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ABSTRACT

Heterogeneity in firm productivity affects the location patterns of firm and agglomeration. Here we provide an economic geography model, involving forward and backward linkages driven by the migration of a footloose entrepreneur (capital owner) with different productivity. As a result we find a sorting equilibrium characterised by co-agglomeration of similar productivity firms, however, in contrast to previous studies, unproductive firms are more likely to agglomerate than their more productive counterparts. This is due to the increasingly severe competition induced by productive firms. Productive firms prevent severe local competition through their co-agglomeration. In terms of social welfare, although the sorting equilibrium involves higher social welfare than a perfectly symmetric pattern of firm location, the market outcome is sub-optimal and induces too much agglomeration.

JEL F15, F23.

Keywords: heterogeneous firms, footloose entrepreneurs, competition, productivity, economic geography

1. INTRODUCTION

Heterogeneity in firm productivity has recently been one of the most important issues in spatial aspects of economics. Not only economic researchers but also national governments and policy makers have drawn attention to how firm heterogeneity affects firm location and geographical concentration, and how high productivity firms can be attracted to a specific area to induce agglomeration in order to boost national average productivity (World Bank, 2009).

There is some empirical evidence regarding firm heterogeneity and location patterns, for example the observation of a core region characterised by severe competition, which pushes lower productivity firms outwards towards the periphery (Syverson, 2004; Asplund and Nocke, 2006). However the precise nature of the relationship between firm location and productivity is still ill-defined. As Duranton and Overman (2005) observe empirically, location patterns are highly heterogeneous across industries. In some sectors large scale firms are dispersed across regions while smaller firms are more concentrated geographically. They suggest that the spatial pattern is influenced by firm or sector characteristics and the way in which sectors/regions are classified. Furthermore, although dense areas are more productive than periphery in some cases,

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this is less obvious, such as the example that downtown Detroit is more productive than suburbs such as Silicon Valley (Glaeser and Kahn, 2004). Similarly Au and Henderson (2005), using data on Chinese cities, found an inverted-U relationship between real wage per worker and urban size, which indicates that larger sized markets (cities) might not attract high productivity firms or might attract more low productivity firms.

Based on the empirical evidence there might be another possible location pattern when firm productivity is heterogeneous. It can be hypothesised that productive firms cause severe local competition due to their lower prices and higher market share and thus they can deter co-agglomeration in the large market and relocate from the large metropolitan area (core) to suburbs (periphery) while maintaining better access to the core region. Urban area locations sometimes cause firm to lower their productivity due to urban congestion effects, such as traffic jams, higher wage and higher land rents. These negative location factors might provide an incentive for productive firms to escape to the suburbs. It is important to study these negative urban congestion effects, but this paper explains firm location patterns by a "self-selection" mechanism, in which severe market competition, which is self-induced by productive firms, promotes their relocation to suburbs. Thus any urban congestion concerns and negative externalities are out of our scope. Our interest is in market competition caused by the geographical concentration of high productivity firms without any urban negative externalities. In order to investigate the hypothesis described, this paper studies the impact of firm heterogeneity on firm location in an economic geography model.

1.1. ***Literature Review and Our Model***

This paper constructs a simple model on the basis of recent advancements of two strands of literature. One strand surrounds the footloose entrepreneur (FE) model in the economic geography literature and the other is concerned with the heterogeneous-firm trade (HFT) model of Melitz (2003) in the international trade literature.

The spatial aspects of economics have been discussed by the "new economic geography" models, initiated by the core-periphery (CP) model of Krugman (1991). These models study the relationship between trade costs and the location patterns of firms, agglomeration processes and the driving forces behind agglomeration and dispersion. However, the CP model has two weak aspects, first is its analytic intractability, the second is that it ignores heterogeneous firm productivity, which is our central focus in this paper.

The first weakness has been resolved by models in subsequent studies. A set of models has been proposed which provide more tractable frameworks such as Ottaviano, Tabuchi and Thisse

(2002). Forslid (1999) Forslid and Ottaviano (2002) and Baldwin et al. (2003, Ch.4) provide an analytically solvable version of CP model called the footloose entrepreneur (FE) model. The FE model involves the migration of capital owners (or entrepreneurs), which causes demand-linked circular causality driven by a migrant expenditure shift as well as cost-linked circular causality driven by a decreasing cost of living due to more local producers in an areas. Later, Pfluger (2004), successfully obtained a simpler model with more analytical solutions, providing the FE model with the quasi-linear utility function.[1] Since the quasi-linear utility function excludes income effects and dampens demand-linked circular causality the agglomeration effect weakens and catastrophic agglomeration never happens, resulting instead in a gradual agglomeration process via trade cost reduction. This paper adopts the FE model of Pfluger (2004) due to its tractability and simplicity as well as for the reasons described below.

A technical reason for using the quasi-linear utility function model is that we conduct an intensive analysis of firm heterogeneity rather than labour heterogeneity and migration of the entrepreneur is a key factor. When firms are heterogeneous, or when entrepreneurs have a varying level of talent, capital returns, via migrant entrepreneurs' income, vary across entrepreneurs. This would incur a heterogeneous income effect across entrepreneurs and a heterogeneous demand shift through their migration, resulting in a mixture of heterogeneous firms with heterogeneous labour migration. Thus the quasi-linear utility function reduces the heterogeneous demand/income effect in order to distinguish it from the previous demand/labour heterogeneity literature (e.g. Tabuchi and Thisse, 2002) and instead highlight the impact of firm heterogeneity on firm location and market competition.[2] Thus in this paper we require that entrepreneurs uniformly spend a unit of demand regardless of their income, although migration still creates demand linkages. A single firm production shift corresponds to a constant uniform shift of a unit of expenditure by a migrated entrepreneur. On the other hand, firms are heterogeneous and thus the impact of a production shift on competition and the cost of living is influenced by the location pattern of firms with varying productivities. Productive firms cause severe competition and improve the cost of living by lowering prices. For this reason, we adopt the FE model with a quasi-linear utility function.

The second weakness, ignorance of firm heterogeneity, can be solved by incorporating the HFT model of Melitz (2003) into an economic geography model. The HFT models focus on the

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[1] Pfluger and Sudekum (2008), an extension of Pfluger (2004) involves welfare analysis with housing market.

[2] Heterogeneity on the demand side is studied by Tabuchi and Thisse (2002) and Okubo and Picard (2008). The sorting of skilled labour is modelled by Mori and Turrini (2005). Empirically Combes et al. (2003) found labour sorting in France.

links between firm productivity and export behaviour on the one hand and the impact of trade liberalisation on industry productivity on the other.[3] Melitz (2003) allows for firm heterogeneity and sunk market entry costs. In his model firms have heterogeneous marginal costs and the most productive firms, those with the lowest marginal costs, enjoy higher market shares and operating profits. As a result the most productive firms will enjoy sales that are large enough to cover the fixed domestic market entry costs and the most productive among these will export to foreign markets and enjoy sales that are large enough to cover the fixed export costs. Trade liberalisation raises only exporters' profits while reducing local producers' profits, this is known as the profit share shifting effect, thus forcing the least efficient local producers to exit the market, allowing the most productive local firms to enter the export market, the co-called selection effect. The location of firms, however, is ignored as firms are assumed to locate in the nation in which they are 'born'. In contrast to Melitz (2003) the aim of this paper is to investigate the way in which firm heterogeneity and firm location interact rather than focus on trade patterns and export behaviour.

Economic geography models with firm heterogeneity are not entirely new, Nocke (2006) first modelled spatial sorting, in which talented entrepreneurs enter a larger market and less talented ones choose to locate in a smaller market. Baldwin and Okubo (2006) extended his idea to "new economic geography" and first developed the Footloose Capital (FC) model of Martin and Rogers (1995) with firm heterogeneity a la Melitz (2003).[4] The main results of Baldwin and Okubo (2006) are 1) firm heterogeneity works as <u>dispersion force</u>, however, firm heterogeneity *per se* never affects the break and sustain points but just the moderate agglomeration process. 2) <u>The most productive firms</u> are the most footloose because the productive firms are more sensitive to profit gap. Accordingly productive firms in smaller markets are more likely to relocate to the larger market. This means that larger market has productive firms, while small market has only unproductive firms ("spatial selection/sorting effect").

The conclusions noted above does not represent all that is known regarding the interaction of firm heterogeneity and location. Baldwin and Okubo (2006) is based on the simplest economic geography model, the FC model, in which capital returns are repatriated to the origin due to the fact there is no labour migration allowed, thus firm location is determined only by nominal profits. By contrast, the inclusion of labour (entrepreneur) migration in the FE model alters the firm location decision. A firm's location is determined by real profits, which means migration

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[3] See also Helpman, Melitz and Yeaple (2004), Melitz and Ottaviano (2008) and Chaney (2008).
[4] Later, Okubo (2009) added intermediate output linkages (the FC vertical linkage model) to Baldwin and Okubo (2006).

takes into account the gap in nominal profit as well as the cost of living. In particular firm heterogeneity impacts upon competition as well as the cost of living. The agglomeration of high productivity firms produce severe competition, resulting in reductions in market shares and profitability, thus deterring these firms from co-agglomeration.

