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Taiji FURUSAWA
Takashi KAMIHIGASHI

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Research Institute for Economics and Business Administration
Kobe University
2-1 Rokkodai, Nada, Kobe 657-8501 JAPAN
Threats or Promises?: A Simple Explanation of Gradual Trade Liberalization∗

Taiji Furusawa† and Takashi Kamihigashi‡

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Abstract

We analyze a infinitely repeated tariff-setting game by two large countries with alternating moves. We focus on the subgame perfect equilibria in which each country chooses its tariff according to a stationary function of the other country’s tariff. We show that there are many equilibria with two steady states, one with higher tariffs (but still lower than the static Nash tariffs), the other with lower tariffs. We also show that there is a special class of equilibria in which there exists a unique, globally stable steady state. In both types of equilibria, one country unilaterally reduces its tariff from the static Nash equilibrium, the other country reciprocates in response to the first country’s implicit “promise” to lower its tariff even further, and this process continues forever, converging to a steady state with tariffs lower than the static Nash tariffs. Therefore it is promises, rather than threats, that induce the countries to gradually reduce their tariffs.

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†Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601 JAPAN. Email: furusawa@econ.hit-u.ac.jp. Tel/Fax: +81-42-580-8866.

‡RIEB, Kobe University, Rokkodai, Nada, Kobe 657-8501 JAPAN. Email: tkamihig@rieb.kobe-u.ac.jp. Tel/Fax: +81-78-803-7015.
1 Introduction

Why does a country sometimes liberalize trade unilaterally? Trade theory suggests that trade liberalization benefits a country as long as it is small. But why does even a large country sometimes liberalize trade unilaterally? A notable example is Britain’s unilateral trade liberalization in the 1840s, including the repeal of the Corn Laws in 1846 (Conybeare, 2002). Did Britain act unilaterally because it believed that unilateral trade liberalization itself would benefit Britain? Or did Britain hope that other countries would follow suit? Bhagwati (2002) argues that the latter idea occurred to British Prime Minister Sir Robert Peel, who showed leadership in abolishing the Corn Laws. Indeed, most European countries gradually liberalized trade from the 1840s to the 1880s, following the continual free trade movement by Britain (Bairoch, 1989; Kindleberger, 1975; Conybeare, 2002, p. 47). History witnessed what is now known as gradual trade liberalization.

In the literature on trade liberalization, threats play an important role in sustaining liberalized trade. Using a repeated tariff-setting game, Dixit (1987) shows that liberalized trade can be sustained by the threat of reverting to the static Nash equilibrium forever after any deviation. The threat of (infinite) Nash reversion is also used to support an entire process of trade liberalization by Staiger (1995), Furusawa and Lai (1999), and Bond and Park (2002). They all consider trade agreements between two countries in which the countries gradually decrease their tariffs while satisfying at all times an incentive constraint such that any deviation triggers Nash reversion. These studies show that optimal reciprocal liberalization must be gradual if skills of workers that are displaced from an import-competing industry dissipates (Staiger, 1995), if there exist sectoral adjustment costs (Furusawa and Lai, 1999), or if the incentive constraint is binding only for one of the countries due to size asymmetry (Bond and Park, 2002).\footnote{Krishna and Mitra (1999) and Coates and Ludema (2001) explain unilateral trade liberalization based on lobbying activities, but they do not consider the gradual feature of liberalization processes.}
However, since punitive retaliatory actions are seldom observed in reality, the threat of reverting to the static Nash equilibrium after a small deviation may not be realistic. Furthermore, such threats are effective only in sustaining an already established (or agreed-upon) process of trade liberalization. Once any deviation occurs, the cooperative process can never be restored.

In this paper, we argue that when a country liberalizes trade unilaterally, what motivates other countries to follow suit is its implicit promise to liberalize trade further if they reciprocate. For this purpose we study a simple tariff-setting game with alternating moves between two large countries. Each country’s one-shot payoff is simply the sum of import and export surplus, and the countries take turns in setting their tariffs: in the first period one country chooses its tariff, in the second period the other country chooses its tariff, in the third period the first country chooses its tariff again, and so on. We focus on the subgame perfect equilibria in which each country, in its turn to move, chooses its tariff according to a stationary function of the other country’s current tariff. A subgame perfect equilibrium in this class is termed an “immediately reactive equilibrium” (IRE) by Kamihigashi and Furusawa (2010), and this class seems particularly suitable for capturing the sequential and reciprocal aspects of trade liberalization.²

We show first that the IREs are versatile enough to encompass familiar equilibria. For example, there is an IRE in which each country keeps choosing its static Nash tariff forever. There is also an IRE in which a steady state with low tariffs is supported by the threat of Nash reversion. We then show that there are many IREs that have two steady states, one with higher tariffs, the other with lower tariffs; the higher steady state is locally stable, and the lower steady state is stable from below but unstable from above. In “effectively efficient” IREs (defined in Section 3), however, there is a unique, globally stable steady state.

²Kamihigashi and Furusawa (2010) show that our model is in fact equivalent to the corresponding model with simultaneous moves as long as the IREs are concerned. In model, however, the dynamics generated by IREs are easier to describe.
In many of these equilibria, including all the effectively efficient IREs, if the initial tariff profile is at the static Nash equilibrium, the countries gradually decrease their tariffs toward a steady state with low tariffs. More specifically, the country that is allowed to move in the first period cuts its tariff first, the second country responds by cutting its tariff, the first country then reacts by further cutting its tariff again, and this process continues and gradually converges to the steady state. Hence these equilibria induce gradual trade liberalization initiated by unilateral tariff reduction. Furthermore, when the first country cuts its tariff in the first period, the second country is not threatened to reciprocate. If it did not reciprocate, the first country would simply keep its tariff unchanged. It is therefore the first country’s implicit promise to lower its tariff even further if the second country reciprocates, that motivates the second country to lower its tariff.

A steady state of an IRE has the property that it is supported by a minimum threat: each country simply makes the other country indifferent between raising its tariff and staying at the steady state. Even after a deviation, each country promises to lower its tariff as long as the other country does so, which makes it possible to restore the steady state in a self-enforcing way. Therefore any stable steady state of an IRE has a built-in mechanism to restore itself after a deviation. This is in sharp contrast to the aforementioned studies on gradual trade liberalization, which use the threat of Nash reversion to support a cooperative process, which could be lost forever in case of any deviation.

