An Assignment Theory of Foreign Direct Investment*

Volker Nocke†  Stephen Yeaple‡
University of Pennsylvania  University of Pennsylvania and NBER

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Abstract

We develop an assignment theory of foreign direct investment (FDI) in which firms conduct FDI by either engaging in greenfield investment or in cross-border acquisitions. Cross-border acquisitions involve firms trading heterogeneous corporate assets to exploit complementarities, while greenfield FDI involves building a new plant in the foreign market. In equilibrium, greenfield FDI and cross-border acquisitions co-exist, but the composition of FDI between these modes varies with firm and country characteristics. Firms engaging in greenfield investment are systematically more efficient than those engaging in cross-border acquisitions. Furthermore, most FDI takes the form of cross-border acquisitions when factor price differences between countries are small, while greenfield investment plays a more important role for FDI from high-wage into low-wage countries. These results capture important features of the data.

JEL Codes: F12, F23.

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†Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia PA 19104, USA. Email: nocke@econ.upenn.edu. URL: www.econ.upenn.edu/~nocke/.

‡Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia PA 19104, USA. Email: snyeapl2@ssc.upenn.edu. URL: www.econ.upenn.edu/~snyeapl2/.
1 Introduction

Multinational enterprises (MNEs) play a dominant role in an increasingly globalized world. In 1999, the domestic operations of the approximately 2,400 U.S. multinational enterprises accounted for approximately 26 percent of U.S. GDP, 63 percent of U.S. exports, 37 percent of U.S. imports, and 68 percent of U.S. R&D expenditures (Mataloni and Yorganson, 2002). That year, nearly half of all U.S. manufacturing workers were employed by U.S. multinationals.

Despite their economic importance, the investment decisions of multinationals are not yet well understood. With few exceptions, the trade literature has not distinguished between the two modes in which a multinational enterprise can engage in foreign direct investment (FDI): cross-border acquisition (entering a foreign market by buying an existing enterprise) and greenfield FDI (entering a foreign market by building a new enterprise). Consequently, the literature has been preoccupied with understanding the volume of FDI, neglecting its composition across modes.

To the extent that MNEs do not view cross-border acquisitions and greenfield FDI as perfect substitutes, trade economists and policy makers should care not only about the volume of FDI, but also about its composition. Indeed, governments of many host countries perceive cross-border acquisitions as being rather different from greenfield FDI. Any change in policies towards FDI and any variation in country characteristics is likely to affect cross-border acquisitions and greenfield FDI differently. Therefore, the volume of FDI cannot fully be understood without first understanding its composition.

If our hypothesis that cross-border acquisitions and greenfield FDI are not perfect substitutes is correct, then this should show up in the data: there should be systematic differences in FDI mode choice across MNEs. Indeed, if the two FDI modes were perfect substitutes, then all firms would be indifferent between the two modes, and so there should be no systematic variation in mode choice across firms within the same industry. In figure 1, we explore the relationship between a U.S. multinational’s efficiency and its propensity to favor cross-border acquisitions over greenfield FDI. This figure is constructed using data from the Bureau of Economic Analysis (BEA) on outward FDI of U.S. multinational enterprises over the period 1994-1998. From this dataset, we construct two measures of a U.S. multinational’s efficiency: (i) the parent company’s sales in the U.S. (relative to its industry mean), and (ii) the parent company’s value added per worker in the U.S. (relative to its industry mean). Based on each measure of firm efficiency, we sort MNE’s into three equally-sized bins: “low”, “medium”, and “high” efficiency. The figure reveals that firms are less likely to choose cross-border acquisitions over greenfield FDI the more efficient they are. The fact that FDI mode choice varies systematically with firm characteristics shows that cross-border acquisitions and greenfield FDI are not perfect substitutes.

FDI mode choice varies not only across firms, but also across host countries with different

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1 See, for example, United Nations Center for Transnational Corporations (2000, p. xxiii).
2 Most models of firm heterogeneity predict a monotonic relationship between firm efficiency and sales.
3 As the econometric analysis in the appendix shows, this relationship is robust to including various control variables that might affect FDI mode choice. Additional information concerning the construction of the data is also provided in the appendix.
Figure 1: The Share of Cross-Border Acquisitions in Total U.S. Outward FDI by Firm Efficiency

Figure 2: The Share of Cross-Border Acquisitions in Total U.S. Outward FDI by the Host Country’s Level of Development
levels of development. In Figure 2, we explore the relationship between a host country’s level of development, as measured by GDP per capita and average wage rate, and the fraction of U.S. multinationals that enter this country through cross-border acquisition rather than greenfield FDI. As the figure reveals, U.S. multinationals are more likely to favor cross-border acquisition over greenfield FDI, the more developed is the host country. Interestingly, more than sixty percent of U.S. outward FDI is directed towards the group of countries with the highest level of development, which is also the group with the highest fraction of cross-border acquisition in total FDI. This suggests that an understanding of the composition of FDI across modes will provide us with a deeper understanding of the volume of FDI.4

In this paper, we develop an assignment theory of foreign direct investment to explain multinationals’ FDI mode choice. According to much of the business literature, acquisitions allow firms to exploit complementarities in their firm-specific assets. This view of mergers and acquisitions is in line with the fact that approximately half of all mergers and acquisitions in the U.S. involve trade in individual plants and divisions rather than entire corporations; see Maksimovic and Phillips (2001).5 These observations show that firms are in the business of buying and selling corporate assets, and that these assets are heterogeneous and complementary. Indeed, as Caves (1998; p. 1963) points out:

“For the reshuffling of plants (or lines of business) among firms to be productive, there must be sources of heterogeneity. [...] These heterogeneities cause assets’ productivities to vary substantially depending on the other business assets with which they collaborate within the firm.”

We therefore model the merger market as a market in which heterogeneous firms buy and sell heterogeneous firm-specific assets to exploit complementarities. The equilibrium of the merger market is the solution of the associated assignment problem: which firm-specific assets should optimally be combined?

In our model, a cross-border acquisition involves purchasing foreign corporate assets that the acquirer lacks. Greenfield FDI, on the other hand, involves building production capacity in the foreign country to allow the firm to deploy its corporate assets abroad. There are two countries that can freely trade with one another. Motivated by our empirical finding that the host country’s level of development is an important determinant of FDI mode choice, we assume that countries differ in factor prices and in the distributions of entrepreneurial abilities (or some other corporate assets). Factor price differences give rise to greenfield FDI (from the high cost to the low cost country) and to cross-border acquisitions (from each country to the other). Cross-country differences in entrepreneurial abilities, however, give rise only to cross-border acquisitions (from each country to the other). Our model thus generates (potentially large) two-way FDI flows even in the absence of transport costs and factor price differences between countries.

In our model, greenfield FDI and cross-border acquisitions co-exist. However, as factor price differences become small, almost all FDI takes the form of cross-border acquisitions. This

4See the appendix for an econometric analysis and a description of the data.
5As Bernard, Redding, and Schott (2004) document, more than sixty percent of U.S. manufacturing firms add or delete entire four-digit product lines within five years.
prediction is consistent with figure 2 since factor price differences between the U.S. and rich, developed countries are arguably much smaller than between the U.S. and poor, developing countries. We also show that the propensity of firms in the high-cost country to engage in cross-border acquisitions rather than greenfield FDI is decreasing in the relative supply of corporate assets in the low-wage country. To the extent that poor, developing countries have fewer attractive corporate assets than rich, developed countries, this result is also consistent with figure 2.

Another prediction of our model is that firms engaging in greenfield FDI are, on average, more efficient than those engaging in cross-border acquisitions. This is consistent with the data summarized in figure 1. The intuition for this result may be summarized as follows. Greenfield FDI necessitates the expense of building a new plant in the foreign country, and such an expense is worthwhile only if the gains from relocating production are sufficiently large. Hence, only sufficiently productive firms will engage in greenfield FDI. In contrast, the market for corporate assets allows even relatively inefficient firms to exploit complementarities.

In our model, foreign direct investment has important implications for aggregate productivity. The existence of an international market for corporate assets allows for a more efficient global assignment of firm-specific assets. However, while the re-assignment of corporate assets improves the distribution of firm efficiencies in the low cost country, it has the opposite effect in the high cost country.

Related literature. Our paper is mainly related to two strands in the theoretical trade literature. A feature common to both strands, and shared by our paper, is the assumption that contracting problems prevent arm’s-length relationships. In our model, FDI arises because of underlying differences across countries, not because of transport costs, and there is a tendency for each firm to locate production in only one country. It is this feature that our paper shares with the literature on “vertical FDI” (e.g., Helpman, 1984, and Neary, 2004). Indeed, recent empirical work by Hanson, Mataloni, and Slaughter (2003) documents a tendency for multinationals to concentrate production in low-wage countries. None of the papers on vertical FDI consider FDI mode choice. In Helpman (1984), there is only greenfield FDI, while in Neary (2004), there are only cross-border acquisitions (motivated by market power).

Our paper is also related to the recent and growing literature on firm heterogeneity which is concerned with the selection of heterogeneous firms into different modes of serving global markets. We extend this literature by introducing an international merger market. Within this literature, the two papers that are most closely related to ours are Helpman, Melitz, and Yeaple (2004), and Nocke and Yeaple (2004). However, in Helpman, Melitz, and Yeaple (2004) there is no motive for firms to engage in cross-border acquisitions, and so greenfield is the only mode of FDI. On the other hand, Nocke and Yeaple (2004) consider cross-border acquisitions

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6There is, however, an interesting recent (but so far empirically untested) literature that explores the trade-offs between in-house production and outsourcing; see Antras (2003), Antras and Helpman (2004), and Grossman and Helpman (2004).

7Other papers in the literature on firm heterogeneity include Melitz (2003), Bernard, Eaton, Jensen, and Kortum (2003), and Eaton, Kortum, and Kramarz (2004).
but analyze the interaction between trade costs (which are absent in the present model) and the source of firm heterogeneity (mobile versus non-mobile capabilities). In the present paper, we are able to analyze general heterogeneity in all corporate assets, which is precluded by the presence of trade costs in Nocke and Yeaple (2004). Another benefit of abstracting from trade costs in the present paper is that it allows us to analyze large country differences.

