a system of differential equations in $i$, $\pi$, and $x$. In the steady state, we have

$$i_s = r + \bar{\pi},$$  \hspace{1cm} (A4)  

$$\pi_s = \bar{\pi},$$  \hspace{1cm} (A5)  

$$x_s = \frac{1 - \gamma}{\{1 + \alpha(r + \bar{\pi})\} y}.$$  \hspace{1cm} (A6)  

By linearizing the system comprising (A1), (A2), and (A3) around the steady state, we have

$$\begin{bmatrix} \dot{i}(t) \\ \dot{\pi}(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & \phi & 0 \\ \frac{\alpha \theta (1-\gamma)}{x_s (1+\alpha x_s)^2} & 0 & \frac{\theta (1-\gamma)}{(1+\alpha x_s) x_s^2} \\ -x_s & x_s & 0 \end{bmatrix} \begin{bmatrix} i(t) - i_s \\ \pi(t) - \pi_s \\ x(t) - x_s \end{bmatrix}. $$

The determinant of the matrix on the right-hand side is denoted as $\Delta$ and has the value

$$\Delta = - \frac{\phi \theta (1 - \gamma)}{(1 + \alpha x_s) x_s} < 0.$$  

The trace of the matrix on the right-hand side is denoted as $Tr$ and has the value

$$Tr = 0.$$  

The determinant and the trace imply that there exist one negative and two positive roots. Note that $i(t)$ is a predetermined variable. Since the number of negative roots is equal to the number of predetermined variables (i.e., $i(t)$), we can pin down the saddle path converging to the steady state for a given value of $i_0$, $\pi_0$, and $x_0$.

Let $\delta_1$ denote the negative root. Let $h_{1j}, j = 1, 2, 3$ denote the elements of the eigenvector associated with the root $\delta_1$. By setting the constants corresponding to the two unstable roots as zero, we can express the solution of this dynamic system as

$$i(t) - i_s = \omega_1 h_{11} \exp(\delta_1 t),$$  \hspace{1cm} (A7)  

$$\pi(t) - \pi_s = \omega_1 h_{12} \exp(\delta_1 t),$$  \hspace{1cm} (A8)  

$$x(t) - x_s = \omega_1 h_{13} \exp(\delta_1 t),$$  \hspace{1cm} (A9)  

where $\omega_1$ is the constant associated with the root $\delta_1$. To obtain $h_{1j}, j = 1, 2, 3$, we solve

$$\begin{bmatrix} -\delta_1 & \phi & 0 \\ \frac{\alpha \theta (1-\gamma)}{x_s (1+\alpha x_s)^2} & -\delta_1 & \frac{\theta (1-\gamma)}{(1+\alpha x_s) x_s^2} \\ -x_s & x_s & -\delta_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. $$

It follows that

$$\frac{h_{12}}{h_{11}} = \frac{\delta_1}{\phi} < 0$$  \hspace{1cm} (A10)  

and

$$\frac{h_{13}}{h_{11}} = \left( \frac{1}{\phi} - \frac{1}{\delta_1} \right) x_s > 0.$$  \hspace{1cm} (A11)
A1.1 Real shocks

Equations (A7), (A8), and (A10) imply that $i(t)$ and $\pi(t)$ will converge to their steady state values from opposite directions. Equations (A7), (A9), and (A11) imply that $i(t)$ and $x(t)$ will converge to their steady state values from the same direction. However, we should note that the real shock of an increase in $\gamma$ does not change the steady state level of $i(t)$. Since $i(t)$ is the only predetermined variable in this system, we do not observe an adjustment process along the saddle path due to the real shock. Hence, as depicted in Figure 5, $i(t)$ remains at its initial steady state level (Panel B in Figure 5). $\pi(t)$, too, remains at its initial steady state level (Panel C in Figure 5). From (A6), we know that $x(t)$ has a new lower steady state value due to an increase in $\gamma$. Since, as argued above, an increase in $\gamma$ does not result in an adjustment process along the saddle path, $x(t)$ jumps to its new steady state level on impact (Panel F in Figure 5). Similarly, as in Section 3.3, the changes in the steady state levels of $x(t)$, $\lambda(t)$, and $c(t)$ due to an increase in $\gamma$ are explained in Appendix A2. From the paths of $\gamma$, $x(t)$, and $i(t)$, and equation (41), we obtain the path of $c^N(t)$ (Panel E in Figure 5) (as explained in Appendix A2). It turns out that the consumption of non-tradables remains unchanged over time.