We also argue that in order to benefit from unilateral liberalization, a country must leave sufficient room for further liberalization; in other words, a mutually beneficial liberalization process must be sufficiently gradual. This is because in our equilibria, a country induces the other country to reciprocate by promising to offer further liberalization. If a country lowers its tariff too much at the beginning, then it cannot lower it much further and thus cannot give the other country much motivation to reciprocate.

We should mention that Johnson (1953-54) studies a similar framework in which two
large countries alternately select their tariffs. In his model, each country chooses its tariff in a myopic way in response to the tariff chosen by the other country in the previous period. The tariff profile then either converges to the static Nash equilibrium or to a cycle around the Nash equilibrium. By contrast, our model is fully rational and has many equilibria in which the tariff profile converges to a steady state with tariffs lower than at the static Nash equilibrium.

The rest of the paper is organized as follows. In Section 2 we describe our tariff-setting game and formally define IREs. In Section 3 we establish some general properties of the IREs of our model based on Kamihigashi and Furusawa’s (2010) results. In Section 4 we describe various IREs of interest and discuss trade liberalization processes. In Section 5 we offer some concluding remarks.

2 The Model

We consider an alternating-move, tariff-setting game between two large countries, 1 and 2. Country $i$ imposes a tariff $\tau_i \geq 0$ on imports from country $j \neq i$. Country $i$’s import demand is assumed to be a strictly decreasing, continuous function of the price of imports such that it is equal to zero at country $i$’s autarkic equilibrium price, whereas its export supply is a strictly increasing, continuous function of the price of exports. Country $i$’s import surplus $m_i(\tau_i)$ is the area below the import demand curve and above the world price level. Country $i$’s export surplus $x_i(\tau_j)$ is the area below the world price level and above the export supply curve; $x_i(\tau_j)$ is a strictly decreasing continuous function of $\tau_j$. The one-shot payoff of country $i$ is its gains from trade, i.e., the sum of its import surplus $m_i(\tau_i)$ and export surplus $x_i(\tau_j)$:

$$u_i(\tau_i, \tau_j) = m_i(\tau_i) + x_i(\tau_j).$$

Optimal tariff theory suggests that $m_i(\tau_i)$ is increasing where $\tau_i$ is small, and decreasing where $\tau_i$ is large. We assume for simplicity that $m_i(\tau_i)$ has a single peak at $\tau_i^N > 0$ and is
strictly increasing for $\tau_i < \tau_i^N$. In Appendix A we derive the surplus functions $m_i(\tau_i)$ and $x_i(\tau_j)$ explicitly in a parametric example based on linear demand and supply functions.

Since $\tau_i^N$ is country $i$’s strictly dominant strategy, $(\tau_1^N, \tau_2^N)$ is a unique static Nash equilibrium. We henceforth restrict the feasible set of country $i$’s tariffs to $[0, \tau_i^N]$, as we are mainly interested in tariff reduction processes. A more general case allowing for $\tau_i > \tau_i^N$ can be analyzed using Kamihigashi and Furusawa’s (2010) results.

Let $T_1 = \{1, 3, 5, \cdots \}$ and $T_2 = \{2, 4, 6, \cdots \}$ denote the sets of periods in which country 1 and country 2 select their individual tariffs, respectively. We focus the subgame perfect equilibria in which country $i$, in its turn to move (i.e., $t \in T_i$), selects its tariff $\tau_{i,t}$ according to a stationary reaction function $f_i(\tau_{j,t})$ of country $j$’s current tariff $\tau_{j,t}$, which was selected in the previous period. Such equilibria are termed immediately reactive equilibria (IREs) by Kamihigashi and Furusawa (2010). Since country $i$ cannot change its tariff in period $t + 1 \in T_j$, we have that $\tau_{i,t+1} = \tau_{i,t}$ for all $t \in T_i$. Let $\delta_i$ denote country $i$’s discount factor. Then, given country $j$’s reaction function $f_j$, country $i$ maximizes the sum of one-shot payoffs from period $i$ (= 1 or 2) onwards:

$$\max_{\{\tau_{i,t}\} \in T_i} \sum_{t=i}^{\infty} \delta_i^{t-i}[m_i(\tau_{i,t}) + x_i(\tau_{j,t})]$$

s.t.

$$\tau_{i,t+1} = \tau_{i,t} \quad \text{for } t \in T_i,$$

$$\tau_{j,t+1} = \tau_{j,t} = f_j(\tau_{i,t}) \quad \text{for } t \in T_j,$$

$$\tau_{j,i} \text{ given.}$$

We say that country $i$’s reaction function $f_i$ is a best response to country $j$’s reaction function $f_j$ if for any $\tau_{j,i} \in [0, \tau_j^N]$, the above maximization problem has a solution $\{\tau_{i,t}\}^{\infty}_{t=i}$ such that $\tau_{i,t} = f_i(\tau_{j,t})$ for all $t \in T_i$. We call a pair of reaction functions $(f_1, f_2)$ an

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3This partial equilibrium setup can be interpreted as a general equilibrium model by assuming that each country $i$ consumes three goods, country $i$’s export good, country $j$’s export good, and a common numeraire good, and that the representative consumer’s utility function is additively separable in the three goods and linear in the numeraire good. The social welfare of each country is then measured by gains from trade and can thus be represented by the total surplus derived from the markets of the non-numeraire goods. See Furusawa and Lai (1999) for another example.
immediately reactive equilibrium (IRE) if $f_1$ is a best response to $f_2$, and vice versa.\footnote{The concept of IRE may or may not be consistent with that of Markov perfect equilibrium (Maskin and Tirole, 1988a, 2001), depending on how “payoff relevant information” is defined.}

Given an IRE $(f_1, f_2)$, we say that $(\tau_1, \tau_2) \in [0, \tau_1^N] \times [0, \tau_2^N]$ is a steady state if $\tau_1 = f_1(\tau_2)$ and $\tau_2 = f_2(\tau_1)$. Needless to say, if the game starts from a steady state $(\tau_1, \tau_2)$, each country $i$ keeps choosing $\tau_i$ forever according to $f_i$.

3 General Properties of IREs

In this section we present some useful properties of IREs. Since the tariff-setting game in this paper is a special case of the general model studied in Kamihigashi and Furusawa (2010), the results of the latter apply here. However, many of them are considerably simplified (and easier to understand) due to the extra assumption that $m_i(\tau_j)$ is strictly increasing. This assumption also enables us to establish some additional results.