Plan of the paper. In the next section, we present the model. Then, in section 3, we derive the equilibrium assignment of corporate assets and the equilibrium location of production. Further, we explore the implications of the assignment for the distribution of firm efficiencies across countries. In section 4, we solve for the FDI flows implied by the equilibrium assignment and location decisions. We show that greenfield FDI and cross-border acquisitions co-exist but that, in the limit as factor price differences vanish, all FDI takes the form of cross-border acquisitions. Next, in section 5, we analyze the link between firm characteristics and FDI mode choice. We show that firms engaging in greenfield FDI are systematically more efficient than those engaging in cross-border acquisitions. Then, in section 5, we investigate the relationship between country and industry characteristics on the one hand, and the composition of foreign direct investment on the other. Finally, we conclude in section 7.

2 The Model

We consider a model of the world economy. There are two countries, 1 and 2, indexed by \( i \), that can freely trade with one another. There are no transport costs or tariffs, and so the price of each good is the same in both countries.

Consumers. The world is populated by a unit mass of consumers with identical CES preferences. The representative consumer’s utility function is given by

\[
U = \ln \left( \int m(\zeta)^{1-\rho} x(\zeta)^{\rho} d\zeta \right)^{\frac{1}{\rho}} + y, \quad \rho = \frac{\sigma - 1}{\sigma}, \quad \sigma > 1,
\]

where \( x(\zeta) \) is the consumption of variety \( \zeta \), \( m(\zeta) \) is the variety-specific mass appeal, and \( y \) the consumption of the homogeneous good. We assume that the representative consumer’s income is sufficiently large so that, in equilibrium, she will consume a positive quantity of the homogeneous good.

Firms. A firm is defined as a triplet \((\tilde{a}, m, i)\), consisting of one (unique) entrepreneur of ability \( \tilde{a} \), a (unique) variety of mass appeal \( m \), and a plant to produce that variety in country \( i \). Each firm can produce (at most) one variety, using entrepreneurial ability \( \tilde{a} \) and labor. An entrepreneur’s ability \( \tilde{a} \) may be thought of as productivity-enhancing headquarter services that can be provided independently of the location of production but only within the firm. In particular, firm \((\tilde{a}, m, i)\) can produce any one unit of its variety using \( 1/\tilde{a} \) units of labor in

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8The variety-specific mass appeal \( m(\zeta) \) should not be confused with intrinsic quality. What we have in mind here is, rather, perceived quality after adjusting for input intensity: a Hyundai arguably is a lower-quality car than a Rolls-Royce but it is considerably cheaper to produce and, therefore, has a greater market share. For expositional simplicity, we abstract from variety-specific input requirements.
country $i$. Hence, the firm’s unit cost of production is given by $\omega_i / \bar{a}$, where $\omega_i$ denotes the wage rate in country $i$. For notational convenience, it will be helpful to use the following (monotone) transform of the entrepreneur’s ability: $a \equiv \bar{a}^{\sigma - 1}$.

A plant is variety-specific in the sense that it must be designed for a specific variety; it has zero value in any other usage. Building a (variety-specific) plant in country $i$ requires $\phi$ units of labor from country $i$.

**Countries.** In each country, producers of the differentiated product face a perfectly elastic supply of labor. The wage rate in country 1 is higher than that in country 2, $\omega_1 > \omega_2$. This assumption can be derived as an equilibrium outcome in a general equilibrium model, where the homogeneous good is produced in both countries by perfectly competitive firms using labor, and firms in country 1 use a more efficient technology in the production of the homogeneous good than firms in country 2.\(^9\) For notational convenience, we normalize the wage in country 2 to one, $\omega_2 \equiv 1$.

In country $i$, there is a mass $E_i$ of atomless producers of the differentiated good. Each firm in country $i$ is endowed with (i) an entrepreneur, (ii) property rights over a unique variety, and (iii) a plant to produce that variety in country $i$. Entrepreneurs and varieties are in fixed supply.

In each country, the empirical distribution function of the varieties’ mass appeal $m$ in the endowments of the population of firms is $H$. We allow for cross-country differences in the distribution of endowments with entrepreneurial abilities. In country $i$, the empirical distribution function of entrepreneurial ability $a$ in the endowments of the population of firms is $G_i$. The world distribution of entrepreneurial ability is then $G \equiv (E_1 G_1 + E_2 G_2) / (E_1 + E_2)$. All distribution functions are continuously differentiable and strictly increasing on $[0, \infty)$. The associated density functions are denoted by $h$, $g_i$, and $g$, respectively. Our formulation does not require us to make any further assumption on the joint distribution of $m$ and $a$ across firms.

**Merger market.** There exists a perfectly competitive global market for corporate assets in which entrepreneurs (firms) can buy and sell varieties and the associated variety-specific plants. Let $W_i(m)$ denote the market price of a variety originating in country $i$ (and its associated plant in country $i$), and $V(a)$ the shadow value of an entrepreneur with ability $a$.

**Foreign direct investment.** There are two modes of foreign direct investment: greenfield FDI and cross-border acquisition. A firm (or entrepreneur) from country $i$ engages in greenfield FDI if (i) it manages a variety that originated in country $i$, and (ii) builds a new plant for that variety in country $j \neq i$.\(^{10}\) Note, however, that a firm engaging in greenfield FDI may not necessarily produce the variety with which it was initially endowed (as the firm may have acquired the variety from some other domestic firm). A firm (or entrepreneur) from country $i$ engages in a cross-border acquisition if it acquires a variety (and its associated plant) in the other country $j$.

\(^9\)Equivalently, we could assume that there are no wage differences, but workers in country 2 are more productive than those in country 1 when producing the differentiated good.

\(^{10}\)A firm that has conducted greenfield FDI will have a variety-specific plant in both countries, but – since product markets are perfectly integrated – will only produce in one of them, namely in the plant located in the low-wage country 2.
3 Equilibrium Assignment

In this section, we turn to the equilibrium analysis of our model. We first derive firms’ equilibrium profit as a function of its assets. We then study (1) the merger market equilibrium, which induces an assignment of abilities to varieties, and (2) the equilibrium location of production.

Preliminaries. Solving the representative consumer’s utility maximization problem, we obtain the following demand for variety $\zeta$:

$$x(\zeta) = \frac{m(\zeta)p(\zeta)\sigma}{m(\zeta)p(\zeta)^{1-\sigma}d\zeta},$$

where $p(\zeta)$ is the price of variety $\zeta$.

Profit maximization implies that each firm charges a fixed markup, and so the price of variety $\zeta$, when produced in country $i$, is given by $p(\zeta) = c_i(\zeta)/\rho$, where $c_i(\zeta)$ is the unit cost of production. Hence, the gross profit of a firm producing variety $\zeta$ in country $i$ is given by

$$Sm(\zeta)c_i(\zeta)^{1-\sigma},$$

where the markup-adjusted residual demand level $S$ is given by

$$S = \left[\sigma \int_{\zeta} m(\zeta)c_i(\zeta)^{1-\sigma}d\zeta \right]^{-1}. \quad (2)$$

Writing gross profits as a function of entrepreneurial ability $a$, mass appeal $m$, demand level $S$, and location of production $i$, we have:

$$\Pi_i(a, m) = \left\{ \begin{array}{ll}
\theta Sam & \text{for } i = 1, \\
S Sam & \text{for } i = 2,
\end{array} \right.$$ \quad (3)

where $\theta \equiv \omega_1^{1-\sigma} < 1$. The parameter $\theta$ captures the cost disadvantage of producing in country 1. Observe that the profit function is supermodular in $a$ and $m$, reflecting that entrepreneurial ability and the variety’s mass appeal are complementary in generating profits. Note also that a firm with an entrepreneur of ability $a$ could generate the same profit when producing a variety of mass appeal $m$ in country 1 and when producing a variety of mass appeal $\theta m$ in country 2.

Equilibrium assignment and location of production. Equilibrium on the merger market induces an assignment of entrepreneurs to varieties. We now investigate the equilibrium assignment of $a$ to $m$, and the equilibrium location of production, holding fixed the markup-adjusted residual demand level $S$. For the equilibrium assignment, the country of origin of an entrepreneur is irrelevant: an entrepreneur can manage production (provide headquarter services) equally well in both countries, independently of his country of origin. Since firms are profit-maximizers and the merger market is perfectly competitive, equilibrium must have the property that the assignment of entrepreneurs to varieties and the location of production maximize firms’ joint profits, holding $S$ fixed.

We first consider the equilibrium location of production of a variety of mass appeal $m$. Two immediate observations can be made. First, no variety that originated in country 2 will be
produced in country 1. To see this, note that no firm would want to move production from a low-cost location to a high-cost location, given that there is a global pool of entrepreneurs. Second, there must exist a threshold $m = \infty$ such that a variety of mass appeal $m$ that originated in country 1 will be produced in country 2 (incuring a fixed cost $\phi$ of building a new variety-specific plant) if $m > m$, and in country 1 if $m < m$.\(^{11}\)

We now turn to the equilibrium assignment of entrepreneurs to varieties. Let $a_i(m)$ denote the set of abilities of entrepreneurs who will manage a variety of mass appeal $m$ originating in country $i$, for any given (endogenous) threshold $m$. It is straightforward to show that $a_2(\cdot)$ is a strictly increasing function: there is positive assortative matching between entrepreneurial ability and the mass appeal of a variety. This follows from three observations. First, $G_2$ and $H$ are strictly increasing and continuous distribution functions. Second, the profit function $\Pi_2(a, m)$ is supermodular in $a$ and $m$. Third, all varieties originating in country 2 will be produced in country 2.

