Discussion Paper Series
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DP2011-11

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March 16, 2011

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January 2011

ABSTRACT

This paper studies anti-agglomeration subsidies in a core-periphery setting when firms are heterogeneous in labour productivity, focusing on the effects of relocation subsidy on firm location in various tax-financing schemes (local versus global). We discuss how subsidy can enhance welfare in periphery. As a result we find that subsidy proportional to profits can induce the relocation of high productivity firms and that a subsidy can increase welfare in periphery. Concerning tax-financing schemes, local tax financing scheme has an optimal level of subsidy.

JEL F21,H32, H25.

Keywords: heterogeneous firms, anti-agglomeration relocation subsidy, global/local tax-financing scheme.

1. Introduction

Low income per-capita and low economic growth in undeveloped peripheral regions within a country is a serious concern in regional policy. To solve this problem, governments often pursue an anti-agglomeration strategy that induces firms to relocate from core to undeveloped peripheral areas

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and creates industrial clusters. One of the main policy programs is the relocation subsidy.[2] For example, the EU Structural Funds have influenced the relocation of industries to peripheral regions of the EU in the last decade.[3] Many previous studies empirically test the impact of Structural Funds on regional growth using regional level data. Some find negative impacts, however some others observe a positive or statistically insignificant effect (Mohl and Hagen, 2010).[4] Much closer to our paper, there are a few empirical studies on the impact of the Structural Funds on industrial location patterns. (Midelfart-Knarvik and Overman, 2002 and Midelfart-Knarvik et al.,2000).[5] Furthermore, one of the recent advancements in relocation subsidy studies is to use micro data and investigates some impacts of a regional subsidy on firm performance and location patterns based on heterogeneous firm productivity. A few current works analyse regional subsidy programs using micro firm/plant level data (Martin et al., 2010 and Okubo and Tomiura, 2010). They examine the impact of relocation subsidy programs on productivity and location patterns in terms of firm productivity.[6] To link with the emerging firm-level empirical research on this issue, our paper uses an economic geography model combined with firm heterogeneity. Our paper has two aims, firstly to

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[2] Relocation subsidy in our paper includes not only direct subsidy in which relocated firms receive subsidy but also indirect subsidy in which public infrastructure in specific areas is developed, local employment is subsidised or taxation is deducted. Even indirect subsidy can reduce cost and give incentive to operate production in periphery.

[3] EU structural funds have several objectives. One of the biggest spending is to support economic development in the less developed regions. In reality, the funds prohibit direct subsidy to relocated firms and intend to promote capital investment in periphery through indirect way such as the regional development of infrastructure.

[4] The divergence of the result is due to different research designs in methodology and data samples. See Table 1 in Mohl and Hagen (2010) for an excellent survey of main results and methodology in previous studies on this issue.

[5] Midelfart-Knarvik and Overman (2002) find that European Structural Funds have some effects on industry relocation, in particular a slightly positive impact for R&D intensive industries in specific regions.

[6] Martin et al. (2010) studies a French subsidy program and Okubo and Tomiura (2010) studies case of Japan. The Industrial Relocation Subsidy started in the 1970s Japan is a kind of direct subsidy, just as in our model. Once firms relocate from core to periphery, they receive a certain amount of subsidy. See Okubo and Tomiura (2010) for more details.

provide theoretical predictions on the impact of a regional subsidy on firm relocation and secondly, to discuss which type of regional subsidy results in a successful outcome in the periphery.

An economic geography model with firm heterogeneity is not entirely new. Baldwin and Okubo (2006) use a model that marries the footloose capital (FC) model of Martin and Rogers (1995) and the heterogeneous-firm-trade model of Melitz (2003).[7] One section in their paper provides a theoretical prediction for the impact of a specific-relocation (per-firm) subsidy. It shows that a fixed amount of relocation subsidy per-firm would always attract the least productive firms since they have the lowest opportunity cost of leaving the agglomerated region. If their theoretical prediction is true, regional policies should unintentionally widen the productivity gap between the core and the periphery.[8] However, their explanation is the first step in explaining the mechanism of relocation subsidies in a simple model. In this paper, we aim to show a contrasting consequence of relocation subsidies, whereby the subsidy reduces the productivity gap and enhances the welfare in periphery. Keeping the basic framework of the economic geography model of Baldwin and Okubo (2006) we integrate schemes of different types of subsidy, including a tax financing scheme. In particular, while Baldwin and Okubo (2006) employ a per-firm relocation subsidy and discuss the change of location caused by the subsidy without welfare analysis, we consider a relocation subsidy proportional to the firm's profits (or productivity), taking account of various tax financing schemes and then find optimal policy scheme based on welfare.[9] As in Baldwin and Okubo (2006), the FC model is employed due to it being the simplest tractable model, i.e. it has analytically solvable

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[7] The previous study on the integration of the economic geography model and heterogeneous-firm-trade model is Okubo (2009).

[8] In another example of theoretical work on the unintended effects of regional policy, Dupont and Martin (2006) show that regional policies, especially subsidies to poor regions proportional to firms' profits that are financed by national taxes, *increase* cross-regional income inequality within a country. Their argument rests on the observation that the effect of the subsidy spills over into rich regions, where many owners of capital (who are the beneficiaries of subsidised profits) reside.

[9] A relocation subsidy proportional to firm profits and size could be more natural than per-firm subsidy, since large and productive firms would be expected to receive more subsidy and to be more likely to benefit from a relocation subsidy than smaller firms.

equilibrium since there is no circular causality, and no backward nor forward linkages. The FC model's assumptions are innocuous to our analysis because our focus is on firm relocation driven by subsidy rather than labour migration.[10]

Our paper makes two main contributions to regional science literature. First, we study the impact of a relocation subsidy in more depth, extending an idea of Baldwin and Okubo (2006) by integrating heterogeneous-firm trade model à la Melitz (2003) to an economic geography model . In regional policy literature, this paper sheds light on how important firm heterogeneity is and how spatial sorting is driven by relocation subsidy.[11] Second, the discussion of tax financing schemes (local or global tax financing scheme) and welfare analysis allow for more precise policy analysis. We can find the optimal relocation subsidy that can relocate high productivity firms to the periphery and can enhance welfare in the periphery.

As a result of analysis, we find that 1) a proportional relocation subsidy attracts the higher productivity firms to the periphery, 2) the periphery has an incentive to provide a proportional subsidy policy financed by a local tax and 3) while a centralised economy chooses zero-subsidy to avoid tax payment, a decentralised economy leads the periphery to choose the optimal level of subsidy, resulting in enhanced welfare in the periphery. These results are either in contrast with or missing in the per-firm relocation subsidy as proposed in Baldwin and Okubo (2006).