We have identified several results which contrast with existing heterogeneous firm economic geography models. Firstly, the earliest movers are <u>unproductive firms</u>; since the migration of productive firms causes more severe competition productive firms are deterred from co-agglomeration, low productivity firms are more footloose in location choice. Secondly, spatial sorting occurs as productive firms will be diversified and unproductive firms create agglomeration in one location. This is also in contrast with previous models in which productive firms are likely to create agglomeration. Thirdly, firm heterogeneity works as an <u>agglomeration force</u>, which is also in contrast with Baldwin and Okubo (2006). Firm heterogeneity is more likely to cause an unstable symmetric initial equilibrium and encouraged full agglomeration, seen through decreased break and sustain points. These results are in contrast with Nocke (2006) and Baldwin and Okubo (2006). Finally, spatial sorting can improve global welfare when compared to a perfectly symmetric equilibrium. However, spatial sorting in market outcomes involves too much agglomeration, compared with the social optimal spatial sorting equilibrium. All of these results cannot be derived from HFT models (Melitz, 2003; Melitz and Ottaviano, 2008), in these models the most productive firms are likely to be more footloose and engage in FDI, while low productivity firms are more likely to exit market. Thus they cannot derive footloose unproductive firms, hence they do not observe co-agglomeration of low productivity firms and spatial sorting in terms of firm productivity.

The rest of the paper is organised in 6 sections, Section 2 presents the basic model. Section 3 explores the long-run equilibrium. Section 4 studies social welfare and optimal equilibrium path. Finally concluding remarks are provided in Section 5.

2. THE BASIC MODEL AND SHORT-RUN EQUILIBRIUM

2.1. *Footloose Entrepreneur Model*

The model works across two regions, the North and the South, two sectors, differentiated manufacturing goods in the "*M*" sector, and a homogenous agriculture good "*A*" sector, and two factors, labour, *L,* and capital, *K*. There are two categories of people, workers, who provides one unit of labour (*L*) and are bound to the land and secondly entrepreneurs, who own one unit of capital (*K*) and is inter-regionally mobile together with their capital/firm. Thus labour, *L* is inter-regionally immobile, while capital, K, is inter-regionally mobile just as workers and entrepreneurs are immobile and mobile respectively. We define the total endowment of resources in the world as $L^w + K^w$ where L^w and K^w are the worldwide endowments of labour and capital exogenously given as $K^w = 1 \quad L^w = \beta$. Thus the total global population, which is simply the sum of workers and entrepreneurs, is 1+β. The two regions are symmetric in all aspects, specifically tastes, technology, openness to trade, and their relative factor endowments of labour and capital. The tastes of the representative consumer in each region are quasi-linear:

(1) $$U = \mu \ln C_M + C_A, \quad C_M \equiv \left(\int_{i\in\Theta} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \quad 0 < \mu < 1 < \sigma$$

where C_M and C_A are consumption of the composite of all differentiated varieties of M goods, and consumption of the homogenous A good respectively. μ measures the share of expenditure spent on M-sector varieties, Θ is the set of varieties available in a typical region, and σ is the constant elasticity of substitution between any two M-sector varieties.

The A-sector is characterised by perfect competition, constant returns to scale, and zero trade costs. The production of the A-sector good involves only labour and good A is taken to be the numeraire. The M-sector is characterised by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trading costs. As usual, $t \geq 1$units must be shipped in order to sell one unit in the foreign market. Following the standard FE model, since one unit of capital, which is owned by an entrepreneur, creates one firm the migration of entrepreneurs/capital corresponds to firm migration. Thus the total number of firms in the world is unity due to $K^w = 1$.

In contrast to standard FE model, firms in M-sector are heterogeneous in terms of labour productivity. For simplicity we have two types of firms with different levels of productivity. One type is unproductive firm and thus requires more units of labour, that is it has a higher marginal cost, we label these as 'H' firms, while the other type is a more productive firm and thus requires less units of labour, corresponding to lower marginal cost firms, which we refer to as 'L' firms. Each firm is required to use one unit of capital, representing as fixed cost. The cost function of a typical Northern firm of type j is:

(2) $$\pi_j + a_j w x_j \qquad 1 \geq \phi \equiv t^{1-\sigma} \geq 0, \quad j \in (H, L)$$

where π_j represents the reward to capital as fixed costs and the second term is variable costs. x_j is firm-level output, a_j is that firm's labour requirement, and w is the labour's reward, or wage. The parameter, ϕ, which plays a critical role in the analysis, is an indicator of the "freeness" of trade in that ϕ ranges from zero, when trade is prohibitively expensive ($t = \infty$) to unity when trade is perfectly free ($t = 1$). The firm-specific unit-input coefficient is a_H for H firms and a_L for L firms, where $a_H > a_L$.

Each region is endowed with an equal share of firms at the initial equilibrium. We define the worldwide mass of varieties/capital to be equal to unity, hence each region's mass of firms/capital is equal to ½ at the initial equilibrium. Each region begins with the same proportion of H- and L-firms; α (1-α) denotes the proportion of H-firms (L-firms) in the composition of the total number of firms across the regions. The share of firm types, α, characterised by productivity distributions, is exogenously given as in Melitz (2003). Since each firm is associated with a particular unit of capital it is natural to assign the source of firm heterogeneity to its capital, i.e. each unit of capital in each region is associated with a particular marginal cost as measured by the firm-specific unit-input coefficients a_H and a_L.

The main focus of this paper is firm heterogeneity. As the difference between a_H and a_L increases firms become more heterogeneous in terms of labour productivity. Firm share is also relevant: for example as α approaches zero or one a single type of firm is dominant and thus firms are almost homogeneous and the setting becomes almost identical to that in the standard FE model. On the other hand, when α is close to 0.5, two types of firms coexist in almost equal proportions. In order to highlight firm heterogeneity we exogenously create a substantial difference between a_H and a_L with $\alpha \approx 0.5$ under the co-existence of two firm types. Specifically we assume that

(3) $$\frac{\alpha}{1-\alpha} < \frac{a_H}{a_L}$$

This means that the cost difference, which determines firm heterogeneity, is larger than the relative proportion of H and L firms.

Utility maximisation gives us the demand function for the i^{th} variety of the M-goods as:

(4) $$c_i = \frac{p_i^{-\sigma}}{\int_{h\in\Theta} p_h^{1-\sigma} dj + \int_{k\in\Theta^*} \phi p_k^{1-\sigma} dk} \mu E$$

where c_i and p_i are the consumption and price of variety *i* and *E* is total expenditure in the North. Southern demand functions are isomorphic. Here we adopt the standard convention of denoting Southern variables with a "*" superscript.

2.2. *The Short-run Equilibrium*

We begin by examining a symmetric equilibrium where the worldwide mass of firms, which we normalise to unity, is evenly split with ½ of all firms locating their production in each of the two regions. We also fix the proportion of each region dedicated to the M and A sectors to be equal with $\alpha / 2$ and $(1-\alpha)/2$ being the type of each firm locating their production in each of the regions.

Due to constant returns, perfect competition, and zero trade costs in the A-sector, the price of the A-sector good is identical in both markets and so equates marginal costs across the regions. The equalisation of prices and marginal costs implies that equilibrium wages must also be identical in the two regions. Choosing a number of units of A, without loss of generality, such that the unit labour-input requirement is unity, we have $w = w^* = 1$ since the price of A is unity.

Utility maximisation implies the usual CES demand functions for each variety of *M*, taken together with the assumed presence of Dixit-Stiglitz monopolistic competition, imply that 'mill pricing' is optimal. That is, a typical M-sector firm based in the North charges a producer price that is a constant mark-up over marginal costs, with all trading costs passed on to consumers in their entirety, specifically:

(5)
$$p_H = \frac{a_H}{1-1/\sigma} \qquad p_H^* = \frac{ta_H}{1\text{-}1/\sigma}$$
$$p_L = \frac{a_L}{1-1/\sigma} \qquad p_L^* = \frac{ta_L}{1\text{-}1/\sigma}$$

We can combine the fact that we are in a symmetric equilibrium, so that each type of firm producing in the North and the South is ½, with expression **(5)** to obtain the marginal cost implied by a_H and a_L, so that price index in North, *P*, can be written as:

$$P = \left(\alpha n_H \left(\frac{a_H}{1-1/\sigma} \right)^{1-\sigma} + \alpha(1-n_H)\left(\frac{ta_H}{1-1/\sigma} \right)^{1-\sigma} + (1-\alpha)n_L\left(\frac{a_L}{1-1/\sigma} \right)^{1-\sigma} + (1-\alpha)(1-n_L)\left(\frac{ta_L}{1-1/\sigma} \right)^{1-\sigma} \right)^{\mu/(1-\sigma)}$$
$$= \left(\frac{1}{(1-1/\sigma)^{1-\sigma}}\right)^{\mu/(1-\sigma)} \left(\alpha n_H a_H^{1-\sigma} + \alpha(1-n_H)\phi a_H^{1-\sigma} + (1-\alpha)n_L a_L^{1-\sigma} + (1-\alpha)(1-n_L)\phi a_L^{1-\sigma} \right)^{\mu/(1-\sigma)}$$

where n_H $(1\text{-}n_H)$ denotes the Northern (Southern) share of H-firms and n_L $(1\text{-}n_L)$ denotes the Northern (Southern) share of L-firms, noting that $n_H = n_L = 1/2$ at the initial equilibrium. Simplifying this and using an analogous approach to the Southern region yields

(6)
$$\Delta = \alpha n_H a_H^{1-\sigma} + \alpha(1-n_H)\phi a_H^{1-\sigma} + (1-\alpha)n_L a_L^{1-\sigma} + (1-\alpha)(1-n_L)\phi a_L^{1-\sigma}$$
$$\Delta^* = \alpha n_H \phi a_H^{1-\sigma} + \alpha(1-n_H) a_H^{1-\sigma} + (1-\alpha)n_L \phi a_L^{1-\sigma} + (1-\alpha)(1-n_L) a_L^{1-\sigma}$$

where, for notational convenience, we define

$\Delta \equiv (1-1/\sigma)^{1-\sigma}(P)^{(1-\sigma)/\mu}$ *and* $\Delta^* \equiv (1-1/\sigma)^{1-\sigma}(P^*)^{(1-\sigma)/\mu}$. Note that Δ is the denominator of the CES demand function.