In this section we assume only that $m_i(\tau_i)$ is strictly increasing and continuous, and that $x_i(\tau_j)$ is strictly decreasing and continuous. Additional assumptions will be introduced in the next section.

Let us define the function $w_i : [0, \tau_i^N] \times [0, \tau_j^N] \rightarrow \mathbb{R}$ by

$$w_i(\tau_i, \tau_j) = m_i(\tau_i) + \delta_i x_i(\tau_j).$$ (3.1)

We call this function country $i$’s effective payoff since country $i$ in effect seeks to maximize the discounted sum of effective payoffs. Indeed, country $i$’s discounted sum of payoffs from period $i$ onward is written as

$$\sum_{t=i}^{\infty} \delta_i^{t-i} [x_i(\tau_{j,t}) + m_i(\tau_{i,t})]$$ (3.2)

$$= x_i(\tau_{j,i}) + \sum_{t=1}^{\infty} \delta_i^{t-1} [m_i(\tau_{i,t}) + \delta_i x_i(\tau_{j,t+1})]$$ (3.3)

$$= x_i(\tau_{j,i}) + \sum_{t=1}^{\infty} \delta_i^{t-1} w_i(\tau_{i,t}, \tau_{j,t+1}).$$ (3.4)
Since country \( i \) has no influence on \( \tau_{j,i} \), its problem is equivalent to maximizing the discounted sum of effective payoffs. This implies that each country’s best responses are characterized by a static maximization problem. To state this result, given country \( j \)’s reaction function \( f_j \), let \( w^*_i(f_j) \) denote country \( i \)’s maximum feasible effective payoff (provided that it exists):

\[
w^*_i(f_j) \equiv \max_{\tau_i \in [0, \tau^N_i]} w_i(\tau_i, f_j(\tau_i)).
\]

**Lemma 3.1.** Country \( i \)’s reaction function \( f_i \) is a best response to country \( j \)’s reaction function \( f_j \) if and only if

\[
w_i(f_i(\tau_j), f_j(f_i(\tau_j))) = w^*_i(f_j) \quad \text{for any } \tau_j \in [0, \tau^N_j].
\]

In other words, \((f_1, f_2)\) is an IRE if and only if (3.6) holds for \( i = 1, 2 \).

See Kamihigashi and Furusawa (2010, Lemma 2.1) for a formal proof. Lemma 3.1 suggests that the level curves of effective payoffs are closely related with best responses. Since \( x_i(\tau_j) \) is strictly decreasing, the \( \omega_i \)-level curve of \( w_i \) can be expressed as the graph of a function, which we denote by \( g^\omega_i(\tau_i) \):

\[
\omega_i = w_i(\tau_i, g^\omega_i(\tau_i)) = m_i(\tau_i) + \delta_i x_i(g^\omega_i(\tau_i)).
\]

(3.7)

Without loss of generality we assume that \( x_i(\tau_j) \) can be extended to a strictly decreasing function, denoted \( x_i(\tau_j) \) again, on \( \mathbb{R} \). Given this innocuous extension, \( x_i(\tau_j) \) is well defined and strictly decreasing for all \( \tau_j \in \mathbb{R} \). Solving (3.7) for \( g^\omega_i(\tau_i) \) we obtain

\[
g^\omega_i(\tau_i) = x_i^{-1}(\omega_i - m_i(\tau_i)).
\]

(3.8)

Since \( x_i(\tau_j) \) is strictly decreasing, a lower level curve is associated with a higher value of \( w_i \); see Figure 1. In terms of level curves, Lemma 3.1 can be restated as follows:

**Lemma 3.2.** A pair of reaction functions \((f_1, f_2)\) is an IRE if and only if for \( i = 1, 2 \),

\[
f_j(f_i(\tau_j)) = g^w_j(f_j)(f_i(\tau_j)) \quad \text{for any } \tau_j \in [0, \tau^N_j].
\]

(3.9)
To illustrate Lemma 3.2, let \((f_1, f_2)\) be given by \(f_i(\tau_j) = \tau_i^N\) for all \(\tau_j\) and \(i\); i.e., each country \(i\) chooses its static Nash tariff \(\tau_i^N\). Since \(w_i^*(f_j)\) is the highest feasible effective payoff given \(f_j\), the corresponding level curve \(g_j^{w_i}(f_j)\) is the lowest one that intersects with the graph of \(f_j\); see Figure 2(a). Since the graphs of \(g_j^{w_i}(f_j)\) and \(f_j\) coincide at \((\tau_i^N, \tau_j^N)\), we have \(w_i^*(f_j) = w_i(\tau_i^N, \tau_j^N)\). Note that for any \(\tau_2\), we have \(f_1(\tau_2) = \tau_1^N\) and \(f_2(\tau_1^N) = g_2^{w_1(f_2)}(\tau_1^N) = \tau_2^N\). Thus (3.9) holds, and \((f_1, f_2)\) is an IRE.

As another example, let \((f_1, f_2)\) be such that \(f_i(\tau_j) = \tau_i^N\) if \(\tau_j = \tau_j^N\), and \(f_i(\tau_j) = \tau_i^N\) otherwise, where \(\tau_1\) and \(\tau_2\) are as in Figure 2(b). In this case, as long as country \(j\) chooses \(\tau_j\), country \(i\) chooses \(\tau_i\). However, if either country deviates at all, the other country immediately reverts to the static Nash tariff \(\tau_i^N\). One can easily check that \((f_1, f_2)\) satisfies (3.9).

The above two examples show that the IREs include familiar equilibria such as the static Nash equilibrium and the Nash reversion equilibrium. There are of course other IREs. For example, condition (3.9) is trivial to verify if \(f_j = g_j^{w_j^i(f_j)}\) for both \(j\). Such an IRE is possible if there are \(\omega_1, \omega_2 \in \mathbb{R}\) such that \(g_j^{\omega_i}(\tau_i) \in [0, \tau_i^N]\) for all \(\tau_i \in [0, \tau_i^N]\) and both \(j\). In this case, if we define \(f_j = g_j^{\omega_i}\) for both \(j\), then \(g_j^{\omega_i}\) exactly coincides with \(f_j\) and is thus the lowest level curve that intersects with \(f_j\). Hence we have \(\omega_i = w_i^*(f_j)\), so that (3.9) immediately follows.
(see Kamihigashi and Furusawa, 2010, Proposition 3.2, for a formal proof). See Figure 2(c) for an example constructed this way.

