Discussion Paper Series

RIEB

Kobe University

DP2011-04

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January 20, 2011

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Note on the Interpretation of Convergence Speed in the Dynamic Panel Model

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January 20, 2011

Abstract

Studies using the dynamic panel regression approach have found the speed of income convergence among the world and regional economies to be high. For example, Lee et al. (1997, 1998) report the income convergence speed to be 30% per annum. This note argues that their estimates may be seriously overstated. Using a factor model, we show that the coefficient of the lagged income in their specification may not be the long-run convergence speed, but the adjustment speed of the short-run deviation from the long-run equilibrium path. We give an example of an empirical analysis, where the short-run adjustment speed is about 40%.

JEL Classification: O40;

Keywords: convergence speed, dynamic panel regression, factor model

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1 Introduction

This note focuses on the interpretation of the speed of convergence with which the logarithm of per-capital real income tends to its steady-state value in the dynamic panel regression model. Researchers using the dynamic panel regression approach, e.g., Islam (1995, 1998), Lee et al. (1997, 1998), and Shioji (2001), argue that economies converge at a relatively high β-convergence speed as compared to the speed obtained in the analyses of conventional cross-sectional regression.

In this note, we argue that their estimates of the speed of β-convergence may be seriously overstated. Our point is that their calculations of the speed of β-convergence do not take into account the dynamics of a common component of the per capita real income, which may be highly persistent. Consequently, their estimates are not of the convergence speed in the long run, but the adjustment speed of the short-run deviation from the long-run equilibrium path.

2 Controversies on the convergence speed obtained using dynamic panel regression

We denote the logarithm of the per capita real income of the economy i at time t as y_{it}. Let us start from the most basic model of dynamic panel regression:

$$y_{it} = \mu + b y_{it-1} + \gamma^{'} \Theta_t + \zeta_{it}, \text{ for all } i = 1, \cdots, N, \tag{1}$$

where ζ_{it} is an independent disturbance. μ is an economy-wide intercept that reflects the initial resource endowments. Θ_t is a vector of some time-specific effects (e.g., a linear time trend, an economy-wide average of real income, and a shock common to all economies) that reflects the technological progress across economies.[1]

This regression is a panel analog to the Barro regression and it is natural to interpret the coefficient of the lagged real income term as a function of the speed of convergence β: namely, $b = \exp(-\beta)$. Many cross-sectional analyses employing the Barro regressions,

[1]For simplicity, we exclude some exogenous variables as the determinants of economic growth such as the savings rate, measures of investment in physical and human capital, and so on to focus on the interpretation of the convergence speed using the panel data approach.

e.g., Barro (1991) and Barro and Sala-i-Martin (1992a,b), report that the estimate of β is around 2% per annum.

Islam (1995) points out that allowing for intercept heterogeneity, i.e., μ_i instead of μ in equation (1), increases the estimate of the speed of convergence to 5% per annum. Indeed, if there is a positive correlation between these intercepts and the lagged real incomes, the conventional cross-section estimate of b will be biased upward.

While Islam (1995) estimates the model where only the steady-state level of income is allowed to differ across economies and where the assumption that economies shared the same steady-state growth rate, given as $\gamma_i = \gamma$ for all $i = 1, \cdots, N$ is made. Lee et al. (1998) criticize Islam (1995) for failing to allow for different common trends across the countries.[2] Their model is given as

$$y_{it} = \mu_i + b_i y_{it-1} + \gamma_i^{'} \Theta_t + \zeta_{it}, \text{ for all } i = 1, \cdots, N, \tag{2}$$

which allows for the existence of non-parallel balanced growth paths. Lee et al. (1997, 1998) argue that homogeneity in the growth effects of convergence forces the estimate of b toward 1, because it renders the panel estimator inconsistent.[3] Based on this model, they report that the speed of β-convergence increases to around 30% per annum.

We support the generalization by Lee et al. (1997, 1998), since such a model is flexible to describe various economies. However, we cast doubt on the interpretation that the coefficient of the lagged dependent variable, b_i, represents the convergence speed in the long run, which is discussed in the next section.

[2] In response, Islam (1998) argues that Lee et al. (1997, 1998) are assessing an economically uninteresting form of convergence when they allow for trend differences. Durlauf et al. (2005) comment that "this debate is an excellent example of the issues of interpretation that are raised in moving between specific economic hypotheses and more general statistical models."

[3] We can understand that Lee et al. (1997, 1998)'s specification is appropriate by looking at the recent literature on the panel unit root test, e.g., Phillips and Sul (2003). Phillips and Sul (2003) argue that if the cress-sectional correlation for the error term is not accounted for, the estimates of the autoregressive coefficients will be biased. In order to control the cross-sectional correlation, the error term is given by a three-component model that contains a fixed effect (μ_i), a common factor (Θ_t), and a purely idiosyncratic factor (ζ_t). Then, it is standard practice to use a similar specification for equation (2) in the panel unit root test.