There are five sections in this paper. The next section introduces the basic model and explores the equilibrium. Section 3 examines how subsidy rates are determined and discusses both the

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[10] Since our model is aimed at studying the impact of subsidy on firm location patterns with various tax financing schemes, which affect per-capita income and welfare in periphery, we use the FC model. If we allowed for labour migration, then the model would combine a heterogeneous labour market and so make it more complex, to the extent that it would become impossible to solve the equilibrium and welfare analytically due to circular causality. What is more, the timing of taxation and subsidy might also affect expectations of the migrants. This is out of our scope however is a space for future research.

[11] Previous studies on regional policies presume firms are homogeneous (as in the standard economic geography model). See Baldwin et al. (2003), Dupont and Martin (2006) and Ostbye (2010).

tax-financing of subsidies and socially optimal firm location. Section 4 provides some further discussion. The last section presents our concluding remarks.

2. THE BASIC MODEL

2.1. *The footloose capital model with heterogeneous firms*

Our model works with two regions, two sectors and two factors of production. The regions – referred to as the North (core) and the South (periphery) – have identical technology, tastes, and openness to trade, but they differ in their market size (the North has larger demand by convention). The two sectors are the M-sector (manufactures) and the A-sector (a numeraire sector). The numeraire sector produces a homogenous good using labour subject to constant returns, perfect competition and costless trade. The M-sector produces a differentiated variety of products using labour as well as capital with increasing returns to scale, there is Dixit-Stiglitz monopolistic competition and iceberg trade costs. Specifically, firms' marginal costs are flat with respect to the scale of production and international trade is subject to iceberg trade costs whereby some goods "melt away" during transportation.

We have two factors, capital and labour. Capital can move between the North and the South, but labour is immobile. We assume that each individual is not mobile and owns an endowment of labour and capital. All capital is owned by individuals so while capital is mobile, capital owners (individuals) are not (this cuts out complications that would otherwise arise from backward linkages). We can exogenously give total units of labour (L^w) and capital (K^w) in the world as well as s_L and s_K, which denote the Northern share of labour and capital endowments at the initial equilibrium respectively. Initial endowment ratios equate shares of capital and labour, s_L=s_K, although capital is mobile and firm location is endogenously determined in the long-run equilibrium. Capital returns are repatriated to the origin. For clarity's sake, each individual in the North and the South has a capital endowment consisting of a portfolio comprising a common share of stock of all the world's firms (portfolio assumption). Thus, by sharing all firms' profits over the world as stock holder, the capital reward that each individual receives is the average operating profits of all firms in the world. Due to the portfolio assumption and the quasi-linear utility function (as shown below), it always follows $s_E=s_L$, where $s_E=E/E^w$ is the North's share of the total income in the world (E^w), normalised, i.e. $E^w=1$. Hence, we have $s_L=s_K=s_E$.

Basic set up

The tastes of the representative consumer in each region are given by the quasi-linear utility function:

$$U = \mu \ln C_M + C_A, \quad C_M \equiv \left(\int_{i \in \Theta} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \qquad 1 > \mu > 0, \quad \sigma > 1$$

where C_M represents consumption of the composite of M-sector varieties, C_A stands for consumption of the A-sector good, μ denotes the expenditure share of the composite of M-sector varieties. $\sigma>1$ is the constant elasticity of substitution between any two M-sector varieties and Θ is the set of all varieties produced.[12] We note that quasi-linear utility function excludes an income effect.

On the supply side, firm-level heterogeneity stems from differences between firms' marginal costs. Although all the Dixit-Stiglitz varieties enter consumers' preferences symmetrically, the cost of producing each variety is different. A typical (Northern) firm *j*'s cost function reads:

$$TC_j = \pi_j + wa_j x_j$$

where π_j is the firm *j*'s fixed cost and thus denotes operating profit (capital return), which is due to the requirement of one unit of capital, and $wa_j x_j$ is firm *j*'s variable cost, which is driven by labour costs. x_j is *j*'s output, a_j is unit labour requirement, w is wage. Importantly, firms have heterogeneous

[12] This set of varieties is pre-determined by endowments because each variety requires one unit of capital and the world capital stock is fixed, normalised to unity.

unit labour requirements. This means that the unit labour requirements are variety-specific and thus firm-specific. The distribution of firm-level efficiency is part of each region's endowment. The specific assumption is that the distribution of the *a*'s is subject to the Pareto distribution, which cumulative density function is defined as:

(1) $$G[a] = (\frac{a^{\rho}}{a_0^{\rho}}), \qquad 1 \equiv a_0 \geq a \geq 0, \quad \rho \geq 1$$

Here a_0 is the scale parameter (highest possible marginal cost) and ρ is the shape parameter. A small ρ means more dispersed in '*a*', that is, firms are more heterogeneous. We can normalise a_0=1. The cumulative density function of *a*'s can be depicted in Figure 1. We initially consider full agglomeration in the North with small trade costs (the condition for full agglomeration is specified below). The distribution in the North is given as G[*a*], where the total mass of firms is 1 in the North and 0 in the South. The total mass of firms is unity and the total capital endowment in the world is thus also unity, K^w=1. Note that our model assumes away entry and exit in the M-sector.

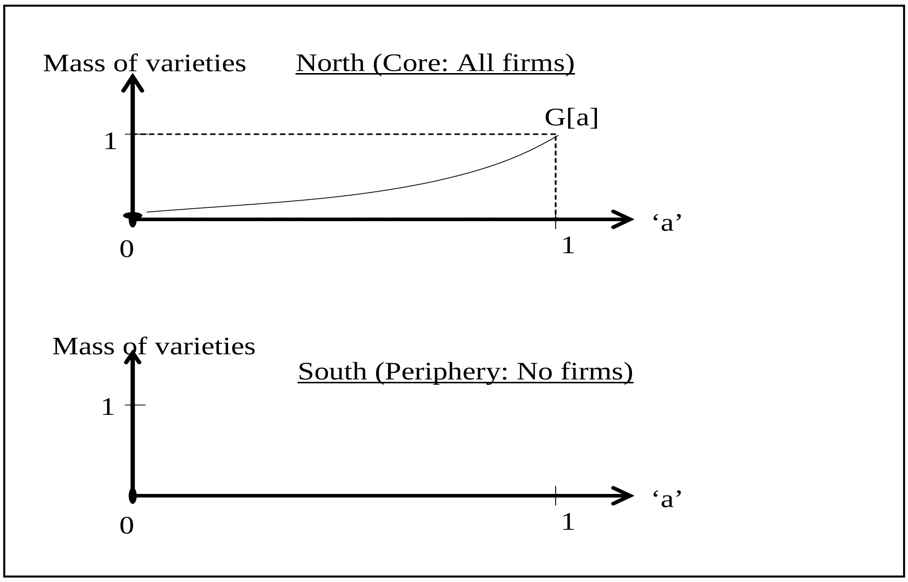

Figure 1: Endowed distribution of capital and the mass of firms (at initial equilibrium).