A firm's operating profit is critical to this analysis since, given the structure of the model, the reward to a unit of capital is the operating profit of the firm with which it is associated. To calculate this we need to know the level of expenditure in each market. For example using the quasi-linear utility function **(1)** Northern and Southern expenditure on all M-goods, *E* and *E** respectively, can be written as:

(7) $$\mu E = \mu(L+K) \qquad \mu E^* = \mu(L^* + K^*)$$

Endowments in each region are thus $L = L^* = \dfrac{\beta}{2}$, $K = \alpha n_H + (1-\alpha)n_L$,

and $K^* = \alpha(1-n_H) + (1-\alpha)(1-n_L)$. Total initial endowments are $K^w = 1 \quad L^w = \beta$. We note that *L* and *L** are exogenously given and equally allocated between regions, while *K* and *K** correspond to firm location and thus are endogenously determined, initially defined as $K = 0.5$ and $K^* = 0.5$ ($n_H = n_L = 1/2$). Using this information, operating profits for a typical North based firm with a unit-input coefficient of a_H and a_L can be written as:

(8) $$\pi[a_H] = \gamma B a_H^{1-\sigma}, \quad \pi[a_L] = \gamma B a_L^{1-\sigma};$$

where $B \equiv (\dfrac{E}{\Delta} + \phi\dfrac{E^*}{\Delta^*})$ $\quad \gamma \equiv \dfrac{\mu}{\sigma}$ $\quad E = \dfrac{\beta}{2} + K$, and $E^* = \dfrac{\beta}{2} + K^*$. The analogous formula for a Southern firm is $\pi^*[a_H] = \gamma B^* a_H^{1-\sigma}$ and $\pi^*[a_L] = \gamma B^* a_L^{1-\sigma}$ where $B^* \equiv (\phi\dfrac{E}{\Delta} + \dfrac{E^*}{\Delta^*})$. Here the *B*'s indicate the market potential in the North and South respectively. Using the indirect utility function corresponding to (1), entrepreneurs are inter-regionally mobile in search of higher utility, or real capital rewards, a fact which is expressed as

(9) $$V_H \equiv \mu(\ln\mu - 1) + \pi[a_H] - \ln P = \mu(\ln\mu - 1) + \gamma B a_H^{1-\sigma} - \frac{\mu}{1-\sigma}\ln\Delta - \mu\ln(1-1/\sigma)$$

$$V_L \equiv \mu(\ln\mu - 1) + \pi[a_L] - \ln P = \mu(\ln\mu - 1) + \gamma B a_L^{1-\sigma} - \frac{\mu}{1-\sigma}\ln\Delta - \mu\ln(1-1/\sigma)$$

3. THE LONG-RUN EQUILIBRIUM

3.1. *Relocation tendencies*

Starting with a situation where firms are evenly divided between the regions we consider the gap in real capital rewards between regions as an incentive for migration for each type of entrepreneur. Since firms are heterogeneous an additional complication must be addressed; whether H-firms or L-firms are the first to relocate. Using **(8)** and **(9)**, the real reward gap that an entrepreneur faces when locating in the North as opposed to the South is:

$$\begin{aligned} V_H - V_H^* &= \pi[a_H] - \ln P - \pi^*[a_H] + \ln P^* = \gamma(B - B^*)a_H^{1-\sigma} - \frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*) \\ V_L - V_L^* &= \pi[a_L] - \ln P - \pi^*[a_L] + \ln P^* = \gamma(B - B^*)a_L^{1-\sigma} - \frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*) \end{aligned} \quad \textbf{(10)}$$

At the initial equilibrium the two regions are identical and thus the gap is zero, that is $V_H - V_H^* = 0$ and $V_L - V_L^* = 0$. The gap in real capital rewards is composed of two terms: the first term is the nominal capital reward gap, which varies between firm types. This term can itself be divided into two parts, the first depending on the market potential gap, *B*-*B**, and the second on the firm-specific productivity term, $a^{1-\sigma}$. Entrepreneur migration will alter the *B*'s via impacts on the Δ's and *E*'s. As $a_H > a_L$ we know that the gap between the L-firms is always larger than that between the H-firms. The second part of the gap in real capital rewards is the living cost effect, which comes from the price index gap and is independent of firm type.

In the symmetric outcome, *B* = *B**, *E*=*E**, and Δ=Δ*, so no firm (or entrepreneur) has an incentive to move. However, if the symmetry in firm location is lost all firms have an incentive to move. At this point the standard economic geography question arises: will the relocation of some firms produce self-reinforcing agglomeration with all firms moving to the North, or will the movement be self-correcting inducing firms to move to restore symmetry? If a slight positive shock to n_H and n_L shifts $V_H - V_H^*$ and $V_L - V_L^*$ from zero to a positive number then the symmetric case is unstable, if the shock turns $V_H - V_H^*$ and $V_L - V_L^*$ negative, symmetry is stable. Importantly, the response of $V - V^*$ to the shock is idiosyncratic due to the heterogeneity of the first term in **(10)**, indicating that there exists a certain trade cost level at which one type of firm is stable and the other is unstable in the symmetric equilibrium.

3.2. *Break point and Spatial Sorting*

We are interested in evaluating the shocks starting from a position of symmetry, where no firms have yet migrated. Following the standard economic geographic procedures we evaluate

$\frac{\partial(V_H - V_H^*)}{\partial n_H}$ and $\frac{\partial(V_L - V_L^*)}{\partial n_L}$ in order to obtain the level of trade "freeness" where this derivative is zero. Unlike in the standard model, the two types of firms have different migration motives. Thus, the break point corresponds to when the first firm breaks the symmetric equilibrium and moves to the other region. Technically this means that we differentiate the real profit gap of each firm in terms of firm share at the symmetric equilibrium, given the other type of firms is fixed. Hence solving $\left.\frac{\partial(V_H - V_H^*)}{\partial n_H}\right|_{n_L=0.5} = 0$ and $\left.\frac{\partial(V_L - V_L^*)}{\partial n_L}\right|_{n_H=0.5} = 0$ in terms of ϕ, the smaller ϕ is the break point.

(11)
$$\begin{aligned} V_H - V_H^* &= \gamma(1-\phi)\left(\frac{E}{\Delta} - \frac{E^*}{\Delta^*}\right)h - \frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*) \\ V_L - V_L^* &= \gamma(1-\phi)\left(\frac{E}{\Delta} - \frac{E^*}{\Delta^*}\right)l - \frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*) \end{aligned}$$

where $h \equiv a_H^{\;1-\sigma}$ $\quad l \equiv a_L^{\;1-\sigma}$, $E = L + K = \frac{\beta}{2} + \alpha n_H + (1-\alpha)n_L$,

$E^* = L^* + K^* = \frac{\beta}{2} + \alpha(1-n_H) + (1-\alpha)(1-n_L), \Delta = \alpha n_H h + \alpha(1-n_H)\phi h + (1-\alpha)n_L l + (1-\alpha)(1-n_L)\phi l$

and $\Delta^* = \alpha n_H \phi h + \alpha(1-n_H)h + (1-\alpha)n_L \phi l + (1-\alpha)(1-n_L)l$.

Solving $\left.\frac{\partial(V_H - V_H^*)}{\partial n_H}\right|_{n_L=0.5} = 0$ and $\left.\frac{\partial(V_L - V_L^*)}{\partial n_L}\right|_{n_H=0.5} = 0$ in terms of ϕ, we get ϕ_H^B and ϕ_L^B respectively and consequently derive that $\phi_H^B < \phi_L^B$.[5] This indicates that H firms are the first movers and break the symmetric equilibrium when trade costs fall. Thus, the break point should be $\phi^B = \phi_H^B$, specifically:

(12)
$$\phi^B = \frac{(\sigma-1)(1+\beta)h - (2\sigma-1)(\alpha h + (1-\alpha)l)}{(\sigma-1)(1+\beta)h + (2\sigma-1)(\alpha h + (1-\alpha)l)}.$$ [6]

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[5] See Appendix 1 for derivation.

[6] In the standard quasi-linear FE model, the homogeneous firm model, *h*=*l* gives the break point as $\phi^B = \frac{(\sigma-1)(1+\beta)-(2\sigma-1)}{(\sigma-1)(1+\beta)+(2\sigma-1)}$, which is identical to that in Pfluger (2004). To keep our analysis interesting, we

Result 1: The first firms to break the symmetric equilibrium are high cost, unproductive firms. Unproductive firms are more footloose.

This result is contrasts with the standard results from economic geography models with firm heterogeneity. In Baldwin and Okubo (2006), productive firms are first movers and are more likely to relocate to the larger market and create agglomeration because they are more sensitive to the nominal profit gap.

When firms are more heterogeneous, i.e. there is a larger *h -l* difference, the break point decreases. This means that firm heterogeneity is more likely to break the symmetric equilibrium and promote spatial sorting. Thus we can conclude that firm heterogeneity works as an agglomeration force.

Result 2: Firm heterogeneity works as an agglomeration force. When firms are more heterogeneous, characterised by a more substantial difference in costs, the break point is lower and the symmetric equilibrium is more likely to be broken.

Once H firms deviate from the South to the North at the break point, the deviation causes the real reward gap of L firms to become negative, i.e. $V_L - V_L^* < 0$, because

(13) $$\left.\frac{\partial (V_L - V_L^*)}{\partial n_H}\right|_{n_H=0.5, n_L=0.5, \phi=\phi^B} = \frac{4\alpha\mu(2\sigma-1)(h-l)}{h(\sigma-1)^2(1+\beta)^3} < 0 .$$

Therefore, the deviation of H-firms to the North at the break point, when starting from the symmetric equilibrium, induces L firms to move to the South. In particular, since L firms supply low price products and thus competition is driven by L firm migration to the South competition becomes more severe in the South. The increased number of L firms causes severe competition in the South, which drives H firm relocation to the North, thus driving spatial sorting.