As one can see from Figure 2(b), an IRE can be discontinuous. Figure 2(d) illustrates another discontinuous IRE. One can easily check that this example also satisfies (3.9).

To study the dynamics generated by IREs, we define an IRE path associated with an IRE \((f_1, f_2)\) as a sequence \(\{(\tau_{1,t}, \tau_{2,t})\}_{t=1}^{\infty}\) satisfying (2.3) and (2.4) for both \(i\):

\[
\tau_{1,2} = \tau_{1,1} = f_1(\tau_{2,1}), \quad \tau_{2,3} = \tau_{2,2} = f_2(\tau_{1,2}), \quad \tau_{1,4} = \tau_{1,3} = f_1(\tau_{2,3}), \quad \cdots .
\]  

The following result shows an important property of IRE paths.

**Lemma 3.3.** Given an IRE \((f_1, f_2)\), let \(\{(\tau_{1,t}, \tau_{2,t})\}_{t=1}^{\infty}\) be any IRE path. For any \(t \geq 2\), if \(t \in T_i\), then \((\tau_{i,t}, \tau_{j,t})\) is on country \(j\)’s optimal level curve:

\[
\tau_{i,t} = g^{w^*_i(f_i)}_i(\tau_{j,t}).
\]  

See Kamihigashi and Furusawa (2010, Theorem 4.1) for the proof. This result shows that any IRE path is characterized by the corresponding level curves \(g^{w^*_1(f_1)}_1, g^{w^*_2(f_2)}_2\) except for the initial period. The initial period must be excluded because \(\tau_{2,1}\) is an arbitrary initial condition that need not be optimal for country 2 given country 1’s reaction function \(f_1\). For example, in the case of Figure 2(a), any \(\tau_{2,1} \neq \tau_2^N\) is not optimal for country 2; thus \((\tau_{1,1}, \tau_{2,1})\) is not on country 2’s optimal level curve for \(t = 1\) unless \(\tau_{2,1} = \tau_2^N\). The situation in Figure 2(b) is similar: \((\tau_{1,1}, \tau_{2,1})\) is not on country 2’s optimal level curve for \(t = 1\) unless \(\tau_{2,1} = \tau_2^N\) or \(\tau_2\). In Figure 2(c), by contrast, any IRE path stays on the optimal level curves for all \(t \geq 1\), which is also consistent with Lemma 3.3.

Lemma 3.3 implies that any steady state must be on both level curves. We state this result as a corollary:

**Corollary 3.1.** Any steady state of an IRE \((f_1, f_2)\) is an intersection between the graphs of the associated level curves \(g^{w^*_2(f_2)}_2\) and \(g^{w^*_1(f_1)}_1\).
Figure 2: Examples of IREs
We say that a pair of level curves \((g_1^ω_2, g_2^ω_1)\) is supported by an IRE if there exists an IRE \((f_1, f_2)\) such that \(ω_i = ω_i^*(f_j)\) for both \(i\). If \((f_1, f_2)\) is an IRE, then \((g_1^ω_1(f_2), g_2^ω_2(f_1))\) is supported by \((f_1, f_2)\). Given a pair of level curves \((g_1^ω_2, g_2^ω_1)\), let \((τ_ω^i_1, ω_2^i_1, τ_ω^i_2, ω_1^i_2)\) denote the lower left corner of the set

\[
\{(τ_1, τ_2) \in [0, τ_1^N] \times [0, τ_2^N] : τ_2 \leq g_2^ω_1(τ_1), τ_1 \leq g_1^ω_2(τ_2)\}.
\] (3.12)

In Figure 2(b), \(τ^ω_ω = τ_0\). In Figures 2(c) and (d), \(τ^ωω = 0\). The following result characterizes all the pairs of level curves supported by IREs.

**Lemma 3.4.** A pair of level curves \((g_1^ω_2, g_2^ω_1)\) is supported by an IRE if and only if (i) the graphs of \(g_1^ω_2\) and \(g_2^ω_1\) have an intersection in \([0, τ_1^N] \times [0, τ_2^N]\) and (ii)

\[
0 \leq g_i^ω_j(τ_i^N) \leq τ_i^N \quad \text{for } i = 1, 2.
\] (3.13)

In particular, under (i) and (ii), the pair of reaction functions \((f_1, f_2)\) defined below is an IRE:

\[
f_i(τ_j) = \max\{g_i^ω_j(τ_j), τ_i^ω_i ω_j\} \quad \text{for } i = 1, 2.
\] (3.14)

This result follows from Kamihigashi and Furusawa (2010, Theorems 5.1 and 5.2). Condition (i) is necessary because if it is violated, there is no path that stays on the level curves forever, which contradicts Lemma 3.3; see Figure 3. Condition (ii) means that each country’s effective payoff must be feasible and no less than its minimax effective payoff.

We say that an IRE satisfying (3.14) is regular. Figure 4 illustrates a typical regular IRE; the IRE depicted in Figure 2(c) is also regular. For the rest of the paper, we mostly focus on regular IREs, which are guaranteed to exist whenever an IRE exists. There are of course other IREs, as we saw in Figure 2, but all IREs have one property in common:

**Lemma 3.5.** For any IRE \((f_1, f_2)\), we have \(f_i(τ_j) ≥ τ_i\) for an \(τ_j \in [0, τ_j^N]\) and \(i = 1, 2\).

See Kamihigashi and Furusawa (2010, Proposition 5.1) for the proof. The next result characterizes the set of steady states supported by IREs:
Figure 3: Level curves not supported by IRE

Figure 4: Regular IRE
Proposition 3.1. Let \((\tau_1, \tau_2) \in [0, \tau_1^N] \times [0, \tau_2^N]\). There exists an IRE such that \((\tau_1, \tau_2)\) is a steady state if and only if

\[
\tau_i \leq g_i^{w_j(\tau_i^N, \tau_j^N)}(\tau_j) \quad \text{for } i = 1, 2.
\] (3.15)

This result is specific to our setting, and is proved in Appendix B. Note that the set of \((\tau_1, \tau_2)\) satisfying (3.15) is the area bounded by the level curves extending from \((\tau_1^N, \tau_2^N)\); see Figure 5.

Recall from Lemma 3.3 that any IRE path stays on the associated pair of level curves except for the initial period. Since both level curves are monotone, any IRE path is also monotone after the initial period, and thus converges to a steady state. We state this observation, which is specific to our setting, as a proposition.