2.2. ***Intermediate results***

Results for the numeraire sector where one has costless trade and perfect competition are well known. Constant returns, perfect competition and zero trade costs equalise nominal wage rates across the regions. We choose units of the numeraire good such that p_A=1=w=w* (where * stands for the South's value). This means that all differences in M-firms' marginal costs are due to differences in their unit labour requirements, a's, so we can refer to the a's as the marginal cost without ambiguity.

The M-sector is marked by all the usual Dixit-Stiglitz results. Firms' prices are a constant mark-up of their marginal selling costs. In the local market, these marginal costs entail only production costs as there are no internal transportation costs. The price in the export market includes iceberg trade costs, t≥1, marked up by the constant Dixit-Stiglitz mark-up. The prices are written as $p_j = \frac{a_j}{1-1/\sigma}$ and $p_j t = \frac{a_j t}{1-1/\sigma}$.

The standard CES demand function for a variety-j produced and sold in the North can be written as

$$c_j = (p_j)^{-\sigma} \frac{\mu E}{\overline{m}}; \qquad \overline{m} \equiv \int_{i\in\Theta} p_i^{1-\sigma} di + \int_{h\in\Theta^*} \phi p_h^{1-\sigma} dh, \qquad \sigma > 1 \geq \phi \equiv t^{1-\sigma} \geq 0$$

where c_j is variety j's consumption and p_j is variety-j's producer price (which equals its consumer price since it is produced locally), E denotes total income in the North. The first term in the definition of $\overline{m}$ reflects the prices of goods that are produced in the North. The second term reflects the imported varieties whose producer prices are p_h; Θ and Θ* are the sets of goods produced in the North and the South, respectively. We note that $\overline{m}$ falls as the prices of individual varieties rise. A parameter that plays a critical role in our paper is ϕ, which we refer to as the free-ness of trade. When iceberg trade costs are prohibitive (t=∞), the freeness of trade is zero(ϕ=0), while when there are no iceberg trade costs (t=1), the freeness of trade is unity (ϕ=1).

2.3. *Initial Equilibrium—Full agglomeration*

Since agglomeration produces rent which holds firms in their own agglomeration region when trade costs become smaller (we call critical trade costs creating rents as "sustain point"), this rent calls for various sorts of public policy incentives with regards to shifting firm location (Baldwin et al. 2003). To exclude the marginal impact of relocation subsidies and focus on the substantial impact of relocation subsidy on firm location, we start with full agglomeration in the North. For this reason, we regard full agglomeration as an initial stable equilibrium. In other words, we start by considering small trade costs so that all firms in the North choose to remain there, i.e. ϕ above the sustain point given as $\phi^{CP} = \dfrac{1-s_E}{s_E}$, where we have introduced s_E as shorthand for the North's share of world expenditure (we adopt the convention that the North is bigger so $s_E > ½$).[13]

Utility maximisation generates the familiar CES demand functions in the manufactures sector. These, together with the standard Dixit-Stiglitz monopolistic competition assumptions on market structure, imply 'mill pricing' is optimal and operating profit earned by a typical firm in a typical market is 1/σ times firm-level revenue.[14] Accordingly, operating profit realised by a firm in the North is:

(2) $$\pi[a] = \frac{\mu}{\sigma}\left(s_E \frac{\left(\frac{a}{1-1/\sigma}\right)^{1-\sigma}}{\int_i p_i^{1-\sigma} di} + (1-s_E)\frac{\phi\left(\frac{a}{1-1/\sigma}\right)^{1-\sigma}}{\int_i p_i^{*1-\sigma} di} \right).$$

The first term in the bracket is the value of firm-specific sales in the Northern market. The second term shows the firm's export sales, which is similar except the firm's price includes the iceberg trade cost raised to 1-σ and the denominator involves p^*'s, the prices of all goods in the Southern market.

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[13] See Martin and Rogers (1995) for the derivation of the sustain point in the FC model.

[14] A typical first order condition is p(1-1/σ)=wa; rearranging, the operating profit, (p-wa)c, equals pc/σ.

The features of **(2)** that matter for our results are 1) all firms earn a positive operating profit, 2) the profitability of firms increases with firm-level efficiency, and 3) the most efficient firms, i.e. firms with low marginal costs, are the largest in the sense that they sell the most.

The operating profit as a function the firm's '*a*' for a firm based in the North can be rewritten as:

(3) $$\pi[a] = a^{1-\sigma}\left(\frac{s_E}{\overline{\Delta}} + \frac{\phi(1-s_E)}{\overline{\Delta}*}\right)\frac{\mu}{\sigma}$$

the $\overline{\Delta}$'s are the denominators of the North's and the South's CES demand functions in the initial equilibrium, i.e. full agglomeration in the North: $\overline{\Delta} \equiv \int_0^1 a^{1-\sigma} dG[a] = \lambda \quad \overline{\Delta}* \equiv \phi\int_0^1 a^{1-\sigma} dG[a] = \lambda\phi$,

where $\lambda \equiv \dfrac{\rho}{1-\sigma+\rho} > 0$, $\rho > 1$. We assume 1-σ+ρ > 0 (which ensures the integrals converge).

2.4. *Relocation subsidy and Deviation Tendency*

A relocation subsidy is employed so as to induce some firms to relocate to the periphery. Once a firm relocates from core to periphery, the firm receives the subsidy proportional to its profit. Specifically while the operating profit in the North can be kept as **(3)**, a proportional subsidy, *u*, is added to the operating profit in the South if a single firm relocates from the North to the South. Operating profit in the South as a function of the relocated firm's '*a*' can be written as:

(4) $$\pi^*[a] = a^{1-\sigma}\left(\frac{\phi s_E}{\overline{\Delta}} + \frac{1-s_E}{\overline{\Delta}^*}\right)\frac{\mu}{\sigma}(1+u)$$

where *u* denotes the rates of a relocation subsidy.[15]

Starting with all firms in the North, the change in operating profit for an atomistic firm moving from the North to the small South would be:

(5)

[15] Note that the subsidy in this paper, which adds to the profits of the relocated firms rather than costs, never affects the mass of varieties and marginal costs and thus never influences prices.

$$\pi[a] - \pi^*[a] = a^{1-\sigma}\left(\frac{s_E}{\bar{\Delta}} + \phi\frac{(1-s_E)}{\bar{\Delta}^*} - \left(\phi\frac{s_E}{\bar{\Delta}} + \frac{(1-s_E)}{\bar{\Delta}^*}\right)(1+u)\right)\frac{\mu}{\sigma} = \frac{a^{1-\sigma}\mu}{\sigma\lambda}\left(1 - \left(\phi s_E + \frac{1-s_E}{\phi}\right)(1+u)\right)$$

Figure 2 plots (5) in terms of *a*. As long as the above equation is positive, no firms have incentive to move to the South. Above a certain level of subsidy, $u^{\min}$, in contrast, the above equation switches from positive to negative. The most efficient firms are those most likely to relocate to the small region, as shown in Figure 2. Note that the minimum subsidy, $u^{\min}$, such that at least one firms move to the South, is satisfied as $1 - \left(\phi s_E + \frac{1-s_E}{\phi}\right)(1+u^{\min}) = 0$ which is derived from (5). On the other hand the subsidy rates to induce all firms to move to the South are given as $u^{\max} = \frac{s_E}{\phi} + \phi(1-s_E) - 1$.