3.3. *Equilibrium*

We now study the long-run equilibrium when trade costs are lower than the break point. As discussed above, H firms relocate to the North at the break point, pushing L firms out to the South. The mechanism for agglomeration involves the same agglomeration and dispersion forces as in the standard FE model. Once the H firms migrate from South to North, the demand shift through entrepreneur migration increases *E* and decreases *E** in **(11)**. As a result *B*

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exclude the case of full agglomeration for any trade cost. As in the standard model, the black-hole condition is given as $\frac{(\sigma-1)(1+\beta)}{(2\sigma-1)} h > \alpha h + (1-\alpha) l$ ·

increases, which is an example of so-called demand-linked circular causality, which attracts more firms to the North, the demand linked circular causality works as an agglomeration force. The migration involves firm relocation and thus the production shift raises Δ and falls Δ^*, which indicates a fall of the Northern price index and a rise of the Southern price index. The improvement in the cost of living in the North attracts more entrepreneurs, which is a cost linked circular causality, which works as an agglomeration force. On the other hand, cost-linked circular causality causes more severe competition due to the lower price index, which decreases *B*, the so-called congestion effect which acts as a dispersion force.

In the long-run equilibrium, real capital rewards to entrepreneurs should be equal across the two regions when firms are dispersed, or when firms agglomerate in one location, real capital rewards are not equal. In contrast to the standard model, we have two types of firms with a differentiated profit gap due to $h<l$, as seen in (11). Thus both types of firm cannot equalise their real capital reward gaps simultaneously, i.e. $V_H - V_H^* = 0$ and $V_L - V_L^* = 0$ with $0 < n_H < 1$ and $0 < n_L < 1$. Based on this, when H firms (L firms) move to the North (the South) there are three possibilities of real reward gap and location patterns in the long-run equilibrium.

Case 1) $V_H - V_H^* = 0$ and $V_L - V_L^* < 0$, thus $0 < n_H < 1$ and $n_L = 0$.

Case 2) $V_H - V_H^* > 0$ and $V_L - V_L^* < 0$, thus $n_H = 1$ and $n_L = 0$.

Case 3) $V_H - V_H^* > 0$ and $V_L - V_L^* = 0$, thus $n_H = 1$ and $0 < n_L < 1$.

However, Cases 1 and 2 never happen and the long-run equilibrium can only be of the type in Case 3.[7] At the break point, all H firms concentrate in the North, hence $n_H = 1$, while L firms locate in both countries. The share of L firms, n_L, is determined by the levelling of the real reward:

(14) $$V_L - V_L^* = \gamma(1-\phi)\left(\frac{E}{\Delta} - \frac{E*}{\Delta*}\right)l - \frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*) = 0$$

where $\Delta = \alpha h + (1-\alpha)n_L l + (1-\alpha)(1-n_L)\phi l$, $\Delta^* = \alpha\phi h + (1-\alpha)n_L\phi l + (1-\alpha)(1-n_L)l$,

$E = \frac{\beta}{2} + \alpha + (1-\alpha)n_L$ and $E^* = \frac{\beta}{2} + (1-\alpha)(1-n_L)$.

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[7] See proof in Appendix 2.

The equilibrium location of the agglomeration of H firms in one region and dispersion of L firms is attributed to the self-induced local competition. Since the co-agglomeration of L firms triggers more severe local competition than that of H-firms, due to lower marginal costs and prices, the self-induced competition deters the co-agglomeration of L firms. At the break point H firms can create full agglomeration immediately, but L-firms cannot. On the other hand, the agglomeration of H firms increases the size of Northern demand, which accommodates some of the L firms.

A simple manipulation using **(3)** yields $\frac{\alpha}{1-\alpha} < \frac{a_H}{a_L} < \left(\frac{a_L}{a_H}\right)^{1-\sigma} = \frac{l}{h}$. Using this condition we always keep $\Delta > \Delta^*$ and $B < B^*$, implying that the demand shift of E and E^* involves a smaller impact from migration than the production shift in Δ and Δ^*. The cost of living is lower in the North due to the agglomeration of H-firms coupled with some the L firms, this also causes more intense competition and reduces market potential and profitability in North. On the other hand, the South is protected from Northern competition and thus has higher market potential, regardless of the higher cost of living.

In this model, demand-linked circular causality as an agglomeration force is independent of firm heterogeneity due to the quasi-linear utility function, thus, the causality is relatively weak. In contrast, firm heterogeneity affects $\Delta > \Delta^*$, because all H firms and some L firms locate in the North. Thus the cost of living effect as an agglomeration force is substantially larger in the North although the North is benefits from lower prices. Simultaneously a congestion effect acts as dispersion force, as we observe higher Δ and decreases in B. A substantial congestion effect and a weak demand-linked effect lead to less market potential in North, i.e. $B < B^*$.

In parallel to the standard FE model n_L does not have an explicit form in the solution. Using some parameter values often adopted in the standard economic geography literature, Figure 1 plots Northern shares of H and L firms at the long-run equilibrium (h=1, l=2, σ=4, μ=0.5, α=0.5 and β=5)[8]. At the break point all H firms concentrate in the North, this case is known as catastrophic agglomeration, while L firms locate in both regions. This creates two asymmetric regions: a bigger Northern market and a smaller market in the South. As trade costs fall, market competition in the two regions is brought closer due to better market access, in addition the difference in the cost of living is smaller. This attracts more firms to the bigger Northern market

[8] $a_H = 1$ and $a_L = 2^{-1/3} \approx 0.79$. We discuss the case of the first deviation of H firms to the North. The opposite could happen if H firms first deviate from the symmetric equilibrium to the South at the break point and thus the figure is a mirror of Figure 1.

and gradual agglomeration happens among L firms. A fall of trade costs gradually raises the number of L firms and all L firms finally concentrate in the North at the sustain point.

Figure 1: Equilibrium in terms of trade costs

Result 3: When trade costs fall below the break point, spatial sorting occurs. All high cost (unproductive) firms concentrate in the North at the break point, whereas low cost (productive) firms diversify their location across both countries. As trade costs fall, low cost firms gradually relocate to North. Finally full agglomeration arises at the sustain point.

This result is a sharp contrast to any previous heterogeneous-firm models specifically in that productive (unproductive) firms are likely to agglomerate (diversify) in terms of their location.

It is also interesting is to see the catastrophic agglomeration of H firms as well as the gradual agglomeration of L firms at the break point.

3.4. *Full Agglomeration and Sustain Point*

To study the stability of the full agglomeration equilibrium, we evaluate the real reward gap at $n_H = 1$ and $n_L = 1$, the point where all firms have located in the North. To find the sustain point, we solve for the ϕ where $V_L - V_L^*$ is just positive. Solving **(8)-(10)** with $n_H = 1$ and $n_L = 1$, we get the sustain point denoted as ϕ^S, which is implicitly defined as[9]:

(15) $$\frac{1}{1-\sigma}\ln\phi^S = \frac{1-\phi^S}{\sigma}\left(\frac{\beta}{2\phi^S} - \frac{\beta}{2} - 1\right)\left(\frac{l}{\alpha h + (1-\alpha)l}\right)$$

More firm heterogeneity, represented by an increased gap in costs between L and H firms decreases the sustain point, $\frac{d\phi^S}{dl} < 0$ and $\frac{d\phi^S}{dh} > 0$.[10] This indicates that firm heterogeneity promotes the agglomeration process and works as an agglomeration force.

Result 4: Firm heterogeneity works as agglomeration force. More firm heterogeneity is more likely to create full agglomeration and thus decrease the sustain point.

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[9] In the standard homogeneous-firm model, using (15) and substituting with *h*=*l*, the sustain point is derived from $\frac{1}{1-\sigma}\ln\phi^S = \frac{1-\phi^S}{\sigma}\left(\frac{\beta}{2\phi^S} - \frac{\beta}{2} - 1\right)$.

[10] See Appendix 3 for the proof.

3.5. *Symmetric Equilibrium and Sorting Equilibrium*

Up until the last section we started from a setting with two symmetric regions with extremely high trade costs and then trade costs fall. At the break point symmetry breaks and catastrophic agglomeration of H firms and gradual agglomeration of L firms arises. In contrast this section starts with free trade and full agglomeration in the North, under these conditions L firms start to relocate to the South at the sustain point and more L firms relocate to the South as trade costs are higher. This is the same argument as in the standard economic geography model, however, the equilibrium with high trade costs is not the same (see Figure 1).

Even at the break point H firms never relocate to the South due to the fact that $V_H - V_H^* > 0$, while L-firms maintain a degree of dispersion with $0 < n_L < 1$ to satisfy $V_L - V_L^* = 0$. Unless all L-firms concentrate in the South, the H firms never break away from being fully agglomerated in the Northern market because the real profit gap is strictly positive, $V_H - V_H^* > 0$. In addition, it is impossible to simultaneously satisfy with both equations $V_L - V_L^* = 0$ and $V_H - V_H^* = 0$ as mentioned above. We note that both $V_L - V_L^* = 0$ and $V_H - V_H^* = 0$ are satisfied only at the symmetric equilibrium and that once location patterns become asymmetric between regions both are not simultaneously satisfied. Thus as trade costs are increase n_L falls, helping to maintain the full agglomeration of the H-firms, i.e. $n_H = 1$. This sorting equilibrium is stable even when $\phi^B > \phi$, in other words even when trade costs are sufficiently high the location pattern cannot replicate the symmetric equilibrium because the sorting equilibrium is always stable, i.e. $n_H = 1$ and $0 < n_L < 1$.