Proposition 3.2. Any IRE path converges to a steady state.

We say that an IRE \((f_1, f_2)\) is effectively efficient if there is no IRE \((\tilde{f}_1, \tilde{f}_2)\) such that 

\[w_1^*(f_2) \leq w_1^*(\tilde{f}_2)\text{ and } w_2^*(f_1) \leq w_2^*(\tilde{f}_2)\]

with at least one of the inequalities holding strictly. In other words, an effectively efficient IRE is not Pareto dominated by any other IRE in terms
of effective payoffs. Effective efficiency can also be characterized graphically:

**Lemma 3.6.** An IRE \((f_1, f_2)\) is effectively efficient if and only if the graphs of \(g_{w_1}^w(f_2)\) and \(g_{w_2}^w(f_1)\) never cross each other (and thus only touch each other).

This result follows from Kamihigashi and Furusawa (2010, Proposition 5.2). Figure 6 illustrates an effectively efficient regular IRE. Effective efficiency has an important dynamic implication, as we will see in the next section.

### 4 Dynamics of Trade Liberalization

With the general results established in the previous section in hand, we now focus on the economic implications of the model. For this purpose, we assume that \(m_i(\tau_i)\) and \(x_i(\tau_j)\) are differentiable on an open interval containing \([0, \tau_i^N]\) and that \(m_i'(\tau_i) > 0\) for all \(\tau_i \in [0, \tau_i^N]\) and \(x_i'(\tau_j) < 0\) for all \(\tau_j \in [0, \tau_j^N]\). Since \(m_i(\tau_i)\) has a single peak at \(\tau_i = \tau_i^N\), we have

\[
m'_i(\tau_i^N) = 0 \quad \text{for } i = 1, 2. \tag{4.1}
\]
Since a tariff on country $i$’s imports creates market distortions, $m_i(\tau_i) + x_j(\tau_i)$ is maximized at $\tau_i = 0$:

$$m_i'(0) + x_j'(0) = 0. \tag{4.2}$$

This is an implication of the well-known result that free trade $(\tau_1, \tau_2) = (0, 0)$ is Pareto efficient, i.e., the contract curve passes through the origin of the tariff space.

The slope of the $\omega_i$-level curve of $w_i$, or the graph of $g_{ji}^{\omega_i}$, is calculated from (3.8) to be

$$(g_{ji}^{\omega_i})'(\tau_i) = -\frac{m_i'(\tau_i)}{\delta x_i'(g_{ji}^{\omega_i}(\tau_i))}. \tag{4.3}$$

It follows from our assumptions on $m_i'(\tau_i)$ and $x_i'(\tau_j)$ that

$$(g_{ji}^{\omega_i})'(\tau_i) \begin{cases} > 0 & \text{if } \tau_i \in [0, \tau_i^N), \\ = 0 & \text{if } \tau_i = \tau_i^N. \end{cases} \tag{4.4}$$

As we saw in Figure 2(b), if $w_i(0, 0) < w_i(\tau_i^N, \tau_j^N)$ for both $i$, then tariffs lower than the static Nash tariffs can be sustained by a threat to revert to the static Nash equilibrium. If it happens that $w_i(0, 0) = w_i(\tau_i^N, \tau_j^N)$, then free trade can be sustained by the same threat; see Figure 7. In such IREs, even a small deviation is punished to a maximum degree: once any deviation occurs, both countries choose the static Nash tariffs forever, and the initial, “cooperative” steady state is never stored.

Therefore, threats are useful in maintaining already low tariffs, but they may not be effective in restoring low tariffs once any deviation occurs, or to initiate a tariff reduction process when the tariffs are already high.\footnote{Of course it is possible to support an entire decreasing path by Nash reversion, but such an equilibrium also fails to restore low tariffs once any deviation occurs.} We argue below that it is promises, rather than threats, that induce both countries to gradually lower their tariffs.

To simplify the exposition, we assume for the rest of the paper that

$$w_i(\tau_i^N, \tau_j^N) < w_i(0, 0) \quad \text{for } i = 1, 2. \tag{4.5}$$

This means that both countries prefer free trade to the static Nash equilibrium in terms of effective payoffs.
Since (4.5) can be written as $m_i(\tau^N_i) - m_i(0) < \delta_i(x_i(0) - x_i(\tau^N_i))$, it is never satisfied if $\delta_i$ is close to zero. If both $\delta_i$ are close to one, then the inequality in (4.5) must be satisfied at least for $i = 1$ or 2. This is because $m_i(\tau_i) + x_j(\tau_i)$ is maximized at $\tau_i = 0$ (recall (4.2)), so that the sum of the left-hand sides of (4.5) over $i = 1, 2$ is strictly less than the sum of the right-hand sides when both $\delta_i$ are close to one. This also indicates that (4.5) holds if both $\delta_i$ are close to one in the special case where the countries are entirely symmetric.

Since (4.5) implies that country $i$’s level curve extending from $(\tau^N_i, \tau^N_i)$ is higher than that extending from the origin, we have

$$g^\omega_j(w^N_i, \tau^N_i)(0) > 0 \quad \text{for } j = 1, 2. \quad (4.6)$$

See Figure 8. Hence the cases considered in Figures 2(a), 2(b), and 7 are now ruled out; we can thus focus on cases where low tariffs cannot be sustained by Nash reversion.

We assume further that $g^\omega_j(\tau_j)$ is strictly concave in $\tau_j$ for all relevant values of $\omega_j$, i.e., for all $\omega_j \in [w_j(\tau^N_j, \tau^N_i), w_j(\tau^N_i, 0)]$ for both $j$.\footnote{See Appendix A for a parametric example that satisfies this assumption.} This and the above assumptions are used only to reduce the number of cases we need to consider when we state our results and draw
graphs. Using the results of Section 3, the analysis here can easily be extended to cases where these assumptions are not satisfied.

We start by studying the stability of some natural steady states:

**Proposition 4.1.** There exists a unique regular IRE such that the static Nash equilibrium $(\tau_1^N, \tau_2^N)$ is a steady state. In this IRE, $(\tau_1^N, \tau_2^N)$ is a unique steady state, and is globally stable. More specifically, given any $\tau_{2,1}$, the IRE path converges to $(\tau_1^N, \tau_2^N)$.

This result follows from (4.6), the concavity of the level curves, and Figure 8, which illustrates the IRE given in Proposition 4.1. Even if the initial tariff $\tau_{2,1}$ is close to zero, both countries successively raise their tariffs, and the IRE path converges to the static Nash equilibrium in the long run.