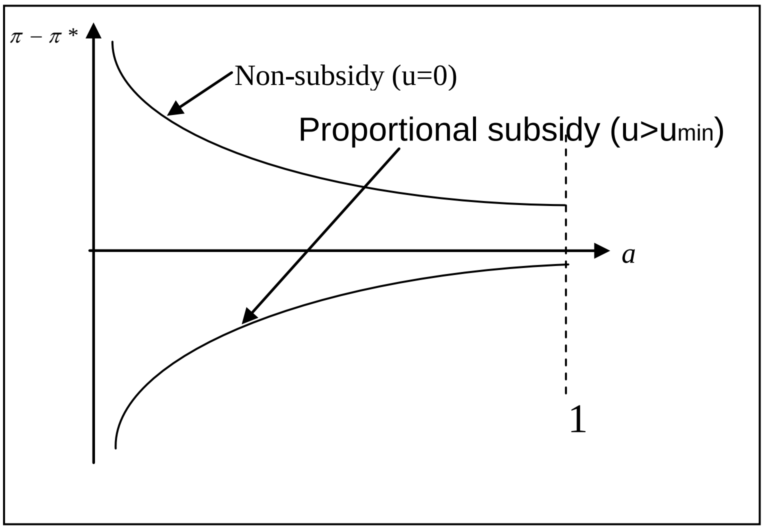

Figure 2: Relocation tendencies with a proportional subsidy.

Result 1: The first firms to respond to a relocation subsidy will be the most efficient firms. The minimum subsidy rate is $u^{\min} = \frac{1}{s_E\phi + \frac{1-s_E}{\phi}} - 1$**, which can induce the most efficient firms to**

move to the small region. The maximum rate is $u^{\max} = \frac{s_E}{\phi} + \phi(1-s_E) - 1$ **which stimulates all firms to move to the small region.**

Note that minimum and maximum rates are independent of firm heterogeneity (λ, ρ), and a maximum rate is finite.[16]

Without relocation subsidies, there is no capital movement from core to periphery. This is a result of small market size and the lack of a cost advantage in the periphery (the production costs are the same as in the core however there is smaller demand in the periphery). However, once substantial subsidies proportional to firm size are introduced, the situation changes: capital moves from core to periphery even with small trade costs. High productivity firms can receive more subsidy than the low productivity ones, implying that the most efficient firms can obtain the largest subsidy, and vice versa. Since the most efficient firms are most sensitive in their location and profits, they are the first ones to relocate to the South benefiting from the largest amount of subsidy.

2.5. *Locational equilibrium*

Based on relocation tendencies, the locational equilibrium is determined to equalise profit between two regions. The profit gap can be written as

(6) $$\pi[a] - \pi^*[a] = a^{1-\sigma}\,\frac{\mu}{\sigma}\left(\frac{s_E}{\Delta} + \phi\,\frac{(1-s_E)}{\Delta^*} - \left(\phi\,\frac{s_E}{\Delta} + \frac{(1-s_E)}{\Delta^*}\right)(1+u)\right)$$

$$\Delta = \lambda\left(\phi a_R{}^{1-\sigma+\rho} + (1-a_R{}^{1-\sigma+\rho})\right), \qquad \Delta^* = \lambda\left(a_R{}^{1-\sigma+\rho} + \phi(1-a_R{}^{1-\sigma+\rho})\right)$$

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[16] The minimum rate of u is always positive. From the full agglomeration condition in the initial equilibrium (non-subsidy): $\pi[a] - \pi^*[a] = \frac{\mu}{\sigma}\left(1 - \phi s_E - \frac{1-s_E}{\phi}\right) > 0$, $1 - (\phi s_E + \frac{1-s_E}{\phi}) > 0$ can be derived. Thus, the minimum subsidy is thus: $u^{\min} = \dfrac{1}{s_E\phi + \frac{1-s_E}{\phi}} - 1 > 0$.

where a_R is the cut-off level of efficiency below which all firms locate in periphery ('*R*' is a mnemonic for relocation cut-off). The cut-off a_R is determined by $\pi[a_R] - \pi^*[a_R] = 0$ and can be solved analytically:

(7) $$a_R = \left(\frac{1-\phi\chi}{(1-\phi)(1+\chi)} \right)^{1/(1-\sigma+\rho)}$$

where $\chi \equiv \frac{(1-\phi(1+u))}{(1+u-\phi)} \frac{s_E}{1-s_E}$. u is positively correlated with a_R, which means that a higher rate of subsidy promotes firm relocation. Figure 3 plots the (Northern) production share, denoted as s_P, and freeness of trade (ϕ).[17]

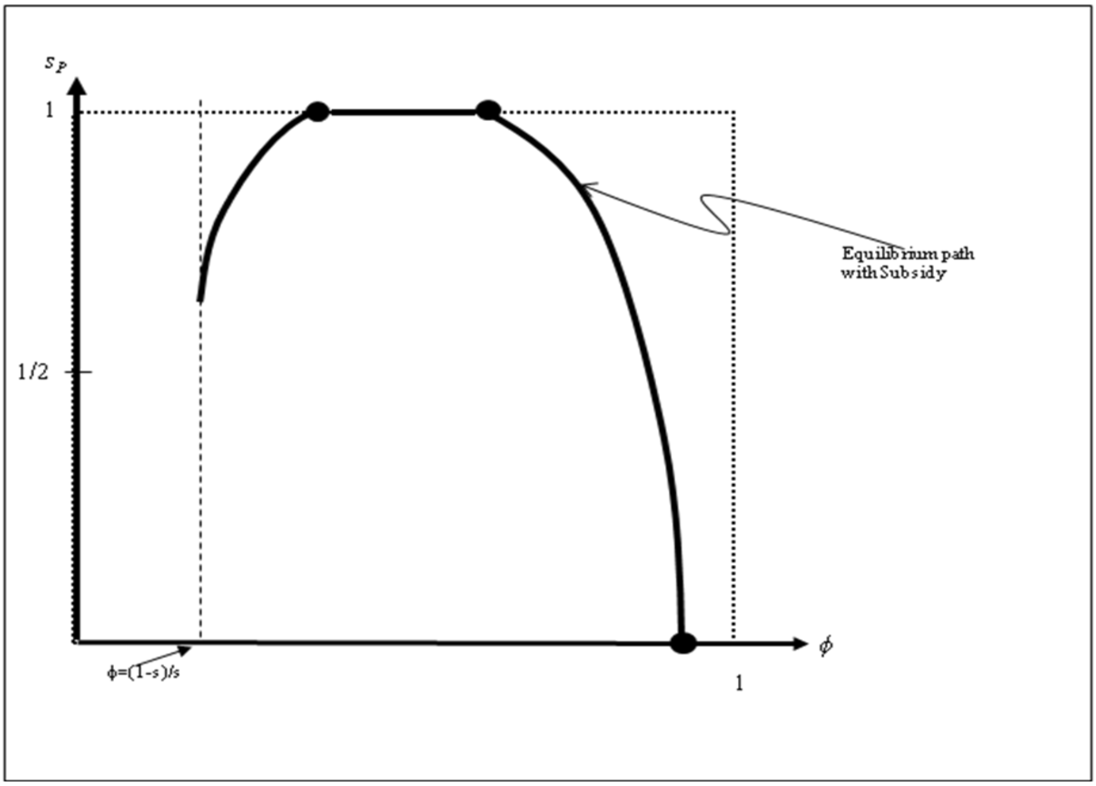

Figure 3: Production Shares.