In contrast to standard economic geography models we have two stable equilibria with high trade costs, as shown in Figure 1. One is the symmetric equilibrium, in which the two regions are symmetric in terms of the firms located within them: each type of firm locates equally locates across regions. While the symmetric equilibrium is stable with high trade costs it is unstable at the break point as lower trade costs induce agglomeration. The other equilibrium is the sorting equilibrium, one region, which becomes the core, has all the unproductive firms and some of the productive firms, while the other, which becomes the periphery, has some productive firms only. Trade cost reduction reduces the intensity of Northern competition and reduces the gap in market competition, thus allowing more productive firms to locate in the core region and consequently all firms co-agglomerate in core.

4. WELFARE ANALYSIS

This section studies social welfare. The central issue in this paper is firm heterogeneity and the impact on firm location. Thus this section mainly discusses regional welfare gaps and profit gaps between two types of firms and later studies socially optimal welfare levels and discusses optimal firm location patterns.

The indirect quasi-linear utility function for a representative consumer can be specified as $\mu(\ln\mu - 1) + y - \ln P$, where y denotes individual labour or an entrepreneur's income (wages or capital returns). Thus, social welfare in the North and South is given as the aggregation of individual's welfare:

$W = \mu(\ln\mu - 1)(L + K) + Y - (L + K)\ln P$ and $W^* = \mu(\ln\mu - 1)(L^* + K^*) + Y^* - (L^* + K^*)\ln P^*$ where $Y = \beta/2 + \alpha n_H \pi_H + (1-\alpha)n_L\pi_L$ and $Y^* = \beta/2 + \alpha(1-n_H)\pi_H^* + (1-\alpha)(1-n_L)\pi_L^*$.

The social welfare gap between two regions, $W - W^*$, is plotted in Figure 2 (h=1, l=2, σ=4,μ=0.5,α=0.5 and β=5). Below the break point, $\phi^B > \phi$, social welfare in the symmetric equilibrium is invariant across the regions. Northern social welfare relatively increases once the symmetric equilibrium breaks and asymmetric equilibrium arises because the North attracts more firms resulting in an increasing population due to increased numbers of entrepreneurs and a better cost of living due to there being more L-firms. Once full agglomeration arises the welfare gap is a function of ϕ, given as $W - W^* = \gamma(1+\beta) + \dfrac{\mu}{1-\sigma}\ln\phi$, in which the second term indicates the regional difference in the cost of living effect. As trade costs fall under full agglomeration, the welfare gap declines due to the improvement of the cost of living in the South.

Figure 2: Regional social welfare gap.

The nominal profit gap between L and H firms $\pi_L - \pi_H$ is plotted in Figure 3 (h=1, l=2, σ=4, μ=0.5,α=0.5 and β=5). The profit gap in the sorting equilibrium initially decreases, then increases as trade costs fall. Above the sustain point, the gap remains constant due to the full agglomeration of firms in the North, hence $\pi_L - \pi_H = \gamma\left(\dfrac{\beta+1}{\alpha h + (1-\alpha)l}\right)(l-h)$. Under gradual agglomeration all H firms concentrate in the North and then more L firms relocate to the North with a rise of Δ. Meanwhile market potential in the North becomes lower due to the smaller increase in *E* relative to the larger increase in Δ. The decreased market potential in the North results in a lowering of nominal profits in the case of gradual agglomeration. Due to the type difference, i.e. *h* and *l*, lowering profit is larger in L firms than in H firms. This is why the profit gap between firm types in Figure 3 first declines in the sorting equilibrium, however as trade costs fall, the gap between Δ and Δ* decreases due to better market access, which moderates the rise of Δ caused by L-firm migration and this raises Northern market potential because of keeping the rise in *E*. This causes the turn from a fall to a rise in the profit gap due to trade cost reduction.

Figure 3: Profit gap between L and H firms.

We now evaluate whether our market equilibrium is optimal in terms of global welfare. To discuss socially optimal welfare, we suppose a social planner can choose the location of each type of firm (i.e. they set n_L and n_H) in order to maximise welfare across the two countries.[11] Figures 4 provides a 3-D plot of the worldwide welfare (plotted on the vertical axes) in terms of n_L and n_H (the two horizontal axes) with high trade costs (ϕ=0.3) and small trade costs (ϕ=0.8), using parameter values, h=1, l=2, σ=4, μ=0.5, α=0.5 and β=3. The global welfare is given as the sum of two regional social welfares: i.e. $W^W \equiv W + W^*$. As seen in Figure 4-a, the global welfare appears saddle-shaped with small trade costs and saddle points at $n_H = 1$ and $0 < n_L < 1$ is higher than any symmetric equilibrium. The maximum welfare point is at $n_H = 1$ and $n_L = 0.5282$ (or $n_H = 0$ and $n_L = 0.4717$ in the case where H firms first deviate to the South).Hence we conclude that the sorting equilibrium dominates the symmetric equilibrium. When firms experience higher trade costs we see that the saddle shape is retained in Figure 4-b, this tells us that $n_H = 1$ and $n_L = 1$ (or $n_H = 0$ and $n_L = 0$ in the transposed case) yield the maximum worldwide welfare, which is higher than that achieved at the symmetric equilibrium, on the other hand $n_H = 1$ and $n_L = 0$ (or $n_H = 0$ and $n_L = 1$ in the transposed case) yields the minimum total welfare. The symmetric equilibrium is thus proved to not represent maxima in

[11] As discussed in Baldwin et al. (2003, Ch.11), the first-best outcome is to exclude all distortion. The planner imposes prices equal to marginal costs and decides firm distributions with a lump-sum transfer from consumers to firms. The second-best outcome is that firms are free to set prices to maximise their profits. Then the social planner chooses firm distribution. In our model, the first and second social optimal equilibrium results in an identical firm distribution, because quasi-linear utility function excludes an income effect, as discussed in Pfluger and Sudekum (2008).

terms of global welfare. The existence of the saddle shape in Figures 3 and 4 is mathematically proved with the Hessian being given as $\Omega(n_H, n_L) \equiv \frac{\partial^2 W^W}{\partial n_H^2}\frac{\partial^2 W^W}{\partial n_L^2} - \left(\frac{\partial^2 W^W}{\partial n_H \partial n_L}\right)^2$. At the symmetric equilibrium $n_L = n_H = 0.5$, $\frac{\partial W^W}{\partial n_H} = 0$ $\frac{\partial W^W}{\partial n_L} = 0$ yields

$\Omega(n_H, n_L) = -\frac{16\alpha^2(1-\alpha)^2\mu^2(l-h)^2(1-\phi)^2}{(\sigma-1)^2(\alpha h+(1-\alpha)l)^2(1+\phi)^2} < 0$. This means that the worldwide function must always be saddle-shaped and the symmetric equilibrium will not be associated with the maximum level of worldwide welfare. Our saddle shape indicates that the socially optimal firm shares are always $n_H = 1$ (or $n_H = 0$).

Thus as shown in Figure 4, the sorting equilibrium, $n_H = 1$ (or $n_H = 0$) and $0 < n_L < 1$ with high trade costs and full agglomeration, $n_H = 1$ and $n_L = 1$ (or $n_H = 0$ and $n_L = 0$) with small trade costs can maximise global welfare. We thus conclude that the symmetric equilibrium is not optimal in terms of maximising worldwide welfare in wither case.

Figure 4-a and Figure 4-b: Social welfare.

We now turn our attention to the sorting equilibrium and its associated socially optimal firm shares which maximise worldwide welfare, this is denoted by $\tilde{n}_L$ and $\tilde{n}_H$, compared with the market equilibrium of n_L and n_H. Figure 5 plots $\tilde{n}_L$ and $\tilde{n}_H$ (h=1, l=2, σ=4, μ=0.5, α=0.5 and β=3) in terms of ϕ. As discussed in previous sections n_H=1 is always a market equilibrium (Section 3) and $\tilde{n}_H$=1 is socially optimal (see Figure 4). As a result of maximising worldwide welfare the socially optimal $\tilde{n}_L$ is always smaller than the market equilibrium and the sustain point is higher than under the market outcome. This indicates that the market sorting equilibrium involves more agglomeration than is socially optimal. The concentration of all H firms in the North creates larger demand due to the concentration of entrepreneurs associated with H firms but lower competition due to their low productivity and higher supply price, which accommodates L firms' location in the North. On the other hand, workers are immobile and are equally distributed across the two regions. This causes there to be too much agglomeration in terms of the socially optimal outcome. For this reason, the market outcome has too many firms in the North, unless full agglomeration occurs. As total entrepreneurs/capital relatively decreases and the total labour increases, this diminishes the discrepancy between the socially optimal and market driven outcomes.

Figure 5: Social optimal firm distribution.

Result 5: The sorting equilibrium would improve global social welfare compared to the symmetric equilibrium. However, the sorting equilibrium is not socially optimal as the socially optimal sorting equilibrium involves less agglomeration than in the market outcome, or more simply the market outcome involves too much agglomeration.

5. CONCLUSIONS

This paper studies the impact of firm heterogeneity on location patterns and constructs an FE model with firm heterogeneity. In contrast to other firm heterogeneity models, this paper takes into account labour (entrepreneur) migration, which creates an expenditure shift from one region to the other. We showed under spatial sorting that the severe competition in the core would be more likely to lead productive firms to locate in the periphery, whereas all unproductive firms concentrate their production in the core.

In this setting we encounter several results which contrast with other firm heterogeneity models. First, spatial sorting occurs, in which low cost/high productivity firms locate in both regions, while high cost/unproductive firms concentrate in a single region. Productive firms are less likely to co-agglomerate due to their self-induced competition and are more likely to diversify their location. Secondly, firm heterogeneity works as an agglomeration force, which decreases the break and sustain points. Thirdly, the symmetric equilibrium is not optimal in terms of worldwide welfare and the sorting equilibrium can improve welfare. However, the sorting equilibrium involves too much agglomeration and is not socially optimal.