**Proposition 4.2.** There exists a unique regular IRE such that free trade $(0,0)$ is a steady state. In this IRE, the steady state $(0,0)$ is unstable. More specifically, given any $\tau_{2,1} > 0$, the IRE path never converges to $(0,0)$.

This result follows from the following result.
Lemma 4.1. In the \((\tau_1, \tau_2)\)-space, the graph of \(g_2^{w_1(0,0)}\) is strictly steeper than that of \(g_1^{w_2(0,0)}\) at the origin. As \(\delta_1\) and \(\delta_2\) both approach one, these slopes converge to each other.

This result is equivalent to saying that \((g_2^{w_1(0,0)})'(0) > 1/(g_1^{w_2(0,0)})'(0)\) and both sides converge to each other as \(\delta_1\) and \(\delta_2\) both approach one. To see this, note from (4.3) that this inequality is equivalent to

\[
\frac{m_1'(0)}{x_1'(0)} \frac{m_2'(0)}{x_2'(0)} > \delta_1 \delta_2.
\]

By (4.2), the left-hand side equals one. Thus the inequality is satisfied, and the right-hand side converges to the left-hand side as both \(\delta_i\) converge to one. This establishes the lemma.

Since the graph of \(g_2^{w_1(0,0)}\) is strictly steeper than that of \(g_1^{w_2(0,0)}\) at the origin, the IRE path moves away from the origin if \(\tau_{2,1}\) is close to zero; see Figure 9. Since both \(g_j^{w_i(0,0)}\) \((= f_j)\), \(j = 1, 2\), are monotone, it follows from Lemma 3.3 that there is no IRE path converging to the origin.

The following result deals with IREs with two steady states, including the IRE in Figure 9 as a special case.

Proposition 4.3. There exist regular IREs with two steady states. In these IREs, the higher

Figure 9: Free-trade steady state
steady state \((\tau_1, \tau_2)\) is locally stable, while the lower steady state \((\bar{\tau}_1, \bar{\tau}_2)\) is stable from below and unstable from above. More specifically, if \(\tau_{2,1} > \bar{\tau}_2\), then the IRE path converges to \((\bar{\tau}_1, \bar{\tau}_2)\). If \(\tau_{2,1} < \bar{\tau}_2\), then the IRE path converges to \((\bar{\tau}_1, \bar{\tau}_2)\) in two periods.

To understand this result, consider the regular IRE in Figure 10, which illustrates how the IRE path converges to the higher steady state if \(\tau_{2,1} > \bar{\tau}_2\). Of particular interest is the case in which \(\tau_{2,1} = \tau_{N,2}\). This can be considered as a situation in which the initial pair of tariffs is at the static Nash equilibrium, and then country 1 unilaterally lowers its tariff to \(\tilde{\tau}_{1,1} < \tau_{N,1}\). At this point there is no threat involved in country 1’s strategy; indeed, if country 2 does not lower its tariff from \(\tau_{2,1}^N\), then country 1 continues to choose \(\tilde{\tau}_{1,1}\). It is therefore country 1’s “implicit promise” to further lower its tariff, depending on country 2’s reaction, that actually gives country 2 an incentive to lower its own tariff. Country 2 on its part makes country 1’s promised reaction optimal for country 1 by promising to reciprocate further in case country 1 further lowers its tariff. These mutually optimal promises result in gradual tariff reduction after country 1’s deviation from the static Nash equilibrium, and the IRE path converges to the higher steady state, which is still lower than the static Nash equilibrium.

It is worth pointing out that this steady state is supported by a minimum “threat.” To be specific, suppose that the initial tariffs of both countries are at this steady state. If country 2 makes a small deviation, then country 1 reacts in such a way as to make country 2’s effective payoff simply unchanged. In other words, instead of punishing country 2, country 1 gives country 2 exactly zero incentive to deviate. Either country thus has nothing to gain as well as nothing to lose by deviating. In contrast to a severe punishment scheme like Nash reversion, this minimum threat is just enough to maintain the steady state and has a built-in mechanism to restore it after a small deviation.

Let us now turn to the lower steady state, which also has an interesting property. Suppose that the initial pair of tariffs is at this steady state. If either country raises its tariff, then
it triggers tariff war: both countries’ tariffs keep rising and converge to the higher steady state, as depicted in Figure 10. However, if either country lowers its tariff rate, the other country does not react at all, for either country’s reaction function is flat from 0 to the lower steady state. This “kinked” feature is not necessarily an artifact of the specific IRE under study here. In fact, Lemma 3.5 implies that in any IRE, neither country sets a tariff lower than its tariff at the lower steady state. Therefore, at the lower steady state, a decrease in either country’s tariff is never matched by a decrease in the other country’s tariff. At this steady state, by lowering its tariff rate, each country only rewards the other country while incurring a loss.

Under our assumption that the level curves are strictly concave, Lemma 3.6 implies that an IRE with two steady states is not effectively efficient, i.e., it is Pareto dominated by another IRE in terms of effective payoffs. As discussed above, the lower steady state of such an IRE is unstable from above; in other words, it is difficult to maintain cooperation to keep the tariffs as low as possible in a regular IRE that is not effectively efficient. The following result shows that an effectively efficient regular IRE always yields stable cooperation.

Proposition 4.4. In any effectively efficient regular IRE, there exists a unique, globally
Figure 11: Globally stable steady state

stable steady state \((\tau_1^*, \tau_2^*)\), which satisfies

\[0 < \tau_i^* < \tau_i^N \quad \text{for} \; i = 1, 2. \quad (4.7)\]

In particular, if \(\tau_{2,1} > \tau_2^*\), then the IRE path gradually converges to \((\tau_1^*, \tau_2^*)\). If \(\tau_{2,1} < \tau_2^*\), then the IRE path converges to \((\tau_1^*, \tau_2^*)\) in two periods.

To see this result, note first that the existence of a unique steady state follows from Lemma 3.6 and the strict concavity of the level curves. The inequalities in (4.7) follow from Propositions 4.1 and 4.2, respectively. The stability properties stated in the proposition should be clear from Figure 11, which illustrates an effectively efficient regular IRE. There is a unique steady state \((\tau_1^*, \tau_2^*)\), which is globally stable. If \(\tau_{2,1} > \tau_2^*\), then the IRE path converges to the steady state, as depicted in the figure with \(\tau_{2,1} = \tau_2^N\). If \(\tau_{2,1} < \tau_2^*\), then the IRE path converges to the steady state in two periods, since country 1 chooses \(\tau_1^*\) whenever \(\tau_{2,1} < \tau_2^*\); i.e., each country faces a kinked reaction curve as at the lower steady state in Figure 10.