[17] The Northern production share, s_P, is defined as the share of aggregated production of Northern firms fraction of total production total production.

3. GLOBAL VS LOCAL TAX-FINANCING SCHEME

This section provides explicit consideration of tax schemes used to finance the relocation subsidy. The subsidy is to relocate to the South as in the last section. We compare two contrasting cases, global and local tax-financing schemes. In the first scheme central government levies a tax on individual in both regions at equal rate. In the latter one, local government can raise tax locally that then subsidises firms from the North to relocate. We make two further assumptions, the subsidy is financed by per-capita taxation and the government budget is always balanced.

3.1. *Centralisation and global tax-financed subsidy*

In the first scheme, we introduce the global tax to finance relocation. Since the central government is assumed to have the authority to implement a subsidy policy, it can be considered to be the coordinator between core and periphery interests, attempting to reduce the wealth gap. Taxation is global and per-capita tax must be the same in both regions, *T*=*T**. With a solution of a_R in (7) for a certain *u*, tax rates are written as:

(8) $$T = T^* = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)a_R^{\;1-\sigma+\rho}u\,.$$ [18]

The measurement of welfare uses the per-capita indirect utility function (*v*),

(9) $$v = \ln(Y - T) + \frac{\mu}{\sigma - 1}\ln\Delta \quad \text{and} \quad v^* = \ln(Y^* - T^*) + \frac{\mu}{\sigma - 1}\ln\Delta^*$$

where *Y* (*Y**) is a per-capita income in the North (South) and *T* (*T**) are per-capita tax in the North (South) to finance subsidies. Since each individual owns one unit of labour and one unit of capital, income can be specified as $Y=Y^*=w+\bar{\pi} = w^*+\bar{\pi} = 1+\bar{\pi}$, where $\bar{\pi}$ is reward per unit of capital, which is average operating profit of all firms due to the portfolio assumption. We notice that the tax

[18] Note that our utility function is quasi-linear, and so there is no income effect. s_E is not influenced by taxation and is not a function of T and u. The total numbers of individuals and firms (capital) in the world are unity.

is refunded as subsidy to relocated firms and then finally returned to all people equally as capital rewards due to the portfolio assumption, $\bar{\pi}$. Since the increase of the capital reward due to the subsidy is equally distributed over all people, the tax payment and the relocation subsidy keep income constant and so just affect firm relocation and thus altering Δ and Δ*. Figure 4 plots welfare scaled to the same total amount of subsidy. Note that the vertical axis represents per-capita welfare, *v*[*u*] in the North, *v**[*u*] in the South, and the horizontal axis depicts subsidy rate, *u*. Without a subsidy, i.e. *u*=0, the northern welfare always exceeds the southern one, *v*[0]>*v**[0], where $v[0]=\left(\ln(1+\frac{\mu}{\sigma})+\frac{\mu}{\sigma-1}\ln(\lambda)\right)$ and $v*[0]=\left(\ln(1+\frac{\mu}{\sigma})+\frac{\mu}{\sigma-1}\ln(\phi\lambda)\right)$.[19] Increasing the subsidy from the minimum levels, i.e. $u^{\min}$, leads to firm relocation from the North to the periphery and thus reduces the northern per-capita welfare while increasing southern per-capita welfare by changing Δ and Δ*. At the extreme, all firms move to the South when the highest subsidy ($u^{\max}$) is provided, at which point the periphery's welfare switches to the initial welfare in the core, and vice versa. $v[u^{\max}]=\left(\ln(1+\frac{\mu}{\sigma})+\frac{\mu}{\sigma-1}\ln(\phi\lambda)\right)$ and $v^{*}[u^{\max}]=\left(\ln(1+\frac{\mu}{\sigma})+\frac{\mu}{\sigma-1}\ln(\lambda)\right)$.

[19] This is correspondent to full agglomeration in the standard footloose capital model (non-subsidy equilibrium). Note that $\bar{\pi}=\mu/\sigma$, $\Delta=\lambda$ and $\Delta^{*}=\lambda\phi$. Note that tax payment, *T*, and subsidy are null due to non-subsidy equilibrium.

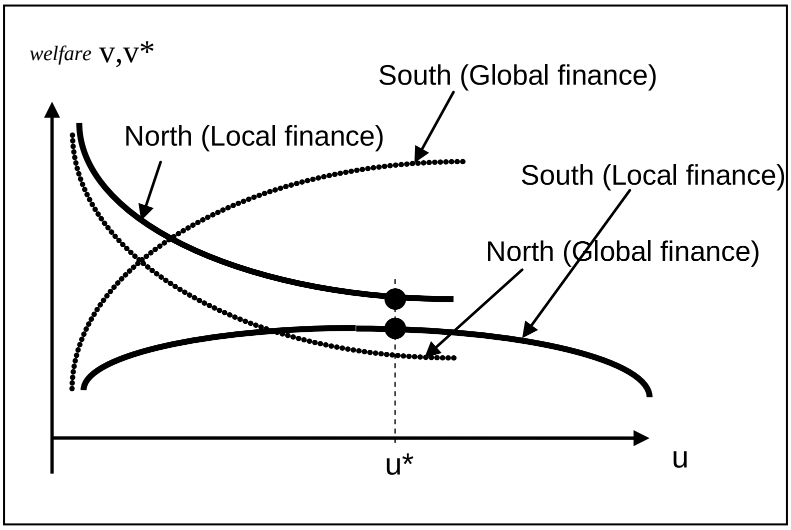

Figure 4: Global vs. local tax-financing scheme and per-capita welfare.