From the profit function (3) and the location of production, we obtain two no-arbitrage conditions that link $a_1(\cdot)$ and $a_2(\cdot)$. All varieties of mass appeal $m < m$ will be produced in their country of origin, and an entrepreneur of ability $a$ makes the same profit when managing a variety of mass appeal $m$ in country 1 and when managing a variety of mass appeal $\theta m$ in country 2. Hence,

$$a_1(m) = a_2(\theta m) \text{ for all } m < m.$$  \(^{(4)}\)

All varieties of mass appeal $m > m$ will be produced in country 2, independently of their country of origin. Hence, $a_1(m) = a_2(m)$ for all $m > m$. All varieties of mass appeal $m > m$ will be produced in country 2, independently of their country of origin. Hence,

$$a_1(m) = a_2(m) \text{ for all } m > m.$$  \(^{(5)}\)

An immediate implication of equation (4) is that the threshold $m$ must be finite.\(^{12}\) From the no-arbitrage conditions (4) and (5), it follows that $a_1(m)$ is single-valued and strictly increasing in $m$ for $m < m$ and for $m > m$. Further, $a_1(m)$ has two elements, namely $a_2(\theta m)$ and $a_2(m)$. Since $a_2(\theta m) < a_2(m)$, it follows that $a_1(\cdot)$ has an upward jump at $m$ in the sense that $\lim_{m \to m} a_1(m) < \lim_{m \to m} a_1(m)$.

We are now in the position to derive $a_2(\cdot)$. In light of the no-arbitrage conditions (4) and (5), which link $a_1(\cdot)$ and $a_2(\cdot)$, we will focus on $a_2(\cdot)$. First, we derive $a_2(m)$ for $m > m$. Since all varieties of mass appeal $m > m$ will be produced in country 2, independently of their country of origin, and since there is positive assortative matching between $a$ and $m$, we have

$$(E_1 + E_2)[1 - G(a_2(m))] = (E_1 + E_2)[1 - H(m)] \text{ for } m \geq m.$$  \({}^{11}\)To see this, suppose otherwise that, in equilibrium, there exist two firms, $(a', m')$ and $(a'', m'')$, whose varieties $m'$ and $m'' < m'$ originated in country 1, and firm $(a', m')$ locates in country 1, while firm $(a'', m'')$ in country 2. The resulting joint profits of the two firms are $S[\theta a' m' + a'' m''] - \phi$. However, this allocation does not maximize joint profits. If the better of the two entrepreneurs, $\max\{a', a''\}$, purchases the more popular variety $m'$ and locates in country 2, while the other entrepreneur purchases variety $m''$ and locates in country 1, then the two firms' joint profits are given by $S[\theta \min\{a', a''\} m'' + \max\{a', a''\} m] - \phi$, which is greater than the joint profit in the candidate equilibrium.

\(^{12}\)To see this, suppose otherwise that no firm from country 1 builds a new plant in country 2, $m = \infty$. However, for $m$ sufficiently large, firm $(a_1(m), m, 1)$ could then make a larger profit by building a new plant in country 2: $\Pi_1(a_1(m), m) = \Pi_2(a_2(\theta m), m) < \Pi_2(a_2(\theta m), m) - \phi$. A contradiction.
where \( G \equiv [E_1G_1 + E_2G_2]/(E_1 + E_2) \) is the global distribution function of entrepreneurial abilities. The term on the l.h.s. represents the mass of entrepreneurs with ability of at least \( a_2(m) \), while on the r.h.s. is the mass of varieties of mass appeal \( m \) and greater. Solving for \( a_2(m) \), yields
\[
a_2(m) = G^{-1}(H(m)) \quad \text{for} \ m \geq \overline{m}. \tag{6}
\]

Next, we derive \( a_2(m) \) for \( \theta \overline{m} \leq m \leq \overline{m} \). From the no-arbitrage condition (4), \( a_1(m) = a_2(\theta m) \) for all \( m \leq \overline{m} \). Hence, all entrepreneurs of ability \( a_2(\theta m) \leq a \leq a_2(\overline{m}) \) will manage production in country 2, independently of their country of origin. We thus have
\[
(E_1 + E_2) [G(a_2(\overline{m})) - G(a_2(m))] = E_2 [H(\overline{m}) - H(m)] \quad \text{for} \ m \in [\theta \overline{m}, \overline{m}].
\]

Observing that \( G(a_2(\overline{m})) = H(\overline{m}) \), we obtain
\[
a_2(m) = G^{-1} \left( \frac{E_1 H(\overline{m}) + E_2 H(m)}{E_1 + E_2} \right) \quad \text{for} \ m \in [\theta \overline{m}, \overline{m}]. \tag{7}
\]

Finally, we derive \( a_2(m) \) for \( m < \theta \overline{m} \). We have
\[
(E_1 + E_2)G(a_2(m)) = E_1 H(m/\theta) + E_2 H(m) \quad \text{for} \ m < \theta \overline{m}.
\]
The term on the l.h.s. represents the mass of entrepreneurs from both countries who have ability less than or equal to \( a_2(m) \). The second term on the r.h.s. is the mass of varieties of mass appeal \( m \) or less that locate (and originate) in country 2. Since \( a_2(m) = a_1(m/\theta) \), the first term on the r.h.s. represents the mass of varieties that locate (and originate) in country 1 and that are managed, in equilibrium, by entrepreneurs of ability \( a_2(m) \) or less. Solving the equation for \( a_2(m) \) yields
\[
a_2(m) = G^{-1} \left( \frac{E_1 H(m/\theta) + E_2 H(m)}{E_1 + E_2} \right) \quad \text{for} \ m \leq \theta \overline{m}. \tag{8}
\]

Summarizing, the function \( a_2(\cdot) \) is defined, over the relevant domains, by equations (6), (7), and (8). The function is strictly increasing, continuous, and differentiable except at \( \theta \overline{m} \) and \( \overline{m} \). Observe also that \( a_2(\cdot) \) depends on \( \overline{m} \); we will later analyze how this function changes with \( \overline{m} \).

The equilibrium assignment of entrepreneurs to varieties originating in country 1, summarized by \( a_1(\cdot) \), follows immediately from \( a_2(\cdot) \) and the no-arbitrage conditions (4) and (5):
\[
a_1(m) = \begin{cases} 
G^{-1} \left( \frac{E_1 H(m/\theta) + E_2 H(\theta m)}{E_1 + E_2} \right) & \text{for} \ m < \overline{m}, \\
G^{-1} (H(m)) & \text{for} \ m > \overline{m}.
\end{cases} \tag{9}
\]

Hence, \( a_1(\cdot) \) is a strictly increasing and differentiable function, except at \( \overline{m} \), where it exhibits an upward jump.

**Equilibrium Price Schedules.** Having analyzed the equilibrium assignment and location of production (as a function of \( S \)), we can now determine the equilibrium price schedules for
varieties and entrepreneurs on the international merger market, for any given markup-adjusted residual demand level \( S \).

Consider first the equilibrium gross profit \( \Pi_2(a_2(m), m) \) of a firm that produces a variety of mass appeal \( m \) in country 2 and is managed by an entrepreneur with ability \( a_2(m) \). The gross profit can be decomposed as

\[
\Pi_2(a_2(m), m) = \Pi_2(a_2(0), 0) + \int_0^m \left[ \frac{\partial \Pi_2(a_2(z), z)}{\partial a} \frac{\partial a_2(z)}{\partial z} + \frac{\partial \Pi_2(a_2(z), z)}{\partial z} \right] dz
\]

where the second equality follows from the observation that \( \Pi_2(a_2(0), 0) = 0 \). On the other hand, the return of an entrepreneur of ability \( a_2(m) \) is the difference between the firm’s gross profit and the equilibrium price of variety \( m \), i.e.,

\[
V(a_2(m)) = \Pi_2(a_2(m), m) - W_2(m).
\]

Since \( \Pi_2(a_2(0), 0) = 0 \), the market values of the worst varieties and entrepreneurs are zero, i.e., \( W_2(0) = 0 \) and \( V(a_2(0)) = V(0) = 0 \). We can now decompose the gross profit as follows:

\[
\Pi_2(a_2(m), m) = V(a_2(m)) + W_2(m)
\]

Comparing the two decompositions, (10) and (12), we obtain

\[
\frac{\partial \Pi_2(a_2(z), z)}{\partial a} = V'(a_2(z)) \quad \text{and} \quad \frac{\partial \Pi_2(a_2(z), z)}{\partial z} = W'_2(z).
\]

Since \( \Pi_2(a_2(m), m) = Sa_2(m) \), the equilibrium price schedules can be written as

\[
W_2(m) = W_2(0) + \int_0^m W'_2(z)dz = \int_0^m \frac{\partial \Pi_2(a_2(z), z)}{\partial z} dz = S \int_0^m a_2(z)dz,
\]

and, using (11),

\[
V(a_2(m)) = S \left[ a_2(m)m - \int_0^m a_2(z)dz \right].
\]

Consider now the equilibrium price schedule for varieties created in country 1. From the no-arbitrage condition (4), a variety of mass appeal \( m < m \) from country 1 generates the same profit as a variety of mass appeal \( \theta m \) from country 2. Hence,

\[
W_1(m) = W_2(\theta m) \text{ for } m \leq m.
\]

Furthermore, we obtain

\[
W_1(m) = W_2(m) - \phi \text{ for } m \geq m
\]
since, in equilibrium, the production of all varieties of mass appeal \( m > \overline{m} \) will be located in country 2, which requires incurring the fixed cost \( \phi \).

From equations (14) and (15), it follows that

\[
W_2(\theta \overline{m}) = W_2(\overline{m}) - \phi. \tag{16}
\]

Consider a variety of mass appeal \( \overline{m} \) originating in country 1. If this variety is produced in country 1, it will be managed by an entrepreneur of ability \( a_2(\theta \overline{m}) \), and its value is thus \( W_2(\theta \overline{m}) \).

On the other hand, if this variety is produced in country 2, a new variety-specific plant needs to be built in that country (at cost \( \phi \)), and production will be managed by an entrepreneur of ability \( a_2(\overline{m}) \). Hence, in this case, the value of the variety is \( W_2(\overline{m}) - \phi \). By definition of \( \overline{m} \), the two usages of the variety must generate the same value, \( W_2(\theta \overline{m}) = W_2(\overline{m}) - \phi \).

Using (13), the indifference condition (16) can be rewritten as

\[
S \int_{\theta \overline{m}}^{\overline{m}} a_2(m) dm = \phi. \tag{17}
\]

The l.h.s. of this equation is the profit increase from a relocation of production of a variety of mass appeal \( \overline{m} \) (originating in country 1) to country 2, taking into account that the equilibrium assignment of entrepreneurs to varieties depends on the location of production. From the viewpoint of maximizing profits (holding \( S \) fixed), moving variety \( \overline{m} \) from country 1 to country 2 implies that an entrepreneur of ability \( a_2(\theta \overline{m}) \) is freed up in country 1 to manage production of variety \( \theta \overline{m} \) in country 2, which in turn allows a re-assignment of entrepreneurs to varieties in country 2: all varieties of mass appeal between \( \theta \overline{m} \) and \( \overline{m} \) will now be managed by more able entrepreneurs, which generates additional profit. On the r.h.s. of equation (17) is the cost of such a relocation of production.