One might wonder why the countries do not lower their tariffs all the way to zero even in an effectively efficient IRE. A short answer is that the first inequality in (4.7) says that
the origin cannot be the steady state of an effectively efficient IRE. To see this intuitively, note that when a country lowers its tariff, it incurs the loss immediately, while it receives the benefit only in the next period, when the other country is expected to reciprocate. Since the future benefit is discounted, a reaction function optimal in terms of effective payoff is different from a reaction function optimal in terms of one-shot payoff. In an effectively efficient IRE, in particular, there is no room for Pareto improvement in terms of effective payoff, so that neither country ever chooses a reaction that would be optimal in terms of one-shot payoff.

In fact, an effectively efficient IRE need not be regular to have a unique, globally stable steady state:

**Proposition 4.5.** Any effectively efficient IRE has a unique, globally stable steady state.

To see this result, note from Proposition 3.2 that any IRE converges to a steady state and thus has at least one steady state. There can be only one by effective efficiency and the strict concavity of the level curves. Thus there is a unique steady state. This steady state is globally stable since any IRE path must converge to this unique steady state by Proposition 3.2.

So far we have only seen symmetric IREs in figures. However, there is no guarantee that trade liberalization is symmetric. Figure 12 illustrates an effectively efficient regular IRE that has an unequal steady state. In this IRE, country 2 enjoys the highest possible effective payoff, while country 1’s effective payoff is unchanged from the static Nash equilibrium. In other words, trade liberalization here is a one-sided effort on country 1’s part.

The asymmetric IRE in Figure 12 has an important implication on unilateral trade liberalization. In this IRE, country 1 chooses the lowest possible response to $\tau^N_2$ in the initial period, offering the highest possible effective payoff to country 2. Given this effective payoff, however, the best that country 2 can do for country 1 is to keep country 1’s effective payoff unchanged from the static Nash equilibrium. Since the IRE in Figure 12 is already effectively
efficient, any higher effective payoff is infeasible for country 1 given its own reaction function $f_1$! This suggests that in order to benefit from unilateral liberalization, a country should leave sufficient room for further liberalization. In other words, a mutually beneficial tariff reduction process should be sufficiently gradual.

Our results so far demonstrate that various steady states can be supported by regular IREs. This is an implication of Proposition 3.1, which shows that the set of all steady states supported by IREs is the area bounded by the pair of level curves extending from the static Nash equilibrium; recall Figure 5. As we have seen, each steady state can be stable from below and unstable from above, locally stable, or globally stable. We can thus divide the set of steady states according to these stability properties.

Figure 13 divides the set of steady states into three regions and one curve. The light gray region is the set of steady states stable from below and unstable from above. A point in this region is surrounded by a pair of level curves extending from a common point on the $\tau_1$ or $\tau_2$ axis. Hence it is the lower steady state of a regular IRE with two steady states and is stable from below and unstable from above by Proposition 4.3. The dark gray region is the set of locally stable steady states. A point in this region is also surrounded by a pair
Figure 13: Classification of steady states supported by regular IREs (light gray = stable from below and unstable from above, dark gray = locally stable, black = globally stable) of level curves extending from a common point on the $\tau_1$ or $\tau_2$ axis. Hence it is the higher steady state of a regular IRE with two steady states and is thus locally stable by Proposition 10. The black region (excluding the entire kinked lower left boundary) and the thick black curve comprise the set of globally stable steady states. The kinked lower left boundary of the black region is the locus of the higher intersection of a pair of level curves extending from a common point on the $\tau_1$ or $\tau_2$ axis. Therefore a pair of level curves having an intersection in the black region has no other intersection in $[0, \tau_1^N] \times [0, \tau_2^N]$; thus a point in this region is a globally stable steady state by the argument of Figures 8. The thick black curve is the locus of points of tangency between a pair of level curves; these points are globally stable steady states by Proposition 4.4.

Although Figure 13 might seem to suggest that a steady state in the black region in the figure cannot be close to the origin, this is not necessarily the case. In fact, if both $\delta_i$ are close to one, a steady state in the black region can be close to the origin. This is an implication of Lemma 4.1, which shows that the slopes of the two level curves extending from the origin converge to each other as both $\delta_i$ approach one. Indeed, if we let both $\delta_i$
approach one, the higher steady state in Figure 9 converges to zero, as the slopes of the two level curves are exactly equal to each other at the origin when both $\delta_i$ are equal to one. It then follows from Figure 13 that if both $\delta_i$ are close to one, the kinked point of the lower left boundary of the black region is close to zero. Therefore, provided that both $\delta_i$ are close to one, a pair of extremely low tariffs can be achieved in the long run as the unique, globally stable steady state of an IRE.

5 Concluding Remarks

In this paper we have analyzed a tariff-setting game between two large countries in which they alternate in setting their individual tariffs. We have focused on the IREs, the subgame perfect equilibria in which each country chooses its tariff according to a stationary function of the other country’s tariff. We have fully characterized the IREs of this model and the set of all steady states. We have shown that there are many IREs with two steady states, one with higher tariffs (but still lower than the static Nash tariffs), the other with lower tariffs. The higher steady state is locally stable, while the lower steady state is stable from below but unstable from above. We have also shown that in effectively efficient IREs, there exists a unique, globally stable steady state. In most IREs, one country unilaterally reduces its tariff from the static Nash equilibrium, the other country reciprocates in response to the first country’s implicit promise to lower its tariff even further, and this process continues forever, converging to a steady state with tariffs lower than at the static Nash equilibrium. We have argued that it is therefore promises, rather than threats, that induce the countries to gradually reduce their tariffs.

We have also argued that trade liberalization must be sufficiently gradual since what motivates a country to decrease its tariff is an expected future decrease in the other country’s tariff. This also implies that to benefit from unilateral liberalization, a country should not decrease its tariff too much at the initial stage.
A steady state of an IRE has the property that it involves only a minimum threat. Each country makes the other country exactly indifferent between raising its tariff and staying at the steady state. Even if a deviation occurs, each country is willing to lower its tariff again provided that the other country does so. Therefore the IREs we have studied have a self-enforcing built-in mechanism to restore a stable steady state as well as to initiate a trade liberalization process. This suggests that an explicit agreement may not be necessary to initiate and continue trade liberalization.