3 Interpretation of the key coefficient in dynamic panel regression in terms of a factor model

The model described as equation (2) is similar to the common factor model by previous studies, e.g., Bai and Ng (2004), in that it allows for heterogeneous (non-parallel) growth paths. Based on the factor model, we will show that the interpretation of b_i as the long-run convergence speed is inappropriate.

Bai and Ng (2004) develop the "Panel Analysis of Nonstationarity in Idiosyncratic and Common Components (PANIC)." The idea of PANIC is to decompose the observed panel data into the common and idiosyncratic components, and then test for stationarity in each component separately. Bai and Ng (2004) consider the following factor model:

$$y_{it} = v_i + \lambda_i' F_t + e_{it}, \tag{3}$$

where F_t is the common component consisting of r factors followed by the economies, λ_i is a parameter vector that represents the different weights assigned to the common factors, and e_{it} for $i = 1, \cdots, N$ is the idiosyncratic component with a zero mean and is orthogonal to the common factors F_t and to itself.

Let us assume that each component, e_{it}, F_t, follows the autoregressive regression model:

$$e_{it} = b_i e_{it-1} + \zeta_{it}, \text{ for all } i = 1, \cdots, N, \tag{4}$$

$$F_t = \phi F_{t-1} + v_t, \tag{5}$$

where ζ_{it} for all $i = 1, \cdots, N$ and v_t are the error terms with a zero mean. An ζ_{it} shock propagates through the autocorrelation structure of equation (4).

From equations (3) and (4), we can derive

$$\begin{aligned} y_{it} &= v_i + \lambda_i' F_t + b_i e_{it-1} + \zeta_{it} \\ &= v_i(1 - b_i) + b_i y_{it-1} + \lambda_i' F_t - b_i \lambda_i' F_{t-1} + \zeta_{it}, \text{ for all } i = 1, \cdots, N, \end{aligned} \tag{6}$$

which is observationally equivalent to the equation (2) with $\mu_i = v_i(1-b_i)$, $\gamma_i = [\lambda_i' \; -b_i\lambda_i']'$, and $\Theta_t = [F_t' \;\; F_{t-1}']'$. As the result shows, the coefficient of the lagged real income b in equation (6), which is the coefficient of the lagged income in equation (2), is nothing but the adjustment speed of the idiosyncratic component e_{it}, as equation (4) shows.

If the idiosyncratic component is stationary, b_i represents the adjustment speed of short-run deviation. Even if the idiosyncratic component is non-stationary, b_i is biased as the long-run convergence speed, as long as the common factor is non-stationary. In other words, the interpretation of b_i as the long-run convergence speed is valid only if the common factors F_t do not exist or they are stationary processes, which seems improbable in most cases.[4]

4 Example

Let us give an example of the adjustment speed of the short-run deviation. The example is an analysis using the per capita real income data for 46 Japanese prefectures, for the period 1955–1999. The estimation of the Bai and Ng (2004) model reveals that there exists one common component F_t, which is nonstationary, and idiosyncratic components e_{it} for $i = 1, \cdots, 46$, which are all stationary (see Shibamoto et al., 2011). Thus, in this example, the estimate of b_i in a type of equation (2) is nothing but the adjustment speed of the short-run deviation from the long-run path. In this case, the per capita incomes of the economies cointegrate with the common component with different long-run weights, which is characterized by $v_i + \lambda_i F_t$.[5] This implies that the economies have heterogeneous balanced growth paths. On the other hand, the idiosyncratic component, e_{it}, which has only a transitory effect on the per capita real income, reflects the short-run deviation from the long-run path for the economy i.

Then, we estimate equation (4) to get the estimate of b_i by ordinary least squares.[6] The mean value of the estimated b_i across prefectures was 0.67, implying that the speed is about 40% ($= -\log(0.67)$) per annum. This estimate is very large if we regard this as the long-run convergence speed, but it is a reasonable value for the adjustment speed of the short-run deviation, because short-run variations in economies, i.e., business cycles,

[4] Although we explain the case wherein e_{it} and F_t follow the first-order autoregressive model in equations (4) and (5) for simplicity, the argument applies for any order of the autoregressive model.

[5] Bernard and Durlauf (1995)'s Definition 2.2 of "common trends in output" embodies this idea.

[6] We add as explanatory variables four lags of the first difference of $\hat{e}_{it}$ in equation (4) to remove the autocorrelation of the residuals.

normally return to their former levels in several years. This figure is comparable to 30% of the "speed of β-convergence" reported by Lee et al. (1997, 1998), which suggests that the figure reported by them is not really the long-run convergence speed, but the adjustment speed in the short-run.[7]

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[7] They admit that the interpretation of this value as the β-convergence speed is highly questionable.

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