**The markup-adjusted residual demand level.** So far, we have derived the equilibrium assignment and location of production for a given markup-adjusted residual demand level \( S \). However, \( S \) is endogenous and determined jointly with the (endogenous) threshold \( \overline{m} \). From equation (2) and the equilibrium assignment of entrepreneurs to varieties derived above, the markup-adjusted residual demand level is given by

\[
S = \sigma^{-1} \left[ E_1 \theta \int_{0}^{\theta \overline{m}} ma_1(m) dH(m) + E_2 \int_{0}^{\overline{m}} ma_2(m) dH(m) \right. \\
\left. + (E_1 + E_2) \int_{\overline{m}}^{\infty} ma_2(m) dH(m) \right]^{-1}. \tag{18}
\]

Combining this equation with the equilibrium condition for \( \overline{m} \), equation (17), we can write the equilibrium threshold \( \overline{m} \) as a function of the exogenous variables \( \theta, \sigma, \phi, \omega_2, E_1, \) and \( E_2 \) (as well as the distribution functions \( G \) and \( H \)):

\[
0 = \sigma \phi \left[ E_1 \theta \int_{0}^{\theta \overline{m}} ma_1(m) dH(m) + E_2 \int_{0}^{\overline{m}} ma_2(m) dH(m) \right. \\
\left. + (E_1 + E_2) \int_{\overline{m}}^{\infty} ma_2(m) dH(m) \right] - \int_{\theta \overline{m}}^{\overline{m}} a_2(m) dm. \tag{19}
\]
It can be established that the r.h.s. of (19) is continuous and strictly decreasing in $m$. Furthermore, the r.h.s. is strictly positive for $m = 0$, and becomes negative for $m$ sufficiently large.\footnote{As $m \to \infty$, the term in brackets remains bounded, while the second term on the r.h.s. tends to $-\infty$.}

Hence, there exists a unique threshold quality $m$, implicitly defined by equation (19).

**Endogenous cross-country differences in the distribution of firm efficiencies.** The equilibrium assignment of entrepreneurs to varieties and the equilibrium location of production have important consequences for cross-country differences in the distribution of firm efficiencies. In our framework, it is natural to define the intrinsic efficiency of a firm of type $(a,m,i)$ at the output stage by $\varphi(a,m,i) = am$ since the firm’s gross profit is proportional to $am$. This definition of intrinsic efficiency controls for wage (and technology) differences across countries.

**Proposition 1** In equilibrium, the efficiency distribution of firms producing in country 2 first-order stochastically dominates that of firms producing in country 1.

**Proof.** See appendix.

This result follows from two observations. First, in equilibrium, the production of all of the most popular varieties will be located in country 2. Second, in equilibrium, a variety of any given mass appeal $m$ will be managed by a more able entrepreneur in country 2 than in country 1, $a_2(m) > a_1(m)$. Hence, it is not only the endogenous change in the location of production that causes aggregate efficiency differences between countries, but also the re-assignment of entrepreneurs to varieties through the market for corporate assets. The proposition shows that the empirical research on the sources of comparative advantage needs to take the “selection effect” of FDI into account.

4 Foreign Direct Investment

In the previous section, we derived the equilibrium assignment of entrepreneurs to varieties and the equilibrium location of production. What still needs to be analyzed is the implied equilibrium pattern of trade and FDI. In this section, we interpret the assignment of entrepreneurs to varieties and the location of production in terms of choice of FDI mode. We then present two key results. First, all greenfield FDI is one-way: from the high cost to the low cost country, while cross-border acquisitions occur in both directions. Second, in the limit as factor price differences between countries vanish, all FDI takes the form of cross-border acquisitions.

**Types of FDI.** In our model, it is natural to assume that the identity of a firm is linked to its entrepreneur. In this sense, it will be the entrepreneurs that buy or sell varieties, rather than the reverse. Greenfield FDI from country $i$ to country $j$ occurs whenever an entrepreneur from country $i$ relocates production of a variety from country $i$ to country $j$ by building a new plant in country $j$. Cross-border acquisition from country $i$ to country $j$ occurs whenever an entrepreneur from country $i$ purchases a variety from country $j$, independently of where this variety will be produced. Two types of cross-border acquisitions are possible, dependent on whether the acquired variety will be produced in its country of origin, or whether local production will be closed down and transferred to the entrepreneur’s home country.
Equilibrium selection. Both the assignment of entrepreneurial types to types of varieties and the location of production by mass appeal of variety are uniquely pinned down in equilibrium. However, in the absence of any mobility costs for entrepreneurs, entrepreneurs of any given ability but originating in different countries are perfect substitutes. Consequently, there is indeterminacy in the equilibrium gross flows of FDI. Since the assumption of no frictions in the market for cross-border acquisitions may be viewed as a limiting case where such frictions become small, we henceforth confine attention to the equilibrium pattern of FDI that minimizes the volume of cross-border acquisitions.

Composition of international commerce. We are now in the position to provide a first characterization of FDI flows. Since all varieties originating in country 2 are produced in that country, any greenfield FDI must be one-way, namely from country 1 to country 2. Any variety of mass appeal \( m > \overline{m} \) that originated in country 1 will be produced in country 2. This relocation of production may be accomplished in two ways: (i) an entrepreneur from country 1 may engage in greenfield FDI in country 2 by building a new plant in that country, or (ii) an entrepreneur from country 2 may purchase that variety, close down production in country 1, and build a new plant in country 2. Since we restrict attention to the equilibrium pattern of FDI that minimizes the volume of cross-border acquisitions, the latter will occur only if there is an insufficient number of entrepreneurs of ability \( a_1(m) \) originating in country 1 to manage production of varieties of mass appeal \( m \) from their home country. In contrast, all varieties of mass appeal \( m < \overline{m} \) from country 1, and all varieties from country 2, will be locally produced. These varieties will be acquired by a local entrepreneur of ability \( a_i(m) \), or else if the local supply of such entrepreneurs is too small, by a foreign entrepreneur.

More formally, let \( \psi_i(m) \) denote the ratio between the number of entrepreneurs, originating in country \( i \), who in equilibrium will be assigned to a variety of mass appeal \( m \), and the number of varieties of that mass appeal originating in the same country. If \( \psi_i(m) < 1 \), then a fraction \( 1 - \psi_i(m) \) of these varieties must be acquired by foreign firms (entrepreneurs). A variety of mass appeal \( m \) from country \( i \) will be managed by an entrepreneur of ability \( a_i(m) \), and so

\[
\psi_i(m) = \lim_{\Delta \to 0} \frac{E_i [G_i(a_i(m + \Delta)) - G_i(a_i(m))]}{E_i [H(m + \Delta) - H(m)]}
= \frac{g_i(a_1(m))}{h(m)} a_i'(m).
\]

Hence, using equations (6) through (9),

\[
\psi_1(m) = \begin{cases} 
\frac{g_1(a_1(m))}{g(a_1(m))} \left[ \frac{E_1}{E_1 + E_2} + \frac{\theta E_2}{E_1 + E_2} h(m/\theta) \right] & \text{if } m \leq \overline{m}, \\
\frac{g_i(a_1(m))}{g(a_1(m))} & \text{otherwise},
\end{cases} \tag{20}
\]

and

\[
\psi_2(m) = \begin{cases} 
\frac{g_2(a_2(m))}{g(a_2(m))} \left[ \frac{E_2}{E_1 + E_2} + \frac{E_1/\theta}{E_1 + E_2} h(m/\theta) \right] & \text{if } m \leq \theta \overline{m}, \\
\frac{g_i(a_2(m))}{g(a_2(m))} \left[ \frac{E_2}{E_1 + E_2} \right] & \text{if } m \in (\theta \overline{m}, \overline{m}]), \\
\frac{g_2(a_2(m))}{g(a_2(m))} & \text{if } m > \overline{m}.
\end{cases} \tag{21}
\]
Observe that both functions \( \psi_1 \) and \( \psi_2 \) are continuous, except that \( \psi_1 \) has a discontinuity at \( \overline{m} \), while \( \psi_2 \) exhibits discontinuities at \( \theta\overline{m} \) and \( \overline{m} \). Note also that the value of \( \psi_i(m) \) depends directly on the relative supply of firms in country \( i \), \( E_i/(E_1 + E_2) \), factor price differences, as summarized by \( \theta \), and the relative supply of managerial abilities in country \( i \), \( g_i(a_i(m))/g(a_i(m)) \); it also depends on the assignment function \( a_i(m) \) but only if there are cross-country differences in the distributions of entrepreneurial abilities, i.e., \( g_i \neq g \).

Using the functions \( \psi_1 \) and \( \psi_2 \), we can derive aggregate statistics of FDI flows. Let \( \gamma_i \) and \( \mu_i \) denote the fractions of entrepreneurs (firms) from country \( i \) who will engage in greenfield FDI and cross-border acquisitions, respectively. Since all greenfield FDI is directed toward the country with the comparative advantage in production, \( \gamma_2 = 0 \). On the other hand,

\[
\gamma_1 = \int_{\overline{m}}^{\infty} \min \{\psi_1(m), 1\} \, dH(m) \quad (22)
\]

since a fraction \( \min \{\psi_1(m), 1\} \) of a variety of mass appeal \( m > \overline{m} \) originating in country 1 will be produced in country 2 as part of greenfield FDI (while the remaining fraction \( \max\{1 - \psi_1(m), 0\} \) will be acquired and relocated by entrepreneurs from country 2).