These results might provide some explanation of the current empirical evidence. For example, small firms in some sectors are more likely to concentrate in one region (Duranton and Overman, 2005). Big Chinese cities witness lower real wage rates than intermediate sized cities, which might be due to low productivity firms' location (Au and Henderson, 2005).

A possible extension to this work is to incorporate the heterogeneous labour literature (Tabuchi and Thisse, 2002) or to add more urban issues (e.g. land rent) or some public policies such as corporate tax and subsidy to the model presented here. Further extension might include a continuum of firm types.[12]

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APPENDIX 1. BREAK POINT

Solving $\left.\frac{\partial(V_H - V_H^*)}{\partial n_H}\right|_{n_L=0.5} = 0$ and $\left.\frac{\partial(V_L - V_L^*)}{\partial n_L}\right|_{n_H=0.5} = 0$ in terms of ϕ, we get

$$\phi_H^B = \frac{(\sigma-1)(1+\beta)h-(2\sigma-1)(\alpha h+(1-\alpha)l)}{(\sigma-1)(1+\beta)h+(2\sigma-1)(\alpha h+(1-\alpha)l)} \text{ and } \phi_L^B = \frac{(\sigma-1)(1+\beta)l-(2\sigma-1)(\alpha h+(1-\alpha)l)}{(\sigma-1)(1+\beta)l+(2\sigma-1)(\alpha h+(1-\alpha)l)}$$

respectively. Taking differences between these yields,

$$\phi_H^B - \phi_L^B = \frac{2(2\sigma-1)(\sigma-1)(1+\beta)(h-l)(\alpha h+(1-\alpha)l)}{\{(\sigma-1)(1+\beta)h+(2\sigma-1)(\alpha h+(1-\alpha)l)\}\{(\sigma-1)(1+\beta)l+(2\sigma-1)(\alpha h+(1-\alpha)l)\}} < 0 .$$

ϕ_H^B is always smaller than ϕ_L^B. Thus we adopt ϕ_H^B as break point.

APPENDIX 2. LOCATION PATTERN IN THE LONG-RUN EQUILIBRIUM

Case 1) $V_H - V_H^* = 0$ and $V_L - V_L^* < 0$ then $0 < n_H < 1$ and $n_L = 0$

Using $n_L = 0$, we can immediately derive $\Delta - \Delta^* = (1-\phi)(\alpha(2n_H - 1)h - (1-\alpha)l) < 0$.

Hence $\Delta < \Delta^*$. The price index effect, $-\frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*)$, is negative. To satisfy $V_H - V_H^* = 0$, the profit gaps $\gamma(B-B^*)l$ and $\gamma(B-B^*)h$ should be positive. Due to $\gamma(B-B^*)l > \gamma(B-B^*)h$, if $V_H - V_H^* = 0$, then $V_L - V_L^* > 0$ should also hold. This represents a contradiction, hence we conclude that this case will not occur.

Case 2) $V_H - V_H^* > 0$ and $V_L - V_L^* < 0$ then $n_H = 1$ and $n_L = 0$

Using $n_H = 1$ and $n_L = 0$, $\Delta - \Delta^* = (1-\phi)(\alpha h - (1-\alpha)l) < 0$. The price index effect, $-\frac{\mu}{1-\sigma}(\ln\Delta - \ln\Delta^*)$, is negative. Suppose that $V_H - V_H^* > 0$, the profit gaps $\gamma(B - B^*)l$ and $\gamma(B - B^*)h$ should be both positive. Due to $\gamma(B - B^*)l > \gamma(B - B^*)h > 0$, if $V_H - V_H^* > 0$, then $V_L - V_L^* > 0$ should also hold. This is a contradiction, hence we conclude that this case does not occur.

Case 3) $V_H - V_H^* > 0$ and $V_L - V_L^* = 0$ then $n_H = 1$ and $0 < n_L < 1$.

n_L is a decreasing function in $V_L[n_L] - V_L^*[n_L]$. Suppose that $\alpha h + (1-\alpha)(2n_L - 1)l = 0$. Then n_L is given as $\underline{n}_L = \frac{1}{2} - \frac{\alpha h}{2(1-\alpha)l}$. $V_L[\underline{n}_L] - V_L^*[\underline{n}_L] = \frac{2(1-\phi)(l-h)\alpha\mu}{(1+\phi)(\alpha h + (1-\alpha)l)\sigma} > 0$. This indicates that the long-run equilibrium, to satisfy $V_L - V_L^* = 0$, should always be more than $\underline{n}_L$, i.e. $n_L > \underline{n}_L$. Thus $\alpha h + (1-\alpha)(2n_L - 1)l > 0$ holds in the equilibrium, resulting in $\Delta > \Delta^*$. Using this relationship, since we discuss the case of a substantial cost gap, i.e. *l* is sufficiently larger than *h*, the gap between Δ and Δ^* is large and thus $B - B^* = -\frac{2(1-\alpha)(1-n_L)(\Delta+\Delta^*) + \beta(\Delta-\Delta^*) - 2\Delta^*}{2\Delta\Delta^*} < 0$ and $V_H - V_H^* > V_L - V_L^* = 0$. We conclude that this case could occur.

APPENDIX 3. FIRM HETEROGENEITY IMPACT

First we define $\chi \equiv \frac{l}{\alpha h + (1-\alpha)l} > 1$. Since we assume a substantial firm heterogeneity, χ should have a large value in our paper. Then we obtain $\frac{d\chi}{dl} = \frac{\alpha h}{(\alpha h + (1-\alpha)l)^2} > 0$, $\frac{d\chi}{dh} = -\frac{\alpha l}{(\alpha h + (1-\alpha)l)^2} < 0$.

When firm heterogeneity is substantial, i.e. there is a larger χ, using (15), we can derive $\frac{d\phi^S}{d\chi} = \frac{-2\sigma\phi^S - \chi(\sigma-1)((\phi^S)^2(2+\beta) - \beta)}{\phi^S(\sigma-1)(1-\phi^S)(\beta\phi^S + 2\phi^S - \beta)} < 0$, because $\phi^S < \frac{\beta}{\beta+2} < \sqrt{\frac{\beta}{\beta+2}}$ should be satisfied in order for the sustain point to exist, resulting in $(\phi^S)^2 < \frac{\beta}{\beta+2} \Leftrightarrow (\phi^S)^2(\beta+2) - \beta < 0$ and we also obtain $\beta\phi^S + 2\phi^S - \beta < 0$. Thus, $\frac{d\phi^S}{dl} < 0$ and $\frac{d\phi^S}{dh} > 0$, hence more firm heterogeneity, i.e. increased an *l* and/or decreased *h* leads to a decrease the sustain point.

APPENDIX 4. THREE FIRM TYPE MODEL

Our model can be extended to include more types of firms. Here we add a third type of firm, intermediate cost/productivity firms (so-called M-firm). For simplicity, we assume the number of each type of firms is equal. In this framework we maintain our key results. Solving $\left.\frac{\partial(V_H - V_H^*)}{\partial n_H}\right|_{n_L=0.5} = 0$, $\left.\frac{\partial(V_M - V_M^*)}{\partial n_M}\right|_{n_M=0.5} = 0$ and $\left.\frac{\partial(V_L - V_L^*)}{\partial n_L}\right|_{n_H=0.5} = 0$ and choosing the smallest one is the break point. As a result, the first movers are H firms. The break point is $\phi^B = \frac{(\sigma-1)(1+\beta)h - \frac{1}{3}(2\sigma-1)(h+l+m)}{(\sigma-1)(1+\beta)h + \frac{1}{3}(2\sigma-1)(h+l+m)}$. Above the break point, spatial sorting occurs. The long-run equilibrium is derived either from Case 1) $V_H - V_H^* > 0$, $V_M - V_M^* > 0$ and $V_L - V_L^* = 0$ or from Case 2) $V_H - V_H^* > 0$, $V_M - V_M^* = 0$ and $V_L - V_L^* < 0$. The first case is likely to occur when *h* and *m* are close. Once the trade costs are at the break point, all H and M firms immediately concentrate in the North, while L firms gradually relocate to the North due to trade cost reduction. This is parallel to the two firm type case discussed in the main text. The second case is likely to occur when *m* is large and close to *l*, that is $n_H = 1$ $0 < n_M < 1$ and $n_L = 0$ above the break point. In this case n_M increases as trade costs fall and all M firms concentrate in the North under low trade costs. Then, as in the first case, L firms gradually move to the North as trade costs fall.

In Case 2, since the North represents less severe competition due to the agglomeration of H firms and the South has more competition due to the agglomeration of L firms, more M firms locate in the North. As trade costs fall, the degree of competition becomes closer and the demand size becomes much more important. Since the North has a higher population the North attracts more M firms. Then M firms agglomerates in the North while, as in Case 1, L firms gradually move to the North.