Here, we introduce one simple voting system concerning pros and cons of anti-agglomeration subsidy to highlight characteristics of the global tax-finance scheme. Then, residents' votes in the core and periphery perfectly reflect subsidy policy-making of the central government: based on each individual's welfare, v and v^*, voting determines the rate of subsidy. Each individual has an equal voting right. The northern residents prefer a zero-taxation and zero-subsidy policy to keep all firms in the North, whereas the southern people prefer positive levels of relocation subsidy to attract firms to the South together with taxation of all people. The North has a larger population than the South. When voting determines the subsidy rates associated with per-capita taxation in both regions (global tax-financing scheme), the non-subsidy outcome is an equilibrium, so the core residents' preference of full agglomeration is dominant. Accordingly, a global tax-financing system using centralised voting could not resolve the gap of per-capita welfare across regions.

Result 2: With centralised taxation, the no subsidy policy is an equilibrium if voting decides subsidy rates. Therefore, no relocation occurs and the gap of welfare remains between core and periphery.

3.2. ***Decentralisation and local tax-financed subsidy***

Now, we suppose the region is a decentralised nation without a central government. Then, since the periphery has authority to implement the subsidy policy, it has an incentive to provide a subsidy financed by southern residents (i.e. local tax-financed subsidy).

Figure 4 plots per-capita welfare, in which the southern welfare is hump-shaped due to the tax collection ("South (Local finance)" in the figure), while the northern welfare, "North (Local finance)", has a similar shape as before. Note that we assume in Figure 4 that the North never provides a subsidy to prevent relocation. The hump-shaped southern welfare induces the South to set an optimal subsidy rate, u^* that maximises the welfare, v^*.[20] In the South, higher rates of subsidy attract more firms but at the same time increase southern tax payment, $T^*>0$, which leads to a reduction in disposable income.

The global tax financing scheme, in which tax is equally levied in both regions and the subsidy is equally distributed via capital returns, has no income effect. However, the local tax-financing scheme is associated with taxation only in the periphery whilst the subsidy is refunded to both regions' people through capital rewards. The North pays no tax, T =0, however receives a part of the subsidy via the rise of capital rewards by firms who receive the relocation subsidy from the South. The per capita tax, levied only in the South, derives from

(10) $$T^* = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)\frac{a_R^{\ 1-\sigma+\rho}u}{1-s_L}$$

where the total amount of subsidy is divided by the southern population 1- s_L (=1- s_E, exogenously given).[21] Thus, a local tax-financed subsidy plays transfers income from the periphery to the core: A

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[20] See Appendix 1 for proof of the hump-shape curve

[21] Note that tax rates are not relevant to s_E because of quasi-linear utility function.

higher level of subsidy reduces disposal income in the periphery (South) and increases income in the core (North). This income transfer raises northern welfare and decreases welfare in the South. The tax payments place a heavy burden on the southern people although South can attract some more Northern firms, and for sufficiently high levels of subsidy, hence local tax, the southern welfare declines.

Result 3: With decentralised taxation, the periphery will choose to independently subsidise firms through locally–financed taxes so as to attract the northern firms. However, since the periphery needs to collect tax locally, there exists an optimal rate of subsidy to maximise welfare in periphery.

As a comparative statistic, we study how the optimal subsidy rates change when trade costs and expenditure shares change. Trade liberalisation increases southern welfare for any *u**. A negative correlation between *u** and ϕ suggests that the South attracts more firms as ϕ decreases (this increases southern welfare), $\frac{du^*}{d\phi} < 0$.[22] Turning to market size, ceteris paribus, an increase in s_E boosts *u**, $\frac{du^*}{ds_E} > 0$. As the northern market is larger, a higher subsidy is necessary to attract firms to the South. Finally, while a higher degree of firm heterogeneity increases welfare in the South at *u**, firm heterogeneity never affects *u**, $\frac{du^*}{d\rho} = 0$ due to the neutrality of $a^{1-\sigma+\rho}$ from firm heterogeneity.[23]

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[22] This result is akin to Dupont and Martin (2006), where the mechanism is such that deeper economic integration causes stronger impact of subsidy on firm relocation due to the home market magnification effect.

[23] Given *u**, $a_R^{1-\sigma+\rho}$ is independent of firm heterogeneity and dependent on s_E and ϕ, as shown in **(7)**. Hence, ρ is neutral in *u*.

One of the largest contrasts with global tax finance, which results in zero-subsidy when using a voting system, is that periphery always has incentive to levy a local tax to employ a subsidy policy. To clarify this, we allow for both regions' subsidy to be locally financed by each region. Then a 'subsidy war' could occur to attract the high productivity firms located in the other region with each local government using a local tax-financed subsidy. Once the South subsidises firms to attract some high productivity firms to relocate to the South, the North attempts to subsidise them to keep full agglomeration as retaliation. Since the North has a wider (per-capita) tax base due to the higher share of population (so it more tax payers), it always wins this subsidy war. In detail, regardless of the same wage and capital returns per capita (average returns), tax per capita in the South always needs to be higher rate than in the North, given the same amount of subsidy, and thus in the end of the subsidy war southern individual's (gross) income would be equal to tax payment, and the South would not be able to increase subsidy rates any more. Hence, the North wins the war. The Nash equilibrium is full agglomeration in the core with no subsidies in either region (u=0) or the subsidy rates such that the total amount of subsidy is equal to southern total income.

3.3. *Social welfare and average productivity*

Then we discuss social welfare under the two tax financing schemes. Using **(9)** the northern/ southern welfares are respectively defined as $V^N = s_L v$ and $V^S = (1-s_L)v^*$. Note that $s_E = s_K = s_L$. Our interest is world welfare, V^W, with respect to subsidy, which is denoted as $V^W = s_L v + (1-s_L)v^*$.[24] Since total income in the world is invariant to the subsidy policy, both types of tax-financed subsidy affect only Δ and Δ* in the world welfare function of V^W without influencing the income part due to invariant total world income (perfectly refunded subsidies). The firm relocation to the South increases Δ* and decreases Δ (as dΔ =-dΔ*).[25] Since the share of

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[24] In this section, we do not consider retaliation nor subsidy wars.

[25] $\frac{d\Delta}{d\Delta *} = \frac{\lambda\alpha(\phi-1)a_R{}^{\alpha-1}}{\lambda\alpha(1-\phi)a_R{}^{\alpha-1}} = -1$ where α=1+ρ-σ

population is smaller in the South ($1\text{-}s_L<0.5$), the increase in welfare caused by the rise in Δ* is dominated by the decreases in Δ. Therefore, the subsidy decreases total world welfare. This social welfare-reducing subsidy boils down to a result in Baldwin et al. (2003): The laissez faire outcome in the FC model is Pareto optimal, when $s_E = s_K = s_L$. This implies that a regional subsidy to change firm location is harmful to the world, so a zero subsidy is socially optimal. In spite of it, zero subsidy would not be equilibrium in a decentralised state, because the periphery would always attempt to introduce a local tax-financed subsidy. Therefore, in order to sustain the socially-optimal zero subsidy, a state should be centralised: the decision making on subsidy policy should be done by a central government, with a voting mechanism, which should prohibit the local tax financing scheme.