As regards cross-border acquisitions, we have

\[
\mu_1 = \frac{E_2}{E_1} \int_{0}^{\infty} \max \{1 - \psi_2(m), 0\} \, dH(m). \quad (23)
\]

To see this, note that if \( \psi_2(m) < 1 \), there is an insufficient number of entrepreneurs from country 2 that have the “right” ability to manage production of varieties with mass appeal \( m \) in country 2, and so a fraction \( 1 - \psi_2(m) \) will to be acquired by entrepreneurs from country 1. Similarly, we have

\[
\mu_2 = \frac{E_1}{E_2} \int_{0}^{\infty} \max \{1 - \psi_1(m), 0\} \, dH(m). \quad (24)
\]

We can now make two important observations. First, the flows of cross-border acquisitions will be balanced in equilibrium,

\[
E_1\mu_1 = E_2\mu_2.
\]

Balancedness obtains since, in each country, the mass of entrepreneurs is equal to the mass of varieties, both before the merger market opens as well as after the merger market closes. Moreover, each greenfield investment involves one entrepreneur and one variety from the same country. Second, all entrepreneurs from country 1 who are of ability \( a \in (a_2(\theta\overline{m}), a_2(\overline{m})) \) will be engaged in cross-border acquisitions in country 2, and so \( \mu_1 = (E_2/E_1)\mu_2 > 0 \).

We summarize our results in the following proposition.

**Proposition 2** In equilibrium, greenfield FDI and cross-border acquisitions co-exist. All greenfield FDI is one-way: from country 1 to country 2, but not in the reverse direction. In contrast, cross-border acquisitions are two-way: from country 1 to country 2, and from country 2 to country 1.

Existing models of vertical FDI (e.g., Helpman, 1984) predict that, at any given production stage, all FDI flows are one-way: the only receiving country is the one that has a comparative advantage in production.
advantage in that stage of production. Yet, there is overwhelming empirical evidence showing that FDI flows are generally two-way. In light of this stylized fact, trade theorists have relied on models with transport costs to generate two-way FDI. As proposition 2 shows, transport costs are not necessary to generate this stylized fact.

There is ample evidence that many governments are wary of foreign acquisitions of domestic establishments that result in the closure of local production. Our model can indeed generate such FDI. The measure of firms involved in such FDI is

\[ E_1 \int_{\overline{\mu}}^{\infty} \max \{1 - \psi_1(m), 0\} dH(m), \]

which is positive if and only if \( g_1(a) < g_2(a) \) for some \( a > a_2(\overline{\mu}) \). Interestingly, if foreign acquisitions result in the closure of local production, they involve the more popular varieties (of mass appeal \( m > \overline{\mu} \)) from country 1.

**Vanishing factor price differences.** As figure 2 has revealed, cross-border acquisitions are the dominant mode of FDI between the richest and most developed countries (where, arguably, factor price differences are not very large). In contrast, a much larger fraction of FDI flows from the rich and developed countries to the poor and developing countries (where, arguably, factor price differences play an important role) involve greenfield.

To explain these findings within our model, we analyze the composition of FDI in the limit as factor price differences become small, i.e., as \( \theta \to 1 \). An immediate observation is that the quality threshold \( \overline{\mu} \to \infty \) as \( \theta \to 1 \). Consequently, greenfield FDI disappears as factor price differences become small: \( \gamma_1 \to 0 \) as \( \theta \to 1 \). Next, as can be seen from equation (21), for any \( m \),

\[ \psi_2(m) \to \frac{g_2(a_2(m))}{g(a_2(m))} \text{ as } \theta \to 1. \]

However, generically, we have \( g_2(a) \neq g(a) \) for (almost) any \( a \). Hence, \( \mu_1 \) and \( \mu_2 \) are bounded away from zero, even as \( \theta \to 1 \). We therefore obtain the following key result.

**Proposition 3** There always exist cross-border acquisitions in both directions, independently of factor price differences. In contrast, greenfield FDI disappears in the limit as factor price differences vanish. Hence, as \( \theta \to 1 \), all FDI takes the form of cross-border acquisitions.

What this propositions highlights is that there are two reasons for cross-border acquisitions, but only one reason for the existence of greenfield FDI. In our model, greenfield FDI occurs only because firms want to exploit factor price differences by relocating production from a high cost to a low cost location. In contrast, cross-border acquisitions not only exist because of factor price differences, but also because the distribution of entrepreneurial abilities (or, more generally, of firm-specific assets) varies from one country to another.\(^{15}\)

More generally, the following features of our model are necessary to obtain two-way cross-border acquisitions in the absence of factor price differences: there must be heterogeneity in

---

\(^{14}\)To see this, suppose otherwise that \( \overline{\mu} \) is bounded from above, independently of \( \theta \). But then, the r.h.s. of equation (19) is strictly positive for \( \theta \) sufficiently close to one, while the l.h.s. is identical to zero; a contradiction.

\(^{15}\)This may be reminiscent of Grossman and Maggi (2000), where trade between countries occurs because of differences in the distributions of workers’ talent.
both types of corporate assets (entrepreneurial abilities and the mass appeal of varities), and these corporate assets must be complementary, and there must be distributional differences across countries in at least one of the two types of corporate assets. This highlights the importance of "two-sided" heterogeneity in our assignment model.

5 Firm Efficiency and Choice of FDI Mode

In this section, we further explore the mapping from a firm’s efficiency to its choice of FDI mode. For this purpose, we impose additional structure on the distributions of entrepreneurial abilities. We then obtain another key result: firms that engage in greenfield FDI are systematically more efficient than those that engage in cross-border acquisitions. Moreover, we show that, under some modest regularity condition on the distribution of varieties’ mass appeal, the probability that an entrepreneur from country 1 engages in FDI is weakly increasing in the entrepreneur’s ability.

Efficiency differences: greenfield FDI vs. cross-border acquisitions. As figure 1 has revealed, U.S. firms engaging in greenfield FDI are systematically more efficient than U.S. firms engaging in cross-border acquisitions. Our model generates this result under a variety of distributional assumptions. A natural restriction on the distribution of entrepreneurial abilities that allows us to obtain this result is the following symmetry condition.

\[(C1) \quad \text{The distributions of entrepreneurial abilities are the same in both countries: } G_1 \equiv G_2.\]

Given this symmetry in entrepreneurial abilities across countries, all FDI is motivated by factor price differences. As can be seen from equations (20) and (21), \((C1)\) implies that \(\psi_1(m) = \psi_2(m) = 1\) for all \(m \geq m_0\). Hence, all cross-border acquisitions involve varieties of mass appeal \(m < m_0\), while all greenfield FDI still involves only varieties of mass appeal \(m > m_0\). Positive assortative matching between \(a\) and \(m\) in each country then implies the following proposition.

**Proposition 4** Firm engaging in greenfield FDI are more efficient than firms engaging in cross-border acquisitions.

If we were to relax the symmetry condition \((C1)\), we would still obtain that firms engaging in greenfield FDI are, *on average*, more efficient than firms engaging in cross-border acquisitions. To violate this weaker prediction, one would need a very strong departure from symmetry, namely that one country has a much larger supply of the very best entrepreneurs than another.

Through the lens of this proposition, it may become apparent why the governments of many countries appear to favor greenfield FDI over cross-border acquisitions. Policymakers may perceive two important differences between the two FDI modes. First, greenfield FDI involves the creation of new plants. Second, greenfield FDI involves the best foreign firms, and therefore a large number of workers.

Efficiency differences: cross-border acquisitions vs. domestic production. To further tighten our predictions on the efficiency differences between firms engaging in different modes of serving
the world market, we impose a mild regularity condition on the distribution of varieties’ mass appeal.

(C2) The elasticity of the density function \( h \),

\[ \frac{mh'(m)}{h(m)} \]

is strictly decreasing in \( m \).

It can easily be verified that condition (C2) is satisfied by a number of standard distributions, e.g., by the two-parameter family of Weibull distributions, \( H(m) = 1 - e^{-(m/\beta)\alpha} \), \( \alpha > 0 \), \( \beta > 0 \) (which includes the exponential distribution as a special case with \( \alpha = 1 \)) and the two-parameter family of Gamma distributions (which includes the chi-squared distribution as a special case). Henceforth, we will assume that both distributional conditions, (C1) and (C2), hold.

(C1) implies that \( \psi_1(0) < 1 < \psi_2(0) \). Moreover, (C1) and (C2) imply that \( \psi_1(\cdot) \) is strictly increasing on \([0, \overline{m}]\), and \( \psi_2(\cdot) \) is strictly decreasing on \([0, \theta \overline{m}]\), has a downward jump at \( \theta \overline{m} \), and is constant on \([\theta \overline{m}, \overline{m}]\). It will prove useful to define a second threshold mass appeal, \( \hat{m} \).

If \( \lim_{m \to \overline{m}} \psi_1(m) > 1 \), the threshold \( \hat{m} \) is such that \( \psi_1(\hat{m}) = 1 \), while \( \hat{m} = \overline{m} \) otherwise. Hence,

\[ \hat{m} = \begin{cases} \psi_1^{-1}(1) & \text{if } \lim_{m \to \overline{m}} \psi_1(m) > 1, \\ \overline{m} & \text{otherwise.} \end{cases} \]  

Using this threshold \( \hat{m} \) and equations (22) to (24), we can summarize the relationship between firm efficiency and FDI mode choice as follows.

**Proposition 5** Consider the mapping from entrepreneurial ability to mode of FDI. For entrepreneurs of ability \( a < a_2(\theta \hat{m}) = a_1(\hat{m}) \), all FDI involves cross-border acquisitions in country 1. For entrepreneurs of ability \( a \in (a_2(\theta \hat{m}), a_2(\overline{m})) \), all FDI involves cross-border acquisitions in country 2. For entrepreneurs of ability \( a > a_2(\overline{m}) \), all FDI involves greenfield FDI in country 2.

Consider an entrepreneur of ability \( a \) from country 1. (i) If \( a \leq a_1(\hat{m}) = a_2(\theta \hat{m}) \), he will not engage in FDI. (ii) An entrepreneur of ability \( a \in (a_2(\theta \hat{m}), a_2(\overline{m})) \) will, with positive probability, acquire a variety in country 2. The probability that a country-2 variety of mass appeal \( m \in (\theta \hat{m}, \theta \overline{m}) \) will be acquired by a foreign entrepreneur is \( 1 - \psi_2(m) \geq 0 \), and is strictly increasing in \( m \). Positive assortative matching between \( a \) and \( m \) then implies that the probability that a country-1 entrepreneur of ability \( a \in (a_2(\theta \hat{m}), a_2(\theta \overline{m})) \) engages in cross-border acquisitions is strictly increasing in \( a \). (iii) If \( a > a_2(\overline{m}) = a_2(\theta \overline{m}) \), the entrepreneur will engage in FDI with probability one, namely in cross-border acquisitions if \( a \in (a_2(\theta \overline{m}), a_2(\overline{m})) \), and in greenfield FDI if \( a > a_2(\overline{m}) \).