Appendix A  A Parametric Example

In this appendix we derive the surplus functions $m_i(\tau_i)$ and $x_i(\tau_j)$ explicitly in a parametric example. We also show that the level curves associated of the effective payoff functions are strictly concave in this example.

Let $p_i$ be the domestic price of country $i$’s import good, which we call good $i$, and $q_i$ be the associated trade quantity. We assume that the import demand and export supply functions are identical across the countries, and that country $i$’s import demand and country $j$’s export supply functions are given by

\begin{align*}
q_i &= 1 + a - p_i, \quad (A.1) \\
q_i &= (p_i - \tau_i) - a, \quad (A.2)
\end{align*}

where $a > 0$ is the autarkic equilibrium price of good $i$ in the exporting country $j$. Note that the autarkic equilibrium price of good $i$ in the importing country $i$ is $1 + a$. In trade equilibrium, $1 + a - p_i = p_i - \tau_i - a$, so that

\begin{align*}
p_i &= \frac{1 + \tau_i}{2} + a, \quad q_i = \frac{1 - \tau_i}{2}. \quad (A.3)
\end{align*}
Consequently, country \(i\)'s import surplus and country \(j\)'s export surplus are given by
\[
m(\tau_i) = \frac{1}{2} (1 + a - p_i)q_i + \tau_i q_j = \frac{(1 - \tau_i)(1 + 3\tau_i)}{8}, \quad (A.4)
\]
\[
x(\tau_i) = \frac{1}{2} (p_i - \tau_i - a)q_i = \frac{(1 - \tau_j)^2}{8}, \quad (A.5)
\]
where we omit the subscript \(i\) by symmetry. The static Nash equilibrium is \((1/3, 1/3)\).

We assume that the discount factor, denoted \(\delta\), is common across the countries. In what follows we show that the \(\omega_i\)-level curve of country \(i\)'s effective payoff is strictly concave for any \(\omega_i \in [w_j(\tau_i^N, \tau_j^N), w_j(\tau_i^N, 0)]\), where
\[
w_i(\tau_i^N, \tau_j^N) = m(1/3) + \delta x(1/3) = (1/6) + (\delta/18), \quad (A.6)
\]
\[
w_i(\tau_i^N, 0) = m(1/3) + \delta x(0) = (1/6) + (\delta/8). \quad (A.7)
\]

Recall that the \(\omega_i\)-level curve of country \(i\)'s effective payoff is given by the function \(g^{\omega_i}(\tau_i)\), which satisfies
\[
\frac{(1 - \tau_i)(1 + 3\tau_i)}{8} + \frac{\delta(1 - g^{\omega_i}(\tau_i))^2}{8} = \omega_i. \quad (A.8)
\]

Differentiating \((g^{\omega_i})'(\tau_i)\) given in (4.3) we have
\[
(g^{\omega_i})''(\tau_i) = -\frac{\delta m''(\tau_i)x'(g^{\omega_i}(\tau_i))^2 + m'(\tau_i)^2 x''(g^{\omega_i}(\tau_i))}{\delta x'(g^{\omega_i}(\tau_i))^3}.
\]

Since \(x'(g^{\omega_i}(\tau_i)) < 0\) (provided that \(g^{\omega_i}(\tau_i) \leq 1/3\)), we have \((g^{\omega_i})''(\tau_i) < 0\) if and only if
\[
0 > \delta m''(\tau_i)x'(g^{\omega_i}(\tau_i))^2 + m'(\tau_i)^2 x''(g^{\omega_i}(\tau_i))
\]
\[
= -3\delta \left(1 - g^{\omega_i}(\tau_i)\right)^2 + (1 - 3\tau_i)^2.
\quad (A.9)
\]

The above inequality is equivalent to
\[
3\delta \left(1 - g^{\omega_i}(\tau_i)\right)^2 > (1 - 3\tau_i)^2. \quad (A.10)
\]

Solving (A.8) for \(\delta(1 - g^{\omega_i}(\tau_i))^2\) and substituting the resulting expression into (A.11), we find that (A.11) reduces to \(\omega_i > 1/6\). This condition is satisfied for any \(\delta \in (0, 1)\) since \(\omega_i \in [(1/6) + (\delta/18), (1/6) + (\delta/8)]\), so we conclude that all the relevant level curves are strictly concave in this example.
Appendix B  Proof of Proposition 3.1

If: Let \((\tau_1, \tau_2)\) satisfy (3.15). Let \(\omega_1 = w_1(\tau_1, \tau_2)\) and \(\omega_2 = w_2(\tau_1, \tau_2)\). Then \(g_1^{\omega_2}\) and \(g_2^{\omega_1}\) satisfy both conditions (i) and (ii) in Lemma 3.4. Define \((f_1, f_2)\) by (3.14). Then \((f_1, f_2)\) is an IRE by Lemma 3.4. Since \((\tau_1, \tau_2) \geq (\underline{\tau}_1, \underline{\tau}_2)\), we have \(f_1(\tau_2) = g_1^{\omega_2}(\tau_2)\) and \(f_2(\tau_1) = g_2^{\omega_1}(\tau_1)\). This together with condition (i) in Lemma 3.4 shows that \((\tau_1, \tau_2)\) is a steady state.

Only if: Let \((f_1, f_2)\) be an IRE such that \((\tau_1, \tau_2)\) is a steady state. Then by Lemma 3.3, we have \(\tau_1 = g_1^{\omega_2}(\tau_2)\) and \(\tau_2 = g_2^{\omega_1}(\tau_1)\). Let \(\omega_1 = w_1^*(f_2)\) and \(\omega_2 = w_2^*(f_1)\). By Lemma 3.2, we obtain condition (i) in Lemma 3.4. Since \(\omega_1 \geq w_1(\tau_1^N, \tau_2^N)\) and \(\omega_2 \geq w_2(\tau_1^N, \tau_2^N)\) by Lemma 3.1, we have \(\tau_1 = g_1^{\omega_2}(\tau_2) \leq g_1^{w_2(\tau_1^N, \tau_2^N)}(\tau_2)\) and likewise \(\tau_2 \leq g_2^{w_1(\tau_1^N, \tau_2^N)}(\tau_1)\). Hence we obtain (3.15).
References