However, we note that zero-subsidy is not always the best policy. For instance, our heterogeneous firm geography model gives one more aspect, i.e. average productivity. If the objective of the government is to boost average productivity in periphery, an optimal policy has a different nuance. As in Melitz (2003), average productivity is defined as a frequency weighted mean of individual productivities, written as $\varphi = \Delta^{\frac{1}{\sigma-1}}$ and $\varphi^* = \Delta^{*\frac{1}{\sigma-1}}$. As above mentioned, since the firm relocation to the South due to subsidy increases in Δ* and decreases in Δ, average productivity decreases in the North and increases in the South. If the regional policy is aimed at promoting average productivity in the South, a positive rate of subsidy is effective. So there are contrasting policy implications for using a productivity objective rather than a social welfare objective.

4. DISCUSSION

Finally, we discuss our results and derive implications. In terms of social welfare, a policy of non-subsidy with centralisation is socially optimal. Importantly, however we note that maximising social welfare is not always a sole objective of subsidy policy in the real world. As many previous empirical studies discuss, regional subsidies are aimed at promoting employment in periphery, boosting average productivity and promoting regional economic growth. If the objective is to attract firms to periphery for these aims, the subsidy policy is still worthwhile to exercise. When a regional subsidy is introduced, it should be consider what type relocation subsidy program should be undertaken (e.g. a proportional or a specific (per-firm) subsidy) together with the type of tax-financing scheme used and method of decision making (e.g. local or centralised government).

One aspect is average productivity. It is important to attract productive firms to periphery for the purpose of boosting average productivity in periphery. In order to do so, a proportional subsidy should be adopted. A per-firm subsidy, as in Baldwin and Okubo (2006), attracts only unproductive firms. Thus this indicates that optimal subsidy policies should involve schemes that more productive firms receive more benefits. For example, deduction of profit/business taxes alongside the construction of transport infrastructures could be better at raising productivity than a per-firm subsidy in which each relocated firm receives a equal amount of subsidy directly.

The other aspect is the centralisation or decentralisation of decision making. Regional interests conflict. As we have seen in the last section, the worst outcome obtains if a subsidy war occurs. Thus it is essential that central government coordinates the different regional interests, even if the objective of the regional policy is to boost the average productivity in that region and subsidy policy is taken in a decentralised state. The subsidy policy program and taxation for that subsidy should be coordinated by central government. In particular, taxation to fund the subsidy should be global in order to prevent tax competition and the distortion of disposal income between the core and the periphery.

5. CONCLUDING REMARKS

This paper studies the impact of relocation subsidies in a heterogeneous firms model with a core-periphery structure. Our main findings are 1) a proportional subsidy induces the most efficient firms to move to the periphery, and 2) non-subsidy is the Nash equilibrium in the repeated game but the periphery in a decentralised state has an incentive to implement local tax-financed subsidies in the stage game. 3) While centralised economy chooses zero-subsidy, decentralised economy leads periphery to choose optimal level of subsidy, resulting in enhancing welfare in periphery.

This paper aims to provide the simplest model of an anti-agglomeration subsidy together with firm heterogeneity. Of course, this paper cannot cover many important issues and aspects related to the public policy. Possible extensions of this paper are the many other forms of taxation and the different types of subsidy policies, including transfer mechanisms and heterogeneous households. In addition, our model is static without free entry and exit of firms. This feature might rule out interesting dynamic dimensions of subsidisation that may be easily analysed within the same framework. For instance, firm relocation implies a lower (greater) mass of active firms in the core (periphery). This will generate a dynamic impact on productivity growth that may more than compensate the sorting effect under subsidies.

REFERENCES

Baldwin, R. and R. Forslid, P. Martin, G. Ottaviano and F. Robert-Nicoud (2003), *Economic Geography and Public Policy*, Princeton University Press, Princeton.

Baldwin, R. and T. Okubo (2006), Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting, *Journal of Economic Geography*, 6, pp.323-346.

Dupont, V. and P. Martin (2006) 'Subsidies to Poor Regions and Inequalities: Some Unpleasant Arithmetic'. Journal of Economic Geography, vol. 6(2), pages 223-240.

Martin, P, T.Mayer, F. Mayneris (2010) "Public support to clusters A firm level study of French "Local Productive Systems"", Regional Science and Urban Economics, forthcoming.

Martin,P and C.A Rogers (1995) `Industrial location and public infrastructure' Journal of International Economics 39: pp335-351.

Melitz, M. (2003). "The impact of trade on intraindustry reallocations and aggregate industry productivity," Econometrica, 71, pp 1695-1725.

Midelfart-Knarvik, K.H., H. Overman, S Redding and A Venables (2000) "The Location of European Industry", European Commission, D-G for Economic and Financial Affairs, Brussels. Economic Papers no 142.

Midelfart-Knarvik, K.H. and H.Overman (2002) "Delocation and European integration: is structural spending justified?" Economic Policy vol. 17 Issue 35, pp. 321-359.

Mohl, P and T. Hagen (2010) "Do EU structural funds promote regional growth? New evidence from various panel data approaches", Regional Science and Urban Economics, 40: pp353-365.

Okubo, T (2009) "Trade Liberalisation and Agglomeration with Firm Heterogeneity -Forward and Backward Linkages" Regional Science and Urban Economics, 39 (5): pp.530-541.

Okubo, T and E. Tomiura (2010). "Industrial Relocation Policy and Heterogeneous Plants Sorted by Productivity: Evidence from Japan," RIETI Discussion papers 10016.

Ostbye, S (2010) "Regional policy analysis in a simple general equilibrium model with vertical linkages", Journal of Regional Science, 50 (3), pp. 756-775.

APPENDIX 1. HUMP-SHAPED WELFARE FOR LOCAL TAX-FINCNCE

We prove the hump-shaped curve with respect to u for a southern local tax-financed subsidy. Specifically, since we cannot solve the welfare function in terms of u in an explicit form, we show $\left.\frac{dv^*}{du}\right|_{u=u^{\min}} > 0$ and $\left.\frac{dv^*}{du}\right|_{u=u^{\max}} < 0$, which implies at least one hump-shape in the function. The per-capita welfare is written as $v^* = \ln(Y^* - T^*) + \frac{\mu}{\sigma - 1}\ln\Delta^*$, where $Y^* = w^* + \bar{\pi} = 1 + \bar{\pi}$,

$T = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s}{\Delta} + \frac{1-s}{\Delta^*}\right)a_R^{1-\sigma+\rho}\frac{u}{1-s}$ and

$\bar{\pi} = \frac{\mu\lambda}{\sigma}\left(\frac{s}{\Delta} + \phi\frac{1-s}{\Delta^*}\right)(1 - a_R^{1-\sigma+\rho}) + \frac{\mu\lambda}{\sigma}\left(\frac{s}{\Delta} + \phi\frac{1-s}{\Delta^*}\right)a_R^{1-\sigma+\rho}(1+u) = \frac{\mu\lambda}{\sigma}\left(\frac{s}{\Delta} + \phi\frac{1-s}{\Delta^*}\right)$. Thus we can be rewritten as $Y^* - T = 1 + \bar{\pi} - T = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s}{\Delta} + \frac{1-s}{\Delta^*}\right)\left(1 - a_R^{1-\sigma+\rho}\frac{u}{1-s}\right) + 1$.