We thus obtain the following monotonicity result.

**Proposition 6** The probability that an entrepreneur from country 1 engages in FDI is weakly increasing in his ability \( a \).

Proposition 6 implies, in particular, that in the high-cost country, firms engaging in cross-border acquisitions are, on average, more efficient than firms producing domestically.
6 Comparative Statics

In this section, we further explore the workings of our model. We generate a number of additional predictions by analyzing the effects of changing various exogenous variables on the equilibrium assignment and location of production, and the aggregate statistics of FDI.

Throughout this section, we assume that conditions (C1) and (C2) hold. Using equations (20) and (25), the aggregate statistics of FDI, (22) to (24), then simplify to

\[
\mu_1 = \frac{E_2}{E_1 + E_2} \mu_2 = \frac{E_2}{E_1 + E_2} [H(\hat{m}) - H(\theta\hat{m})],
\]

(26)

and

\[
\gamma_1 = 1 - H(\overline{m}).
\]

(27)

The fixed cost of building a plant. We now want to explore the effects of changing the fixed cost \( \phi \) of building a new plant in country 2. Intuitively, one would expect that an increase in \( \phi \) raises the threshold \( \overline{m} \), and hence reduces the share \( \gamma_1 \) of entrepreneurs from country 1 engaging in greenfield FDI in country 2. Indeed, this can easily be established using equation (19). Further, \( \overline{m} \to 0 \) as \( \phi \) becomes small, and \( \overline{m} \to \infty \) as \( \phi \) becomes large. Next, note that if \( \hat{m} < \overline{m} \), \( \hat{m} \) is implicitly defined by \( \theta h(\theta\hat{m})/h(\hat{m}) = 1 \), and hence (locally) independent of \( \phi \); otherwise \( \hat{m} = \overline{m} \), and so \( \hat{m} \) is strictly increasing in \( \phi \). We thus have the following lemma.

**Lemma 1** The threshold \( \overline{m} \) is strictly increasing in \( \phi \), \( \overline{m} \to 0 \) as \( \phi \to 0 \), and \( \overline{m} \to \infty \) as \( \phi \to \infty \). Further, if \( \hat{m} < \overline{m} \), then \( \hat{m} \) is (locally) independent of \( \phi \). Hence, there exists a unique \( \phi > 0 \) such that \( \hat{m} < \overline{m} \) for all \( \phi > \hat{\phi} \), and \( \hat{m} = \overline{m} \) for all \( \phi \leq \hat{\phi} \).

Since \( \overline{m} \) is strictly increasing in \( \phi \), it follows immediately from equation (27) that the fraction \( \gamma_1 \) of entrepreneurs from country 1 engaging in greenfield FDI is decreasing in \( \phi \). The effect of \( \phi \) on the cross-border acquisition volume, however, depends on whether or not \( \phi > \hat{\phi} \), as the following proposition shows.

**Proposition 7** A small increase in the fixed cost \( \phi \) of building a plant, \( d\phi > 0 \), has the following effects:

\[
d\gamma_1 < 0, \quad d\mu_1 = \frac{E_2}{E_1 + E_2} d\mu_2 \begin{cases} > 0 & \text{if } \phi < \hat{\phi} \\ = 0 & \text{otherwise.} \end{cases}
\]

**Proof.** See appendix. \( \blacksquare \)

This proposition shows that the total number of cross-border acquisitions can be independent of the cost and volume of greenfield FDI, namely whenever the fixed cost \( \phi \) is sufficiently large (and the volume of greenfield FDI small). However, even in this case, the composition of cross-border acquisitions from country 1 to country 2 is not independent of \( \phi \) in the sense that a change in \( \phi \) affects the set of entrepreneurs from country 1 who will engage in cross-border mergers.

**Country characteristics.** We now want to investigate the effects on the equilibrium of varying country characteristics: the factor price differential, as measured by \( \theta \), and the numbers
of entrepreneurs in each country, $E_1$ and $E_2$. For simplicity, we will confine attention to the case $\phi > \bar{\phi}$.

Consider first an increase in $\theta$, i.e., a reduction in the factor price difference between the two countries. Under condition (C1), FDI is ultimately driven by factor price differences. Hence, one would expect the volume of FDI to decrease as the countries become more similar. Indeed, this intuition is correct, as the following proposition shows.

**Proposition 8** A reduction in the factor price difference between the two countries, $d\theta > 0$, has the following effects:

$$d\gamma_1 < 0 \text{ and } d\mu_1 = \frac{E_2}{E_1} d\mu_2 < 0.$$ 

**Proof.** See appendix. □

Consider now a small increase in the number of entrants in country 1, $E_1$, holding the total number of entrants in the two countries, $E_1 + E_2$, fixed. That is, the aggregate mass of varieties and entrepreneurs is held constant, while varieties and entrepreneurs from country 2 are becoming relatively more scarce. This implies that a smaller fraction of entrepreneurs from country 1 will engage in cross-border acquisitions, while a larger fraction of entrepreneurs from country 2 will acquire varieties in country 1. Holding $S$ fixed, this also implies that a larger fraction of entrepreneurs from country 1 will engage in greenfield FDI. Further, the shift in endowments in favor of the high-cost country results in a higher markup-adjusted residual demand level, which further increases the incentive to engage in greenfield FDI in the low-cost country. Clearly, a small increase in $E_2$, holding $E_1 + E_2$ fixed, has the opposite effects.

**Proposition 9** A small decrease in the number of entrants in country 2, $E_2$, holding the total number of entrants, $E_1 + E_2$, fixed, has the following effects:

$$d\gamma_1 > 0 \text{ and } d\mu_1 = -d\mu_2 < 0.$$ 

**Proof.** See appendix. □

Intuitively, a relative decrease in the number of entrants in country 2 implies that there are fewer attractive corporate assets in country 2 that entrepreneurs from country 1 can acquire. Hence, as $E_2/(E_1 + E_2)$ decreases, entrepreneurs from country 1 will substitute away from cross-border acquisitions in favor of greenfield FDI. In the limit as the relative number of entrants from country 2 goes to zero, all FDI from country 1 to country 2 will be in the form of greenfield FDI, i.e., $\mu_1 \to 0$, while $\gamma_1$ is bounded away from zero. This is an alternative for the observation (summarized figure 2) that U.S. MNEs are much more likely to choose greenfield FDI when engaging in FDI in poor, developing countries (where, arguably, there are fewer attractive target firms) than when engaging in FDI in rich, developed countries.

A small decrease in the fraction of entrants who originate in country 2 makes the countries “more similar” if $E_1 < E_2$, and “more dissimilar” if the reverse inequality holds. An interesting question is whether the aggregate number of cross-border acquisitions, $E_1\mu_1 + E_2\mu_2$, increases or decreases as the two countries become more similar. From equations (23) and (24), we have

$$E_1\mu_1 + E_2\mu_2 = \frac{2E_1E_2}{E_1 + E_2} [H(\hat{m}) - H(\theta\hat{m})].$$
Recall that $\bar{m}$ is independent of the number of entrants. Hence, the aggregate volume of cross-border acquisitions follows a gravity-type equation: it increases as the two countries become more similar in terms of their populations of entrants.

**Corollary 1** Consider a small increase in the number of entrants in country 1, $E_1$, holding the total number of entrants, $E_1 + E_2$, fixed. Then, the aggregate number of cross-border mergers, $E_1\mu_1 + E_2\mu_2$, increases if $E_1 < E_2$, and decreases if $E_1 > E_2$. Hence, the aggregate number of cross-border mergers is maximized when $E_1 = E_2$.

Does the aggregate volume of cross-border acquisitions increase or decrease as the two countries become more similar? As proposition 8 and corollary 1 show, the answer depends on which set of country characteristics one considers. As countries become more similar in terms of production costs, the aggregate two-way volume of cross-border acquisitions decreases, whereas the opposite result obtains as countries become more similar in terms of their populations of entrants.

### 7 Conclusion

In this paper, we have developed an assignment theory to analyze both the volume of foreign direct investment and its composition between cross-border acquisitions and greenfield investment. In our model, a firm consists of a bundle of heterogeneous and complementary corporate assets. The merger market allows firms to trade these corporate assets to exploit complementarities. A cross-border acquisition involves purchasing foreign corporate assets, while greenfield FDI involves building production capacity in the foreign country to allow the firm to deploys its corporate assets abroad. Equilibrium in the merger market is the solution to the associated assignment problem.

There are two countries that can freely trade with one another. Factor price differences between countries give rise to greenfield FDI and to cross-border acquisitions, while cross-country differences in entrepreneurial abilities give rise only to cross-border acquisitions. In equilibrium, greenfield FDI is always one way: from the high-cost to the low-cost country, while cross-border acquisitions are always two-way. Hence, our model can generate two-way FDI even in the absence of transport costs and factor price differences.

Firms’ choice between the two modes of FDI, and the re-assignment of corporate assets on the international merger market have an important impact on aggregate productivity. While FDI may positively affect the distribution of firm efficiencies in one country, it may have the opposite effect in the other country. Indeed, if productivity is measured at the plant level, cross-border acquisitions tend to reduce the observed productivity of the plants in the high-cost country.

We have derived the following key predictions. (1) Firms engaging in greenfield FDI are systematically more efficient than those engaging in cross-border acquisition. As we have shown, this is consistent with the data. (2) As factor price differences between countries vanish, all FDI takes the form of cross-border acquisitions. This prediction is consistent with our observation that U.S. multinationals are more likely to favor cross-border acquisitions over greenfield FDI in
high-wage rather than low-wage countries. (3) As the relative supply of corporate assets in the low-wage country decreases, firms in the high-wage country substitute away from cross-border acquisitions in favor of greenfield FDI. Again, this is consistent with our observation that the share of cross-border acquisition in total U.S. FDI is decreasing in the host country’s level of development.