A1.2 Monetary shocks

As argued in Section A1, equations (A1), (A2), and (A3) constitute the dynamic system under the sticky interest rate rule. We should note that the monetary shock does not change the steady state level of $i(t)$. Since $i(t)$ is the only predetermined variable in this system, we do not observe an adjustment process along the saddle path. Hence, $i(t)$ remains at its initial steady state level (Panel B in Figure 8). From (A5), $\pi(t)$, too, remains at its initial steady state level (Panel C in Figure 8). From (A6), we get that $x(t)$ has a new lower steady state value due to an increase in $\alpha$. Since, as argued above, an increase in $\alpha$ does not result in an adjustment process along the saddle path, $x(t)$ jumps to its new lower steady state level on impact (Panel G in Figure 8). Similarly as in Section 4.3, the new steady state levels of $x$, $\lambda$, and $e$ due to an increase in $\alpha$ are explained in detail in Appendix A3. From the paths of $\alpha$ and $i(t)$ and by substituting (A21) into (41), we can get that the new steady state level of $c^N(t)$ is equal to its initial level (Panel F in Figure 8) (as explained in Appendix A3). From (15), the steady state level of real money balances $n(t)$ increases on impact (Panel E in Figure 8).

A2 Shifts in $x$, $\lambda$, and $e$ due to an increase in $\gamma$

Let $\gamma'$ denote a higher level of $\gamma$ (i.e., $\gamma \rightarrow \gamma'$). Correspondingly, we denote the initial steady state levels of $x(t)$, $\lambda(t)$, $e(t)$ and $c^N(t)$ as $x$, $\lambda$, $e$, and $c^N$, and denote their new steady state levels as $x'$, $\lambda'$, $e'$, and $c^{N'}$. We now show how their steady state levels change (or does not change) in response to the increase in $\gamma$ in Sections 3.3 and A1.1.
From equation (A6) (equation (46)), the following must hold

\[ x' = \frac{1 - \gamma'}{\{1 + \alpha'(r + \bar{r})\} \hat{y}^N}, \]  
(A12)

in the new steady state. Then, we have

\[ \frac{x'}{x} = \frac{1 - \gamma'}{1 - \gamma} < 1. \]  
(A13)

Substituting the definition of \( x(\equiv \frac{1}{e}) \) into (A13) gives

\[ x' = \frac{1 - \gamma' \lambda}{1 - \gamma e}. \]  
(A14)

The substitution of (24) into (A14) gives\(^{18}\)

\[ x' = \lambda \left( \frac{1 - \gamma'}{\gamma'} \right) \left( \frac{rk_0 + y^T}{\hat{y}^N} \right). \]  
(A15)

From the definition of \( x(\equiv \frac{1}{e}) \), we also have \( x' = \frac{\lambda'}{e} \). Using (24) again, we obtain

\[ x' = \left( 1 - \gamma' \right) \left( \frac{rk_0 + y^T}{\hat{y}^N} \right). \]  
(A16)

Hence, it follows from (A15) and (A16) that

\[ \frac{\lambda'}{\lambda} = \frac{\gamma'}{\gamma} (> 1). \]  
(A17)

From the definition of \( x(\equiv \frac{1}{e}) \) and (A13), equation (A17) implies that

\[ \frac{e'}{e} = \frac{x'}{x} \lambda' x' = \frac{\gamma'}{\gamma} \left( 1 - \gamma \right) \left( 1 - \gamma' \right) (> 1). \]  
(A18)

From equations (41), (A13), and the fact that \( i \) remains unchanged, we can confirm that

\[ c^N = c'^N \left( = \hat{y}^N \right). \]  
(A19)

### A3 Shifts in \( x, \lambda, \) and \( e \) due to an increase in \( \alpha \)

In this appendix, we show how \( x, \lambda, \) and \( e \) change (or does not change) in response to the increase in \( \alpha \) in Sections 4.3 and A1.2.

From equation (A6) (equation (46)), the following must hold

\[ x' = \frac{1 - \gamma}{\{1 + \alpha'(r + \bar{r})\} \hat{y}^N}, \]  
(A20)

\(^{18}\)Equation (24) holds under both interest rate rules.
in the new steady state. Then, we have

\[ \frac{x'}{x} = 1 + \alpha \left( r + \bar{\pi} \right) \left( 1 + \alpha' \left( r + \bar{\pi} \right) \right) \left( < 1 \right). \]  (A21)

From equation (11), we have

\[ e = e' = \left( \frac{\gamma}{1 - \gamma} \right) \left( \frac{\bar{y}^N}{rk_0 + \bar{y}^L} \right). \]  (A22)

By substituting the definition of \( x \left( \equiv \frac{1}{e} \right) \left( x' \left( \equiv \frac{1}{e'} \right) \right) \) into (A21) and using (A22), we have

\[ \frac{x'}{x} = 1 + \alpha \left( r + \bar{\pi} \right) \left( 1 + \alpha' \left( r + \bar{\pi} \right) \right) \left( < 1 \right). \]  (A23)

From equations (41), (A21), and the fact that \( i(= r + \bar{\pi}) \) remains unchanged, we can confirm that

\[ C^N = C'' \left( = \bar{y}^N \right). \]  (A24)

References


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