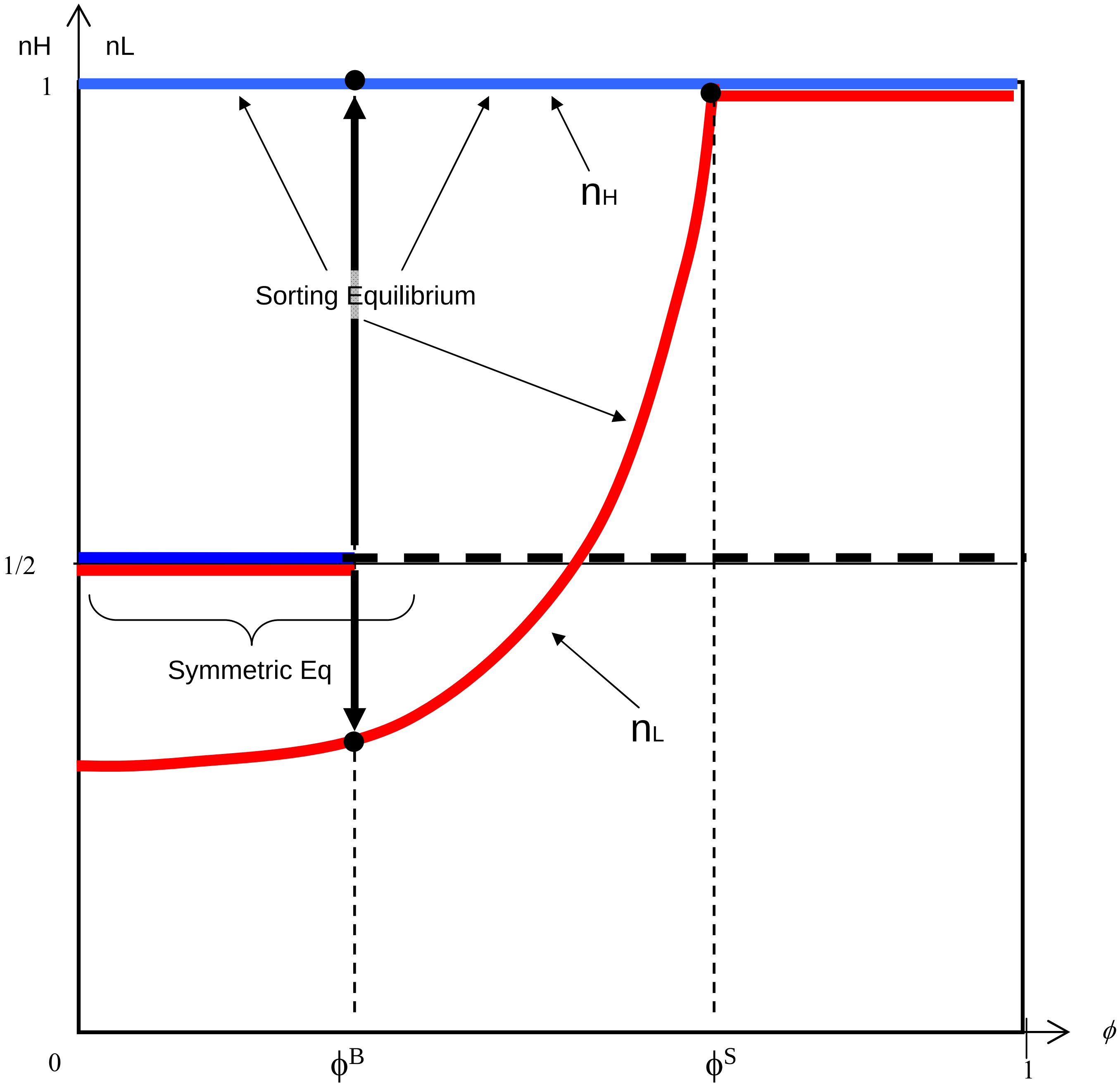

Figure 1: Equilibrium in terms of trade costs

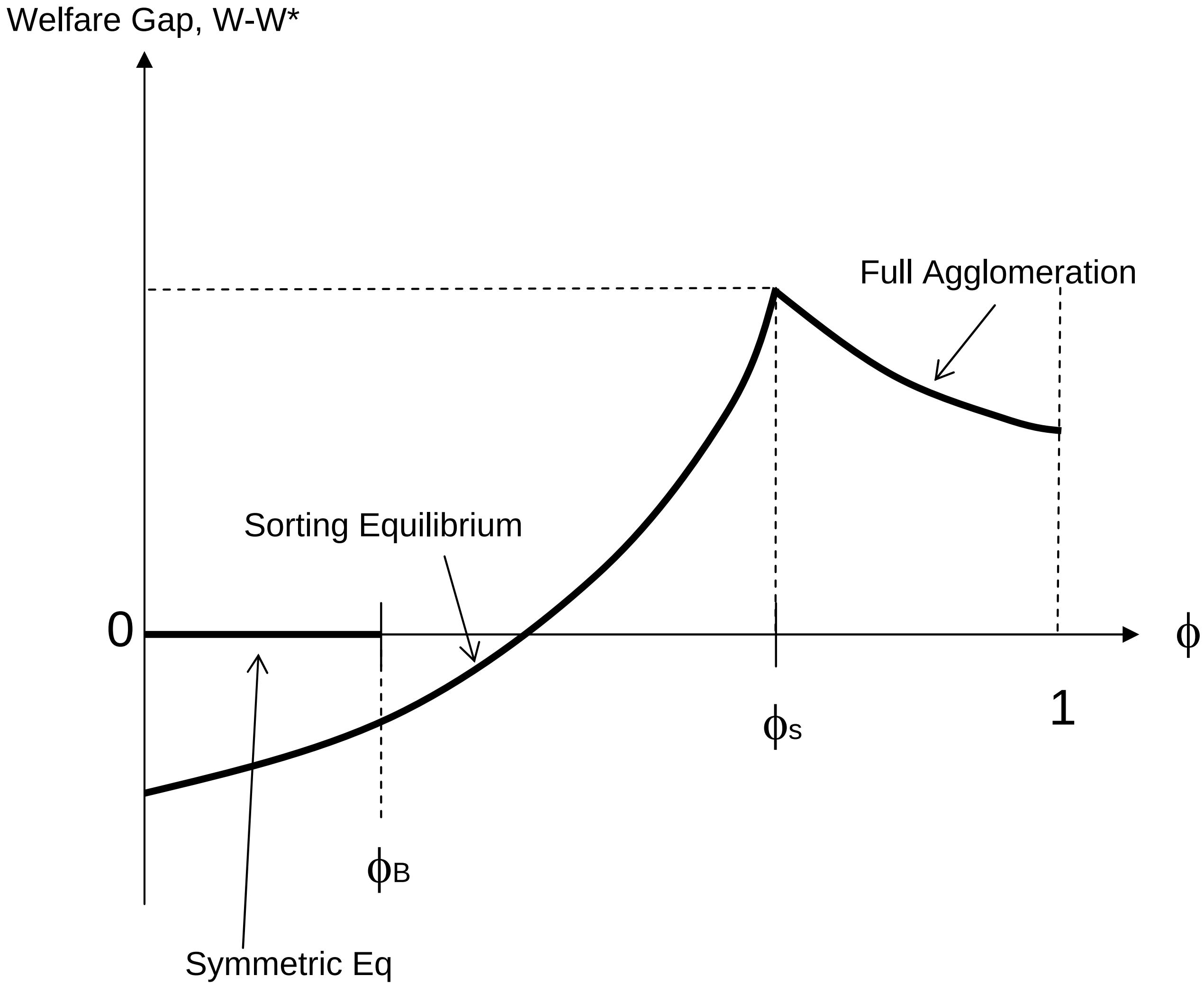

Figure 2: Regional social welfare gap

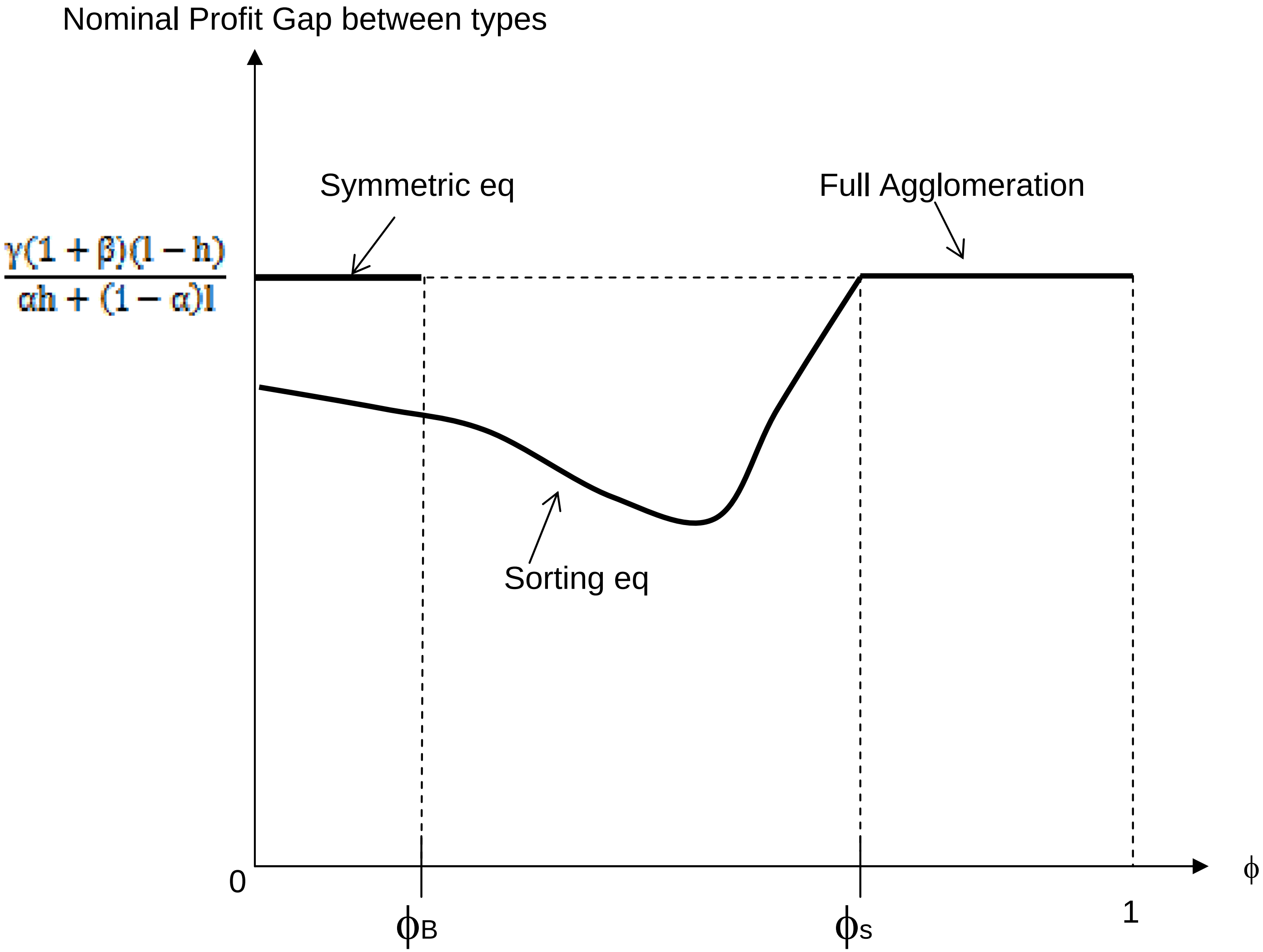

Figure 3: Profit gap between L and H firms