While outside the scope of this paper, our model may also fruitfully be used as a framework for policy analysis. For instance, it would be interesting to compute the welfare implications of various policy experiments, such as restrictions on cross-border acquisitions or greenfield FDI. From the host country’s point of view, cross-border acquisitions involve a change in ownership of local production (and may even lead to the closure of local production), while greenfield FDI involves the opening of a new establishment. In this sense, cross-border acquisitions bring “less” to the host country’s economy than greenfield FDI. Moreover, greenfield FDI involves better foreign firms than cross-border acquisitions. Hence, the optimal government policy toward foreign direct investment should be tailored to the particular mode of FDI: greenfield FDI vs. cross-border acquisitions. We believe this to be an interesting avenue for future research.
Appendix: Empirics

This appendix serves two purposes. First, we explain the data used to construct figures 1 and 2. Second, we present an econometric analysis of the composition of a U.S. multinational's mode choice as a function of firm and country characteristics.

Our firm-level data come from the Bureau of Economic Analysis (BEA), which each year conducts a mandatory survey of all U.S. firms with foreign affiliates above a certain size threshold. Firms that come to own a new enterprise abroad are required to report (i) whether that enterprise was obtained through cross-border acquisition or greenfield FDI, (ii) in which industry that enterprise produces, and (iii) in which country that enterprise is located. From this database, we collected every recorded investment by those multinationals whose mainline of business is a traded good over the five-year period 1994-1998. The BEA dataset also contains a wide range of data on the characteristics of the parent firms that are conducting FDI abroad, which are used as explanatory variables in our analysis.

Construction of figures 1 and 2. Our two measures of a firm's efficiency are (i) the logarithm of sales of the U.S. parent firm in the United States (USSALE), and by (ii) the logarithm of the parent firm’s value-added per employee (VADDPW). In order to sort firms in different industries into efficiency groups, we must make efficiency data comparable across industries. Using data for 1994, we calculate the difference between the efficiency of firm i in industry k and the industry k average, and normalize this difference by the standard deviation in efficiency across firms in industry k. On the basis of relative efficiency, we sort firms into the three equally-sized groups, aggregate investment counts over firms within a group, and calculate the share of investments taking the form of cross-border acquisitions by efficiency group.

To construct the data in figure 2, countries are sorted into three equally-sized groups on the basis of their real GDP per capita in 1994, and on the basis of the average wage paid by multinationals in that country in 1994. These data come from the Summers and Heston dataset and the Bureau of Economic Analysis, respectively.

Econometric model. To assess the robustness of the relationships shown in figures 1 and 2, we estimate a logit model that relates a firm’s mode choice across countries and industries to a variety of firm and country characteristics. The dependent variables takes the value of one if the firm chooses cross-border acquisition, and zero otherwise. The key explanatory variables are the two measures of firm efficiency described above (USSALES and VADDPW) and the host country’s level of development. In addition to including a variety of firm and country control variables, we also control for fixed effects by parent industry.

In the construction of our sample, we classified investments to facilitate interpretation of the results. In particular, we aggregated a firm’s investments over the sample period 1994-1998 so that, for each firm, a country-industry pair appears at most once. For firms that made more than one investment in a particular country and industry, a country-industry observation for a firm was coded as a cross-border acquisition only if all investments made over the five-year period in that country-industry cell were cross-border acquisitions, and was coded as a greenfield investment otherwise. To ensure comparability across firms, the 1994 value of each

---

16 Almost all industry-country pairs were either entirely characterized by cross-border acquisition or greenfield. Only a handful of country-industry pairs in the sample involved both cross-border acquisitions and greenfield
Table 1: Descriptive Statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACQ</td>
<td>0.435</td>
<td>0.496</td>
<td>Indicator for Acquisition</td>
</tr>
<tr>
<td>USSALE</td>
<td>15.2</td>
<td>1.61</td>
<td>Sales in U.S.</td>
</tr>
<tr>
<td>VADDPW</td>
<td>4.45</td>
<td>0.523</td>
<td>(Gross Product)/Employees</td>
</tr>
<tr>
<td>EMP</td>
<td>10.0</td>
<td>1.44</td>
<td>Employees</td>
</tr>
<tr>
<td>DIV</td>
<td>-1.01</td>
<td>0.658</td>
<td>Herfindahl Index for Parent Firm Sales</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>-0.389</td>
<td>1.32</td>
<td>(R&amp;D Expenditure)/Sales</td>
</tr>
<tr>
<td>EXP</td>
<td>0.696</td>
<td>0.460</td>
<td>Indicator for Previous Country Experience</td>
</tr>
<tr>
<td>INTRAIM</td>
<td>0.610</td>
<td>0.336</td>
<td>(Intra-Firm Imports)/(Total Imports)</td>
</tr>
<tr>
<td>RGDPPC</td>
<td>9.81</td>
<td>0.723</td>
<td>Real GDP per Capita</td>
</tr>
<tr>
<td>POP</td>
<td>16.7</td>
<td>1.38</td>
<td>Population</td>
</tr>
<tr>
<td>OPEN</td>
<td>3.94</td>
<td>0.648</td>
<td>(Exports plus Imports)/GDP</td>
</tr>
</tbody>
</table>

All continuous variables (except INTRAIM) in logarithms.
All firm data is from the BEA’s confidential dataset.
Country level data is from the Penn World Tables.

parent firm’s characteristic is used to predict that firm’s investment behavior over the whole five-year period.

We consider six specifications. In three of these specifications, our measure of firm efficiency is USSALES. In the other three, we measure firm efficiency using VADDPW and control for firm size using EMP, the number of workers employed by the parent. We include four firm-level controls. First, we include RDSALE, which is the logarithm of a firm’s R&D expenditures to its total sales. Second, we include DIV, which is a measure of a firm’s diversification across industries. Third, to quantify a parent firm’s previous experience in a foreign country, we include an indicator variable, EXP, which is equal to one if the parent firm owned an enterprise in that country prior to the sample period, and is zero otherwise. Fourth, we include the variable INTRAIM, which is the ratio of a parent firm’s intra-firm imports to total imports so as to measure the extent of the firm’s international vertical integration. In all six specifications, we include measures of three country characteristics: a host country’s level of development, its market size, and its degree of openness to international trade. To gauge a host country’s level of development in a parsimonious fashion, we include RGDPPC, which is the logarithm of real GDP per capita. A country’s market size is captured by POP, which is the logarithm of the

FDI. The results that obtain from estimating the same model on the raw data offer very similar results.

17 The correlation between USSALES and VADDPW is 0.46.

18 DIV is defined as:

$$\log \left( \sum s_{p,j} \right)^{-1},$$

where $s_{p,j}$ is the share of the parent firm’s sales in industry $j$, and the sum is over the eight largest industries in which the parent firm sells.
country’s population. Finally, we include OPEN, which is a measure of a country’s openness to international trade (the sum of exports and imports, divided by GDP). Descriptive statistics are presented in table 1.

The results are shown in table 2, where the heading of each column indicates the measure of productivity used in that specification. The columns are organized by the number of controls included in the analysis with the first two corresponding to the most parsimonious specifications, the next two corresponding to specifications including the firm level controls, and the last two corresponding to specification that also include fixed effects by affiliate industry. All six specifications include parent industry fixed effects. Standard errors (shown in parentheses) are robust to heteroskedascity and clustering by firm.

The results provide evidence that the relationships reported in figures 1 and 2 are robust to a wide range of firm and industry controls. In all six specifications, the coefficient on the efficiency measure is negative and statistically significant, indicating that the more efficient firms enter foreign markets through greenfield FDI rather than through cross-border acquisition. Firms do not perceive cross-border acquisitions and greenfield FDI as perfect substitutes.\footnote{Our empirical results complement those of Blonigen (1997), who provides indirect evidence suggesting that greenfield FDI and cross-border acquisitions are different in nature.} Further, in all six specifications the coefficient on RGDPPW is positive and statistically significant, indicating that firms tend to enter rich, developed countries through cross-border acquisition rather than through greenfield FDI.
Table 2: Greenfield FDI vs. Cross-Border Acquisitions as a Function of Firm and Country Characteristics.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>USSALE</td>
<td>VADDPW</td>
<td>USSALE</td>
<td>VADDPW</td>
<td>USSALE</td>
<td>VADDPW</td>
</tr>
<tr>
<td>Efficiency</td>
<td>-0.211</td>
<td>-0.932</td>
<td>-0.280</td>
<td>-1.128</td>
<td>-0.301</td>
<td>-1.287</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.308)</td>
<td>(0.103)</td>
<td>(0.374)</td>
<td>(0.122)</td>
<td>(0.504)</td>
</tr>
<tr>
<td>EMP</td>
<td>-0.141</td>
<td>-0.111</td>
<td>-0.111</td>
<td>-0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.115)</td>
<td>(0.167)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIV</td>
<td>0.026</td>
<td>0.298</td>
<td>0.431</td>
<td>0.766</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.317)</td>
<td>(0.317)</td>
<td>(0.412)</td>
<td>(0.437)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.038</td>
<td>0.099</td>
<td>0.009</td>
<td>0.068</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.143)</td>
<td>(0.170)</td>
<td>(0.196)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXP</td>
<td>0.187</td>
<td>0.190</td>
<td>0.539</td>
<td>0.542</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.243)</td>
<td>(0.244)</td>
<td>(0.282)</td>
<td>(0.281)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INTRAIM</td>
<td>-0.306</td>
<td>-0.569</td>
<td>-0.783</td>
<td>-1.062</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.459)</td>
<td>(0.487)</td>
<td>(0.617)</td>
<td>(0.663)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGDPPC</td>
<td>0.662</td>
<td>0.613</td>
<td>0.564</td>
<td>0.533</td>
<td>0.550</td>
<td>0.502</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.154)</td>
<td>(0.175)</td>
<td>(0.173)</td>
<td>(0.214)</td>
<td>(0.219)</td>
</tr>
<tr>
<td>POP</td>
<td>0.037</td>
<td>0.057</td>
<td>0.004</td>
<td>0.015</td>
<td>-0.134</td>
<td>-0.120</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.097)</td>
<td>(0.100)</td>
<td>(0.103)</td>
<td>(0.124)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>OPEN</td>
<td>-0.626</td>
<td>-0.551</td>
<td>-0.591</td>
<td>-0.573</td>
<td>-0.947</td>
<td>-0.918</td>
</tr>
<tr>
<td></td>
<td>(0.185)</td>
<td>(0.189)</td>
<td>(0.202)</td>
<td>(0.207)</td>
<td>(0.280)</td>
<td>(0.284)</td>
</tr>
<tr>
<td>FE: Parent Industry</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>FE: Affiliate Industry</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Obs.</td>
<td>856</td>
<td>849</td>
<td>724</td>
<td>725</td>
<td>641</td>
<td>642</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-488</td>
<td>-480</td>
<td>-412</td>
<td>-407</td>
<td>-305</td>
<td>-302</td>
</tr>
</tbody>
</table>

Standard errors (shown in parenthesis) are robust to heteroskedasticity and allow for clustering by parent firm. The correlation between USSALE and VADDPW is 0.46. The efficiency measure corresponding to each specification is shown under the column number.
Appendix: Proofs

**Proof of proposition 1.** Let $\lambda_i(\varphi)$ denote the fraction of firms producing in country $i$ that are of efficiency less than or equal to $\varphi$. We need to show that $\lambda_1(\varphi) > \lambda_2(\varphi)$ for all $\varphi > 0$.