Using this, we differentiate v^* in terms of u:

$$\frac{dv^*}{du} = \left(\frac{d\bar{\pi}}{d\Delta}\frac{d\Delta}{da_R}\frac{da_R}{du} + \frac{d\bar{\pi}}{d\Delta^*}\frac{d\Delta^*}{da_R}\frac{da_R}{du}\right)\left(1 - a_R^{\alpha}\frac{u}{1-s}\right) + \bar{\pi}\left(-\alpha a_R^{\alpha-1}\frac{da_R}{du}\frac{u}{1-s} - \frac{a_R^{\alpha}}{1-s}\right) + \frac{\mu}{\sigma-1}\frac{1}{\Delta^*}\frac{d\Delta^*}{da_R}\frac{da_R}{du}$$

First, we show $\left.\frac{dv^*}{du}\right|_{u=u^{\min}} > 0$. At $u = u^{\min}$ ($a_R = 0$), we can rewrite as

$\frac{d\pi}{d\Delta} = -\frac{\mu}{\sigma}\frac{s}{\Delta^2} = -\frac{\mu}{\sigma}\frac{s}{\lambda^2}$, $\frac{d\pi}{d\Delta^*} = -\phi\frac{\mu}{\sigma}\frac{1-s}{\Delta^{*2}} = -\frac{\mu}{\sigma}\frac{1-s}{\phi\lambda^2}$, $\frac{d\Delta^*}{da_R} = \lambda\alpha(1-\phi)a_R^{\alpha-1}$,

$\frac{d\Delta}{da_R} = \lambda\alpha(\phi - 1)a_R^{\alpha-1} = -\frac{d\Delta^*}{da_R}$, $\frac{da_R}{du} > 0$ and $\bar{\pi} = \frac{\mu}{\sigma}$. Thus, we can derive

$$\left.\frac{dv^*}{du}\right|_{u=u^{\min}} = \left(-\frac{d\bar{\pi}}{d\Delta}\frac{d\Delta^*}{da_R}\frac{da_R}{du} + \frac{d\bar{\pi}}{d\Delta^*}\frac{d\Delta^*}{da_R}\frac{da_R}{du}\right) + \bar{\pi}\left(-\alpha a_R^{\alpha-1}\frac{da_R}{du}\frac{u^{\min}}{1-s}\right) + \frac{\mu}{\sigma-1}\frac{1}{\Delta^*}\frac{d\Delta^*}{da_R}\frac{da_R}{du}$$

$$= \left(s - \frac{1-s}{\phi}\right)\frac{\mu}{\sigma\lambda^2}\frac{d\Delta^*}{da_R}\frac{da_R}{du} + \frac{\mu}{\sigma\lambda}\left(-\alpha a_R^{\alpha-1}\frac{da_R}{du}\frac{u^{\min}}{1-s}\right) + \frac{\mu}{\sigma-1}\frac{1}{\lambda\phi}\frac{d\Delta^*}{da_R}\frac{da_R}{du}$$

$$\cong \left\{ \left(s + \frac{s}{\phi} \right)(1-\phi) - \frac{u^{\min}}{1-s} \right\} \frac{\mu \alpha a_R^{\alpha-1}}{\sigma \lambda} \frac{da_R}{du} > (2s(1-\phi)(1-s)^2 - u^{\min}) \frac{\mu \alpha a_R^{\alpha-1}}{\sigma \lambda (1-s)} \frac{da_R}{du}$$

$$= (2s(1-\phi)(1-s)^2 - \frac{1}{s\phi + (1-s)/\phi} + 1) \frac{\mu \alpha a_R^{\alpha-1}}{\sigma \lambda (1-s)} \frac{da_R}{du}$$

$$> (2s(1-\phi)(1-s)^2 - \frac{1}{2\sqrt{s(1-s)}} + 1) \frac{\mu \alpha a_R^{\alpha-1}}{\sigma \lambda (1-s)} \frac{da_R}{du} > 0$$

$\left. \frac{dv^*}{du} \right|_{u=u^{\min}} > 0$ always holds.

Similarly, we derive the sufficient conditions for $\left. \frac{dv^*}{du} \right|_{u=u^{\max}} < 0$ at $u^{\max}$ ($a_R = 1$). We can write as

$\frac{d\bar{\pi}}{d\Delta} = -\frac{\mu}{\sigma} \frac{s}{\Delta^2} = -\frac{\mu}{\sigma} \frac{s}{\phi^2 \lambda^2}$, $\frac{d\bar{\pi}}{d\Delta^*} = -\phi \frac{\mu}{\sigma} \frac{1-s}{\Delta^{*2}} = -\frac{\mu}{\sigma} \frac{1-s}{\lambda^2} \phi$, $\frac{d\Delta^*}{da_R} = \lambda \alpha (1-\phi)$, $\frac{d\Delta}{da_R} = \lambda \alpha (\phi - 1)$, and $\frac{da_R}{du} > 0$.

(11)

$$\left. \frac{dv^*}{du} \right|_{u=u^{\max}} = \left(-\frac{d\bar{\pi}}{d\Delta} \frac{d\Delta^*}{da_R} \frac{da_R}{du} + \frac{d\bar{\pi}}{d\Delta^*} \frac{d\Delta^*}{da_R} \frac{da_R}{du} \right) \left(1 - \frac{u^{\max}}{1-s} \right) + \bar{\pi} \left(-\alpha \frac{da_R}{du} \frac{u^{\max}}{1-s} - \frac{1}{1-s} \right) + \frac{\mu}{\sigma - 1} \frac{1}{\Delta^*} \frac{d\Delta^*}{da_R} \frac{da_R}{du}$$

The second term in (11) is always negative. Due to $\frac{d\bar{\pi}}{d\Delta} > \frac{d\bar{\pi}}{d\Delta^*}$, if $1 - \frac{u^{\max}}{1-s} < 0$ is satisfied, then the first term is always negative. The negative two first terms indicate that marginal income effect by subsidy is negative due to tax payment. When the negative marginal income effect (the first and the second terms in (11)) is larger than marginal price index effect (the third term in (11)), the impact of subsidy on v^* is negative. The sufficient condition for this can be derived as $\frac{s}{\phi} - 1 - (1-s)(1-\phi) > 0$.

As s is larger, $1 - \frac{u^{\max}}{1-s} < 0$ becomes more negative and $\left. \frac{dv^*}{du} \right|_{u=u^{\max}} < 0$ is more likely to occur. This indicates that smaller population in the South would have a greater burden of tax per-capita, which results in a greater negative marginal income effect.