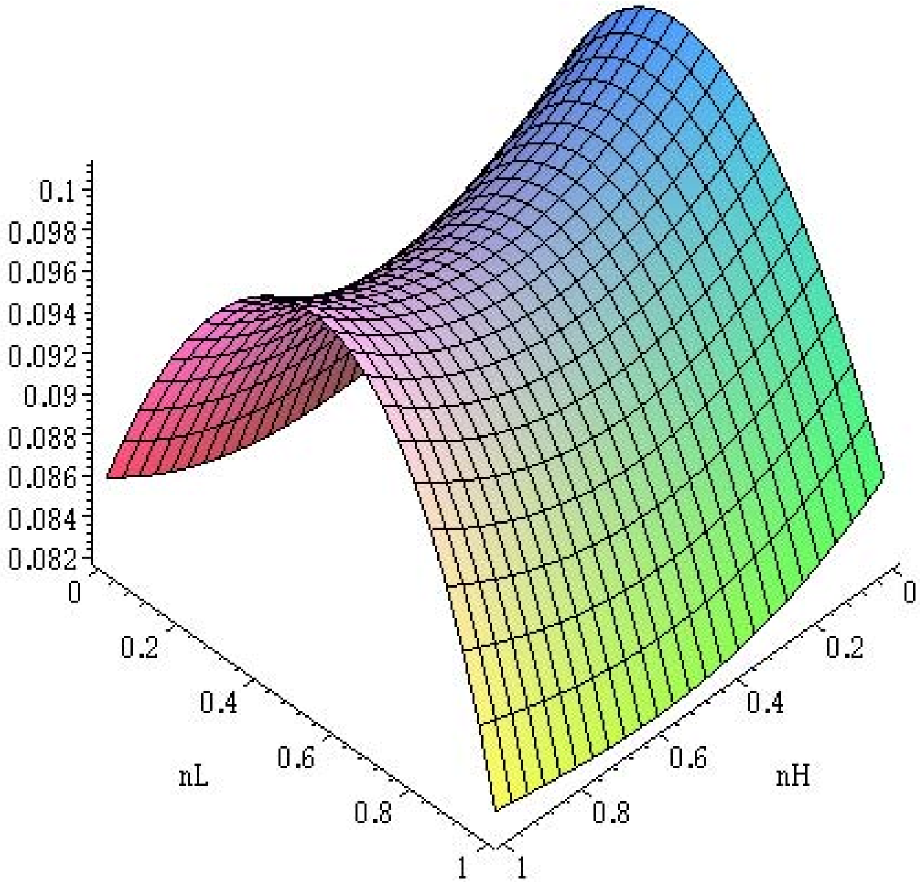

Figure 4-a Social welfare.

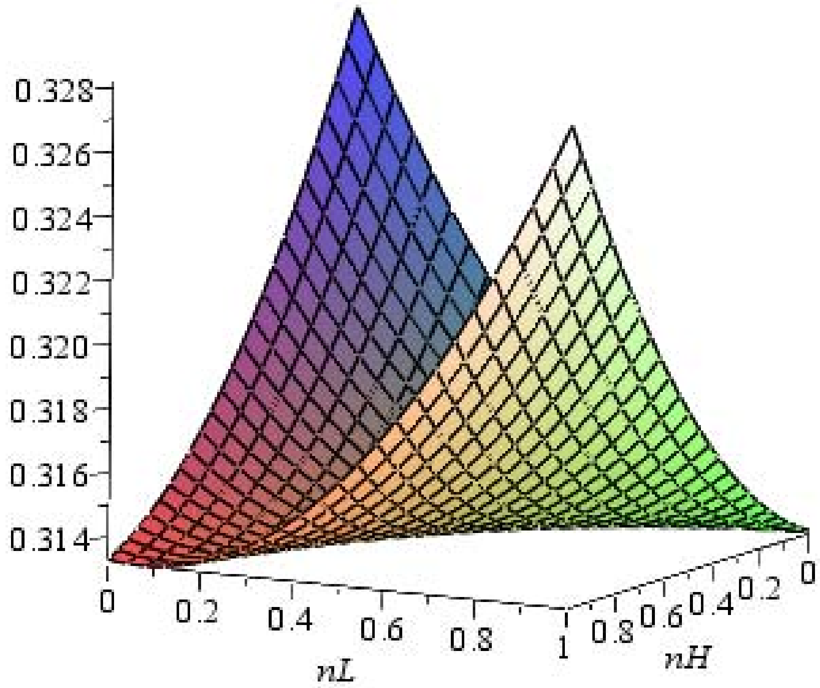

Figure 4-b: Social welfare

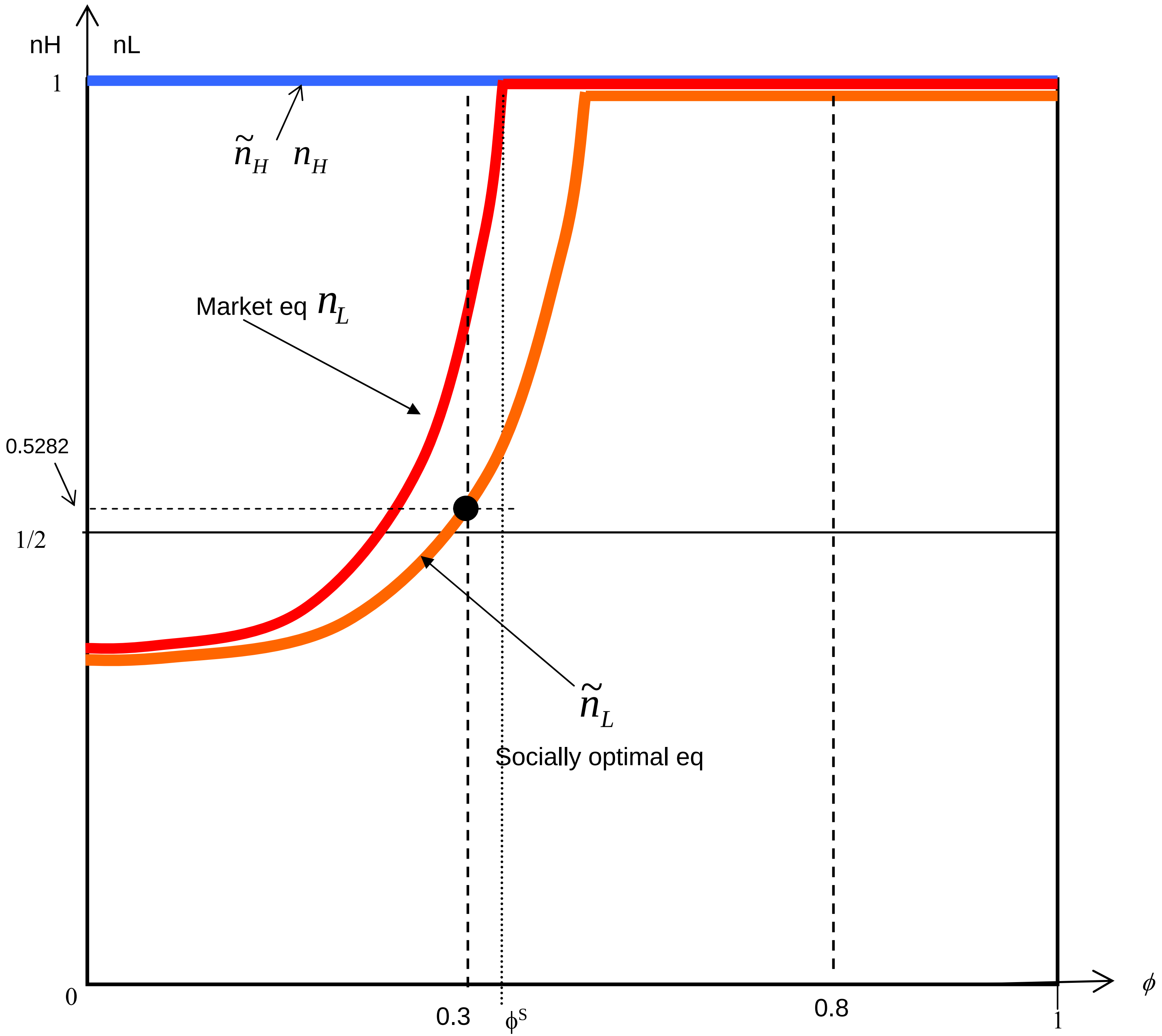

Figure 5: Social optimal firm distribution