For $\varphi \geq \overline{m}a_1(\overline{m})$, we have 
\[
\lambda_1(\varphi) = 1 > \lambda_2(\varphi).
\]
Consider now $0 < \varphi < \overline{m}a_1(\overline{m})$, and let $m_i(\varphi)$ be such that $\varphi = m_i(\varphi)a_i(m_i(\varphi))$, i.e., $m_i(\varphi)$ is the mass appeal of the variety that will, in equilibrium, be produced by a firm of efficiency $\varphi$ in country $i$. We have 
\[
\lambda_1(\varphi) = \frac{E_1 H(m_1(\varphi))}{E_1 H(\overline{m})} > \frac{1 + (E_1/E_2)[1 - H(\overline{m})]}{E_2 + E_1[1 - H(\overline{m})]} \equiv \lambda_2(\varphi),
\]
where the second inequality follows from the observation that $m_1(\varphi) > m_2(\varphi)$.

**Proof of proposition 7.** An increase in $\phi$ induces an increase in the threshold $\overline{m}$ (see lemma 1), which in turn leads to an increase in $\gamma_1$ (see equation (27)).

Consider now the effect of an increase of $\phi$ on $\mu_i$. Let $\varphi(m) \equiv h(m) - \theta h(\theta m)$. Further, let $\tilde{m}$ be defined by $\varphi(\tilde{m}) = 0$. To see that $\tilde{m}$ is unique, note that 
\[
\varphi'(\tilde{m}) = h'(\tilde{m}) - \theta^2 h'(\theta \tilde{m}).
\]
Further, 
\[
h'(\tilde{m}) < \theta^2 h'(\theta \tilde{m})
\]
if and only if 
\[
\frac{\tilde{m}h'(\tilde{m})}{h(\tilde{m})} < \frac{\theta h'(\theta \tilde{m})}{h(\theta \tilde{m})}
\]
since $h(\tilde{m}) = \theta h(\theta \tilde{m})$. However, the last inequality must hold by condition (C2). Hence, $\varphi'(\tilde{m}) < 0$, and so $\varphi(m) > 0$ for any $m < \tilde{m}$, and $\varphi(m) < 0$ for any $m > \tilde{m}$.

From (20) and (25), we have $\tilde{m} = \min\{\overline{m}, \overline{m}\}$. If $\phi \geq \hat{\phi}$, then $\hat{m} = \tilde{m}$, and so an increase in $\phi$ has no effect on $\hat{m}$, and hence (by equation (26)) no effect on $\mu_i$. If $\phi < \hat{\phi}$, then $\hat{m} = \overline{m} < \tilde{m}$. From lemma 1, it follows that a small increase in $\phi$ leads to an increase in $\overline{m}$. Using equation (26) and the implicit function theorem, an increase in $\overline{m}$ induces an increase in $\mu_i$ if $\varphi(\overline{m}) \equiv h(\overline{m}) - \theta h(\theta \overline{m}) > 0$. However, since $\overline{m} < \tilde{m}$ if $\phi < \hat{\phi}$, it follows indeed that $\varphi(\overline{m}) > 0$.

**Proof of proposition 8.** We first want to show that $\overline{m}$ is increasing in $\theta$. Let $\varphi(\theta, \overline{m})$ denote the r.h.s. of equation (19). Since $\partial \varphi(\theta, \overline{m})/\partial \overline{m} < 0$, we need to show that $\partial \varphi(\theta, \overline{m})/\partial \theta > 0$. 
From (19), we obtain
\[
\frac{\partial \varphi(\theta, \bar{m})}{\partial \theta} = \sigma \phi \left[ E_1 \int_0^{\bar{m}} ma_1(m) dH(m) + E_1 \theta \int_0^{\bar{m}} m \frac{\partial a_1(m)}{\partial \theta} dH(m) 
+ E_2 \int_0^{\theta \bar{m}} m \frac{\partial a_2(m)}{\partial \theta} dH(m) \right] + a_2(\theta \bar{m}) \bar{m}
\]
\[
= \sigma \phi \left[ E_1 \int_0^{\bar{m}} ma_1(m) dH(m) + E_1 \theta \int_0^{\bar{m}} m^2 \frac{E_2 h(\theta m) h(m)}{[E_1 + E_2] g(a_2(\theta m))} dm 
- E_2 \int_0^{\theta \bar{m}} (m/\theta)^2 \frac{E_1 h(m/\theta) h(m)}{[E_1 + E_2] g(a_2(\theta m))} dm \right] + a_2(\theta \bar{m}) \bar{m}.
\]
Changing variables, this expression can be rewritten as
\[
\frac{\partial \varphi(\theta, \bar{m})}{\partial \theta} = \sigma \phi E_1 \int_0^{\bar{m}} ma_1(m) dH(m) + a_2(\theta \bar{m}) \bar{m}.
\]
Hence, \(d\bar{m}/d\theta > 0\). From (27), it then follows immediately that \(d\gamma_1/d\theta < 0\).

Consider now the share \(\mu_1\) of entrepreneurs from country 1 who engage in cross-border mergers in country 2. From equation (26), we obtain
\[
\frac{d\mu_1}{d\theta} = \frac{E_2}{E_1 + E_2} \left\{ -\tilde{m} h(\theta \tilde{m}) + [h(\tilde{m}) - \theta h(\theta \tilde{m})] \frac{d\tilde{m}}{d\theta} \right\}.
\]
Since \(\phi > \tilde{\phi}\) by assumption, \(\tilde{m}\) is defined by \(h(\tilde{m}) - \theta h(\theta \tilde{m}) = 0\). Hence, the above expression simplifies to
\[
\frac{d\mu_1}{d\theta} = -\frac{E_2}{E_1 + E_2} \tilde{m} h(\theta \tilde{m}) < 0.
\]
Similarly,
\[
\frac{d\mu_2}{d\theta} = -\frac{E_1}{E_1 + E_2} \tilde{m} h(\theta \tilde{m}) < 0.
\]

**Proof of proposition 9.** Observe that the effect of an decrease in \(E_2\), holding \(E_1 + E_2\) fixed, is equivalent to the effect of increasing \(E_1\), holding \(E_1 + E_2\) fixed. Hence, we need to prove that \(d\gamma_1/dE_1 > 0\) and \(d\mu_1/dE_1 < 0\). To this end, we first show that \(\bar{m}\) is decreasing in \(E_1\), holding \(E_1 + E_2\) fixed. Let \(\varphi(E_1, \bar{m})\) denote the r.h.s. of equation (19). Since \(\partial \varphi(E_1, \bar{m})/\partial \bar{m} < 0\), we
integrating by parts yields

$$\sigma \int_{0}^{\bar{m} \theta} \theta \int_{0}^{m} m a_1(m) dH(m) - \int_{0}^{\bar{m} \theta} m a_2(m) dH(m)$$

$$+ E_2 \int_{0}^{\bar{m} \theta} m \frac{\partial a_2(m)}{\partial E_1} dH(m)$$

$$= \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$

$$- \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$

$$= - \theta \int_{0}^{\bar{m} \theta} a_1(m) [H(m) - H(\theta m)] dm - \theta \int_{\ominus \bar{m} \theta} m a_1(m) dH(m)$$

$$+ \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$

$$- \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$

where the equality follows again from integrating by parts. Substituting this expression for the third and fourth terms in brackets in equation (28), we obtain that

$$\sigma \theta \int_{0}^{\bar{m} \theta} \theta \int_{0}^{m} m a_1(m) dH(m) - \int_{0}^{\bar{m} \theta} m a_2(m) dH(m)$$

$$+ E_2 \int_{0}^{\bar{m} \theta} m \frac{\partial a_2(m)}{\partial E_1} dH(m)$$

$$= - \sigma \theta \left[ \int_{0}^{\bar{m} \theta} a_1(m) [H(m) - H(\theta m)] dm - \int_{\ominus \bar{m} \theta} m a_1(m) dH(m) + \int_{\ominus \bar{m} \theta} a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)ight]$$

$$- \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$

$$- \int_{\ominus \bar{m} \theta} \sigma \theta [H(\bar{m}) - H(\theta m)] a_2(\theta \bar{m}) \theta \int_{0}^{m} m a_1(m) dH(m)$$
which is clearly negative. From (27), it then follows that $\gamma_1$ is increasing in $E_1$, holding $E_1 + E_2$ fixed.

Observing that $\hat{m}$ is independent of $E_1$ (since $\phi > \hat{\phi}$ by assumption, we obtain from (26) that

$$
\frac{d\mu_1}{\partial E_1}\bigg|_{E_1+E_2=\text{const.}} = -\frac{H(\hat{m}) - H(\theta \hat{m})}{E_1+E_2} < 0,
$$

and

$$
\frac{d\mu_2}{\partial E_1}\bigg|_{E_1+E_2=\text{const.}} = \frac{H(\hat{m}) - H(\theta \hat{m})}{E_1+E_2} > 0.
$$

References


